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ABSTRACT

A common approach in the literature, whether the investigation is about futures price risk premiums or biases in option-based implied volatility coefficients, is to use samples in which consecutive observations can be regarded as uncorrelated. That will be the case for non-overlapping forecast horizons constructed by either focusing on short time-to-maturity contracts or excluding some data. In this article we propose a parametric bootstrap procedure for uncovering futures and options biases in data characterized by overlapping horizons and correlated prediction errors. We apply our method to test hypotheses that futures prices are efficient and unbiased predictors of terminal prices, and that squared implied volatility, multiplied by time left to option expiry, is an unbiased predictor of terminal log-price variance. We apply the test to corn, soybean meal and Class III milk futures and options data for the period 2000-2011. We find evidence for downward bias in soybean meal futures, as well as downward volatility bias in Class III milk options. Importance of these results is illustrated on the example of premium determination for Livestock Gross Margin Insurance for Dairy Cattle (LGM-Dairy).

Keywords: parametric bootstrap, risk premium, volatility bias, revenue insurance, LGM-Dairy

Introduction

Most crop insurance products supported by the U.S. federal government are priced based on historical information on variables such as crop yields, market prices and rainfall indices. Traditionally the use of forward looking information embedded in the market traded financial derivatives was limited to futures prices used as a forecast of the harvest-time crop price. More recently information regarding expected volatility embedded in options premiums has been utilized in ratemaking procedures for option-like insurance products. One such product is the Livestock Gross Margin Insurance for Dairy Cattle (LGM-Dairy), an Asian basket option-type insurance product that allows dairy farmers to protect their dairy-based income over feed cost margin.

A key feature of LGM-Dairy is the contract design rule that stipulates premiums must be actuarially fair. An actuarially fair premium is one for which the calculated premium equals the expected contract indemnity. That premium equals expected indemnity holds only insofar as the accuracy of the assumptions supporting the rating method. The LGM-Dairy rating method is based on assumptions of unbiasedness of both futures prices and implied volatilities inferred from at-the-money options. However, both these assumptions are strongly contested in the literature, with no consensus having been reached. For example, while Kolb (1992), Deaves and Krinsky (1995), and McKenzie and Holt (2002) find risk premiums in futures prices for at least some commodities they examined, Frank and Garcia (2009) find no evidence of time-varying risk premiums in any markets they investigated. Likewise, current evidence regarding biases in agricultural options on futures is also mixed. Some researchers have found implied volatilities to be upward biased estimates of realized volatility (McKenzie, Thomsen, and Phelan 2007; Brittain, Garcia and Irwin 2011). Others find no evidence to support such hypothesis (Urcola and Irwin 2011; Egelkraut, Garcia and Sherrick, 2007).

After four years of pilot-program status LGM-Dairy has generated premium revenue that exceeds indemnity payments by more than thirty to one. Some dairy economists have started to question if LGM-Dairy premiums are indeed actuarially fair (Novakovic, 2012). The primary objective of this analysis is to determine if the observed discrepancy between collected premiums and paid indemnities can be explained by potential unaccounted biases in futures prices or volatility risk premiums in agricultural options.

A common approach in the literature, whether the investigation is about price or volatility risk premiums, is to focus on short time-to-maturity horizons that allow consecutive observations in the sample to be uncorrelated by choosing non-overlapping forecast intervals. Such an approach is not feasible for our task as LGM-Dairy allows farmers to insure revenue up to 11 months into the future. Class III milk futures contracts have only been traded for 13 years and restricting our sample to non-overlapping horizons would thus effectively leave us with a sample size of 14 observations. However, forecast intervals for consecutive deferred futures contracts are strongly overlapping, and consequently prediction errors are indeed highly correlated. The main contribution of this paper is the development of a new parametric bootstrap method for testing unbiasedness of futures and options with long time-to-maturity horizons that explicitly addresses prediction error correlation. Application of this method to the rating method of LGM-Dairy allows us to answer questions regarding the actuarial fairness of this revenue insurance product.

We begin this analysis with a brief description of the LGM-Dairy program, with a focus on assumptions regarding marginal distributions informed by futures and options data. In the second section we propose a parametric bootstrap procedure for uncovering futures and options biases in data characterized by overlapping horizons and correlated prediction errors. In the third section we investigate how biases in futures prices and volatility risk premiums would influence the price of LGM-Dairy contracts under alternative insurance contract designs. Finally, in the concluding section we discuss the implications of our findings for the LGM-Dairy rating method.

A Brief Overview of the LGM-Dairy Rating Method

LGM-Dairy is an Asian basket option-type revenue insurance product that compensates participating dairy farmers for unexpected declines in their gross margin defined as the difference between milk revenue and purchased feed costs (Gould and Cabrera 2011, Gould 2012). LGM-Dairy contracts can be purchased once a month after the CME futures markets close on the last business Friday of each month. Only one LGM-Dairy contract can be purchased by a dairy operation per month, and a farmer may insure at most 10 months of gross margin under any one insurance contract, not including the first month after the sales date.

Let t represent the month of insurance contract purchase and i the future month insured relative to the month of contract purchase $\left(i = 2,...,11\right)$. Expected milk revenues under LGM-Dairy are based on the previous three day average of Class III futures settlement prices, $\left(f_{t,i}^{M}\right)$ including the purchase day prices, and declared milk marketings, $\left(M_{t,i}\right)$ in each of up to 10 insurable months. Expected feed costs are based on the previous three day average of futures prices for corn and soybean meal $\left(f_{t,i}^{C}; f_{t,i}^{SBM}\right)$ and declared corn $\left(C_{t,i}\right)$ and SBM $\left(SBM_{t,i}\right)$ equivalents of livestock feed expected to be purchased over the coverage period to produce milk insured under the LGM-Diary contract. For those months for which corn or SBM futures are not traded, the associated prices are defined as the weighted average of the CME futures settle prices obtained from surrounding months.[1]

[1] For example, when purchasing an LGM-Dairy contract at the end of July, the expected October corn price is the weighted average of September and December corn futures prices where the weights are 0.667 and 0.333, respectively. This is not a problem for Class III milk as future contracts exist for all months.

In addition to monthly milk marketings and purchased feed use, a farmer must decide on the Gross Margin Deductible (D), i.e. the threshold decline in expected gross margin for insured milk after which LGM-Dairy will begin paying indemnities. Allowable deductible amounts range from \$0.00 to \$2.00 per cwt of milk and once chosen, deductible does not change across insured months. Not surprisingly, the higher the deductible, the lower the insurance premium as it decreases the probability of payouts and level of indemnities when they are forthcoming.

Given the decision on the quantity for milk marketings, purchased feed usage and deductible level, the gross margin guarantee (G_t) is calculated as:

$$G_t = \sum_{i=2}^{11} \left(f_{t,i}^{M} - D \right) \times M_{t,i} - \sum_{i=2}^{11} f_{t,i}^{C} \times C_{t,i} - \sum_{i=2}^{11} f_{t,i}^{SBM} \times SBM_{t,i} \tag{1}$$

LGM-Dairy premiums are determined by the United States Department of Agriculture's Risk Management Agency (RMA) using Monte Carlo simulation methods. They use draws from a joint distribution of the final prices, conditional on the information available at contract purchase. With 10 insurable months and three commodities involved, the joint distribution of interest consists of 30 marginal distributions and a copula that ties them together. Of the 30 marginal distributions, up to 24 are fitted directly from options and futures data, and the rest are interpolated through weighted averaging of surrounding marginal distributions.

All marginal price distributions are assumed to be lognormal. Futures prices are taken as conditional means of the terminal futures prices, and implied volatilities determine their conditional variance. Let us focus on a particular commodity and contract with nearby index j, expiring at time T.[2] Conditional on information set at time t, given annualized time to terminal price determination $\tau = \frac{T-t}{252}$, futures price $f_{t,j}$ and implied volatility $\sigma_{t,j}$, the marginal distribution of the terminal price p_T is

$$F\left(\ln p_{T,j}; f_{t,j}, \sigma_{t,j} \right) = N\left(\ln f_{t,j} - \frac{1}{2}\sigma_{t,j}^{\ 2}\tau, \sigma_{t,j}^{\ 2}\tau \right) \tag{2}$$

The marginal distributions of milk and feed prices are joined together through the Iman-Conover (1982) procedure equivalent to the Gaussian copula method (Mindenhall, 2006). The LGM-

[2] It may help reader if we summarize all information regarding subscripts in one footnote. Insurable months, relative to LGM-Dairy sales date, are always subscripted with $i = 2,...,11$. Commodity-specific nearby index of a futures contract is its relative ranking on a particular day, in terms of time left to maturity, with contract with the lowest time to maturity being 1st nearby, etc. Nearby indices are always subscripted with j, with $j = 2,...,11$ for Class III milk, $j = 1,...,5$ for corn, and $j = 1,...,6$ for soybean meal. We can also order contracts in the sample by their expiry dates. When needed, contract indices are subscripted by $l = 1,...,N$, where N is the total number of contracts of a particular commodity in the sample.

Dairy premium is estimated as the simple average of simulated indemnities observed over the 5,000 simulations (RMA, 2005).

Parametric Bootstrap Tests of Unbiasedness in Futures Prices and Implied Volatilities

From equation (2) it is clearly seen that the LGM-Dairy rating method is based on the assumption that futures price and associated implied volatility are unbiased determinants of the moments of the conditional terminal price distribution. In this section we examine if these assumptions are valid. In particular, we devise a formal test of two hypotheses:

H1: Futures prices are efficient and unbiased predictors of terminal prices,
H2: Squared implied volatility, multiplied by time left to maturity is an unbiased predictor of terminal log-price variance.

Under H1 we have

$$f_{t,j} = E_t\left(p_{T,j}\right) \quad (3)$$

To standardize, we can divide (3) by $f_{t,j}$, and obtain

$$E_t\left(\frac{f_{t,j} - p_{T,j}}{f_{t,j}}\right) = 0 \quad (4)$$

We shall denote the expression in the brackets, multiplied by 100, as prediction percentage error (PPE)

$$PPE_{t,j} = \frac{f_{t,j} - p_{T,j}}{f_{t,j}} \times 100 \quad (5)$$

where $f_{t,j}$ is the j^{th} nearby futures price at the time of LGM-Dairy contract purchase and $p_{T,j}$ is the terminal price for a futures contract in question. Over N contracts with unbiased futures prices, we would expect PPEs to average to zero. Therefore, an appropriate sample equivalent of hypothesis (4) is

$$\frac{1}{N}\sum_{l=1}^{N}\frac{f_{l,j} - p_{l,j}}{f_{l,j}} \times 100 = \frac{1}{N}\sum_{l=1}^{N} PPE_{l,j} = 0 \quad (6)$$

where $f_{l,j}$ is the futures price for contract *l*, observed at a time where it had nearby index j, and p_l is the terminal price.

As evident from (2), an implication of H2 is that

$$E_t\left(\sigma_{t,j}^2\tau-\left(\ln p_{T,j}-\left(\ln f_{t,j}-\frac{1}{2}\sigma_{t,j}^2\tau\right)\right)^2\right)=0 \tag{7}$$

To standardize, we divide by the conditional variance $\sigma_{t,j}^2\tau$ and obtain

$$E_t\left[\frac{\ln p_{T,j}-\left(\ln f_{t,j}-\frac{1}{2}\sigma_{t,j}^2\right)}{\sigma_{t,j}\sqrt{\tau}}\right]^2=1 \tag{8}$$

We denote the expression in the brackets as square standardized prediction error (SSPE). Summing up SSPEs over N contracts, and taking root mean, we obtain the testable implication of H2:

$$\sqrt{\frac{1}{N}\sum_{l=1}^{N}\left[\frac{\ln p_{l,j}-\left(\ln f_{l,j}-\frac{1}{2}\sigma_{l,j}^2\right)}{\sigma_{l,j}\sqrt{\tau}}\right]}=1 \tag{9}$$

A majority of previous analyses of the efficiency of commodity futures prices have focused on short time-to-maturity horizons. The principal reason behind such econometric strategy is the desire to avoid residual autocorrelation by using only non-overlapping contracts. However, if we want to utilize all available j^{th}-nearby futures contracts to test the hypothesis of interest at distant time-to-maturity horizons, we need to explicitly account for the fact that for consecutive deferred contracts, prediction errors will likely be strongly positively correlated. To our knowledge, only Kolb (1992), and Deaves and Krinsky (1995) attempted to address this issue in the context of commodity futures. The Kolb (1992) method uses random draws of historic data, to be followed by a regression of realized prediction errors on time to maturity. Kolb then tested for autocorrelation in residuals using the Durbin-Watson test and found none. We assert that his finding is due to failure to sort data in ascending date-of-trade order before testing for autocorrelation. Deaves and Krinsky (1995) use regression methods to test if the mean of log-difference returns at given time-to-maturity horizon is statistically different than zero.[3] They

[3] Deaves and Krinsky do not correctly interpret results of their model. It does not follow from the assumption of unbiasedness. If $f_t=E_t\left(p_T\right)$ then $\ln f_t=E\left(\ln p_T\right)$. Jensen's inequality postulates that $\varphi E\left(X\right)\leq E\left(\varphi X\right)$. For example, in the context of Geometric Brownian motion, which is the assumed stochastic process for futures prices that underpins classical Black's option pricing model, it follows from Ito's lemma that

explicitly account for autocorrelation in residuals when calculating the standard error of the intercept using a procedure developed by Hansen and Hodrick (1980) in their analysis of forward exchange rates as predictors for future spot rates. These methods rely on asymptotic theory and are thus only valid for large enough samples. In addition, correlation-corrected standard errors have only been applied in tests for unbiasedness of futures prices, not volatility biases.

In order to test our stated hypotheses, subject to both correlated prediction errors and relatively small sample sizes, we proceed by utilizing a parametric bootstrap approach to approximate the distribution of test statistics in (6) and (9) under the null hypothesis. We test each hypothesis using the bootstrapped p-values.

To explain our test we first introduce a simple case of non-overlapping horizons followed by the description of the needed changes to account for error autocorrelation. The null hypothesis stipulates efficient and unbiased futures prices, log-normality of terminal prices and unbiasedness of implied volatilities. Under those assumptions, conditional on information available at time t, terminal prices p_T are distributed lognormally as in equation (2). Given a particular futures price and implied volatility, it follows that we can simulate terminal prices by generating normal variables with desired mean and standard deviation, and then taking an exponent of them:

$$p_T = \exp\left(z_t \times \left(\sigma_t \sqrt{\tau}\right) + \left(\ln f_t - 0.5\left(\sigma_t^2 \tau\right)\right)\right) \tag{10}$$

In the case of non-overlapping forecast horizons, z_t are independent draws from a standard normal distribution.

For this analysis to mimic LGM-Dairy purchase day expected prices, we collect the futures prices and calculate implied volatilities on the last business Friday of the month. In calculating implied volatilities σ_t we use standard Cox, Ross and Rubinstein (1979) binomial option pricing model with 500 steps and LIBOR interest rates with maturity matching option time-to-maturity. Denote the number of months in the sample as *N*. Let index *j* represent the nearby count for a particular futures contract, where $j = 1$ denotes the first nearby contract, defined as the futures contract with the lowest number of days to corresponding options expiry. We first select a particular nearby index j. We then obtain a $N \times 1$ vector of futures prices $f_{l,j}$ and expected variances $\sigma_{l,j}{}^2\tau$, and generate K bootstrapped $N \times 1$ vectors of realized prices $p_{l,j}^*(k)$. For

$\ln f_t = E_t\left(\ln p_T\right) + \frac{1}{2}\sigma^2\tau$. In this context, in the absence of risk premiums, log-differences should thus average not to zero, but to half of log-price conditional variance.

consistency and clarity, we denote bootstrapped statistics with asterisk to differentiate them from actual sample-based statistics.

For each of the K bootstrapped samples of realized prices we calculate average PPE, denoted $\overline{\mathrm{PPE}}_j^*(k)$ as

$$\overline{PPE}_j^*(k) = \frac{1}{N}\sum_{l=1}^{N} PPE_{l,j}^*, k = 1,...,K \tag{11}$$

The formal tests of the futures unbiasedness hypotheses consists of constructing bootstrapped confidence interval for the $\overline{\mathrm{PPE}}_j^*(k)$ statistics, and determining if sample $\overline{PPE}_j$ value lies within the critical region. The bootstrapped confidence interval for α level of confidence is found by sorting the bootstrapped $\overline{\mathrm{PPE}}_j^*(k)$ statistics and identifying the critical values as entries at positions $\frac{\alpha}{2}K$ and $\left(1-\frac{\alpha}{2}\right)K$. We set the number of replications $K = 10,000$ and $\alpha = 0.05$ so the critical values of the bootstrapped distribution are found at positions 250 and 9,750. If the sample $\overline{PPE}_j$ is lower than $\overline{PPE}_{j,\alpha/2}^*$ or higher than $\overline{PPE}_{j,1-\alpha/2}^*$ we reject the null hypothesis of unbiasedness of futures prices for j^{th} nearby futures contracts.

For volatility unbiasedness test we shall use bootstrapped standardized square prediction errors, $\overline{SSPE}_j^*(k)$. The bootstrapped distribution of the variance unbiasedness test statistic in (9) depends directly only on the autocorrelation structure and sample size, not futures prices or implied volatilities. To see that, notice that from (10) it follows that square root of average bootstrapped SSPE can be simplified to:

$$\sqrt{\overline{SSPE}}^*(k) = \sqrt{\frac{1}{N}\sum_{l=1}^{N} z_{l,k}^2}, k = 1,...,K \tag{12}$$

In case of non-overlapping forecast intervals, $z_l \sim i.i.d.\, N(0,1)$, thus

$$N\overline{SSPE}^* \sim \chi^2(N) \tag{13}$$

For large enough sample, (13) converges to a normal distribution and asymptotic theory can be used for determination of critical SSPE values. In the case of correlated errors, described in the next section, we must use bootstrap methods to generate a distribution for (12). If sample-based $\sqrt{\overline{\mathrm{SSPE}}}$ falls outside the critical values of the bootstrapped $\sqrt{\overline{SSPE}}^*$ distribution we reject the hypothesis that implied volatility coefficients multiplied by the square root of the time left to maturity are unbiased estimates of the terminal log-price standard deviation for j^{th} nearby contracts.

The Case of Overlapping Prediction Horizons

We now turn to case where realized prediction errors are allowed to be correlated. Such a situation would occur whenever the time of futures price measurement falls before all previous contracts have expired, i.e. whenever nearby index $j>1$. For distant horizons, these correlations may be rather strong. For example, for 11-th nearby Class III milk futures contracts, the PPE autocorrelation at first lag is 0.906. If our bootstrapped distributions are to truly reflect the data generating process we need to explicitly account for these autocorrelations. In time series analysis, the classical method to account for autocorrelated error structures is to estimate an ARMA model. If we are to proceed in such fashion, there are two questions we need to address. First, what variable exactly to model as having an ARMA process, and second, are there any restrictions on the ARMA coefficients that must be imposed in order to properly bootstrap terminal prices under the null hypothesis.

An autocorrelated structure should be reflected in the time series process for z_t in equation (10), so we need to find the way to transform correlations in prediction errors to dependence in the time series process with normal innovations and unit unconditional variance:

$$z_t = \sum_{m=1}^{p} \varphi_i z_{t-m} + \sum_{m=1}^{q} \alpha_j \varepsilon_{t-m} + \varepsilon_t, z_t \sim N(0,1) \tag{14}$$

We start by utilizing the equation (2), obtaining implied quantile $u_{t,j}$ of the contract that was the j^{th} nearby at time t given by

$$u_{t,j} = F\left(\ln p_{T,j}; f_{t,j}, \sigma_{t,j}\right) \tag{15}$$

That is, $u_{t,j}$ tells us where realized price falls in the time-*t* conditional distribution that is based on futures price and implied volatility. For example, if implied quantile is 0.9 that means that realized price turned out to be quite higher than was expected at time *t*, i.e. chances of terminal price settling at that particular level or higher were deemed to be only 10%.

Quantiles $u_{t,j}$ lie in the $[0,1]$ interval and cannot be used directly in ARMA modeling. For that reason, we construct a series of standard normal z-scores based on quantiles $u_{t,i}$. The first step is to use the standard normal distribution function and inverse probability integral transform:

$$\tilde{z}_{t,j} = \Phi^{-1}\left(u_{t,j}\right) \tag{16}$$

We must also account for the possibility that unbiasedness of futures-implied mean and options-implied variance may not actually be valid assumptions for the terminal price conditional distribution. Consequently, quantiles $u_{t,j}$, which are distributed uniformly under the null, may not be distributed uniformly in our sample. In order to capture the autocorrelation structure under the null hypothesis, unrestricted z-scores $\tilde{z}_{t,j}$ are standardized to insure zero mean and standard deviation of one. Denote the mean and the standard deviation of unrestricted z-scores $\tilde{z}_{t,j}$ as $\tilde{\mu}_j$ and $\tilde{\sigma}_j$, respectively. Restricted z-scores are then calculated as

$$z_{t,j} = \frac{\tilde{z}_{t,j} - \tilde{\mu}_j}{\tilde{\sigma}_j} \tag{17}$$

We will use these restricted z-scores in the ARMA model. Under the assumption of futures market efficiency, for the j^{th} nearby futures prices, z-score autocorrelations at lags higher than $j-1$ must be zero. If that were not the case, then *observed* past prediction errors could improve the forecasting power of futures prices. In order to implement this restriction we always restrict the number of autoregressive lags to zero, and allow only up to $j-1$ moving average lags in the model. In other words, for $ARMA(p,q)$, the null hypothesis of efficiency and unbiasedness in futures prices implies that $p=0, q<j$.

For a particular nearby index j, estimated coefficients from an $MA(j-1)$ model are then used in simulating z-scores to be used in parametric bootstrap. To insure unit unconditional variance of the simulated z-scores we set the variance of the innovation to be

$$\sigma_{\varepsilon,j}^2 = \frac{1}{1+\sum_{m=1}^{j-1}\hat{\alpha}_m^2} \tag{18}$$

and simulate z-scores via $z_{t,j} = \varepsilon_t + \sum_{m=1}^{q}\alpha_m \varepsilon_{t-m}$ .For each bootstrapped sample of z-scores, we allow the simulation to run for 500 time periods before recording a sequence of length N. In total, k samples of $N \times 1$ vectors of z-scores are simulated. From this point forward, the procedure is the same as in the case of non-overlapping horizons. Bootstrapped terminal prices are simulated as in (10), and as before, formal tests for the presence of bias in futures prices or implied volatilities is conducted by comparing sample average *PPE* and root mean *SSPE* with critical bootstrapped values of $\overline{PPE}^*$ and $\sqrt{\overline{SSPE}}^*$ distributions.

Description of Data Used in the Analysis

We apply the above bootstrap procedures to a set of Class III milk, corn and soybean meal futures and options data from January 2000 through July 2011.[4] For this analysis we need to recognize that Class III milk futures are traded for all twelve calendar months while corn futures trade for March, May, July, September and December, and soybean meal futures trade for January, March, May, July, August, September, October and December. We use 3rd through 11th nearby contract for Class III milk, 1st through 5th nearby contract for corn and 1st through 6th nearby contract for soybean meal.[5] To be consistent with LGM-Dairy, futures prices and implied volatilities are observed on the last business Friday of the month. However, to keep the design of the test simple and relevant to situations outside our immediate application we do not average prices over three trading days. Furthermore, unlike LGM-Dairy rating method, terminal prices are defined as futures prices on the options expiry day.

For corn, we wish to measure futures prices for the 1st nearby contract approximately 2 months before contract expiry, so we use prices observed in February, April, June, September and December. Given the expiration schedule for options on corn futures, first nearby futures contract in February is May, in April it is July, etc, and the time to maturity for the first nearby contract varies between 49 and 63 calendar days. For soybean meal, we observe prices in January, March, May, June, July, August, October and November. Given the expiration schedule for options on soybean meal futures, the first nearby futures contract in January is March, for March it is May, etc, and the time to maturity for the first nearby contract is between 21 and 28 days. This procedure yields a sample size of 139 for Class III milk, 59 for corn, and 93 for soybean meal. Descriptive statistics are listed in Table 1.

[Insert Table 1 about Here]

Results of the Parametric Bootstrap Tests

Results from our testing of unbiasedness in Class III, corn and SBM futures prices and implied volatilities are summarized in Table 2 and graphically displayed via Figure 1. Bootstrapped mean prediction errors are generated under the null hypothesis, and overlap with x-axis so closely that they are barely visible in Figure 1. By comparing sample-based mean prediction errors and shaded 95% bootstrapped confidence interval it is clear that for Class III milk and corn there is no evidence of bias in futures prices. For soybean meal futures, however, we find substantial downward bias. For example, over the past 11 years, 6th nearby soybean meal futures (with a

[4] All futures and options data were obtained from the University of Wisconsin, *Understanding Dairy Markets* website (http://future.aae.wisc.edu)

[5] Due to peculiar rules for Class III milk contract expiry (e.g. February contract expires in the first week of March), 3rd nearby milk contract corresponds to 1st insurable month.

mean of 254 calendar days to maturity) have been on average 10.53% below the terminal price. From Figure 1 we see that for all tested nearby contracts mean prediction errors are very close to the lower boundary of the 95% bootstrapped confidence interval. Unbiasedness of soybean meal futures is formally rejected at 95% confidence level for the 2^{nd}, 5^{th} and 6^{th} nearby contracts, and for all but the 1^{st} nearby contract at 90% confidence level. We conclude that soybean meal futures do exhibit downward bias.

[Insert Table 2 about Here]

[Insert Figure 1 about Here]

The results of our bootstrap tests for unbiasedness of expected variance are illustrated in Figure 2. For both corn and soybean meal, the root mean square standardized prediction errors fall well within the 95% confidence interval. For Class III milk futures, however, we find that expected variance seems to be too low for all tested nearby indices. In previous literature, in situations where implied volatilities have been found biased, the direction of the bias was upward, i.e. implied volatilities were overpredicting the terminal price volatility. In this instance, it seems that implied volatilities have been under-predicting the terminal price variance. It would, however, be too soon to conclude that this result implies that options are in fact too cheap. If options were too cheap, there would be possible to create a trading strategy with positive expected profits over the long run. To properly evaluate if such strategy is feasible we would need to use bid and ask prices, rather than settlement prices.[6]

[Insert Figure 2 about Here]

If implied volatilities of Class III milk contract seem too low, we can ask the question – what are the lowest implied volatility coefficients that would still be consistent with the observed root mean square percentage prediction errors? In other words, we are looking for the lowest volatility coefficients for which we would not be able to reject the null, if the test is based on (non-standardized) root mean squared PPEs. To that end, we find calibrated, i.e. data-fitting average volatility coefficients such that the upper bound of the non-standardized root mean squared PPEs bootstrap confidence interval constructed using those coefficients as the null hypothesis coincides with the sample root mean square PPEs. As can be seen in Figure 3, the term structure of average implied volatilities should have a much more pronounced concave shape than is observed.

[Insert Figure 3 about Here]

These results raise two questions. First, why is the term structure of calibrated volatility coefficients concave? And why is the term structure of options-based average implied volatilities nearly flat?

[6] We plan to conduct such analysis in further research and for now only warn against premature conclusions, as transaction costs and bid-ask spread may make the apparent bias still consistent with market efficiency.

To answer the first question, we multiply calibrated volatility coefficients by the square root of time-to-maturity and find the resulting terminal log-price standard deviation to be nearly constant for horizons longer than 6 months. This can perhaps be explained by extremely low price elasticity of supply of milk in the short run. Under such supply structure, minor demand or weather shocks can quickly result in substantial swings in milk prices, resulting in large increase of price risk for 3rd through 6th nearby milk contract. In addition a highly inelastic market demand for dairy products means that small shifts in supply coming from relatively small dairy herd adjustments induce strong mean-reversion in milk prices.

Past experience has shown that most major weather and demand shocks, whether price-enhancing or price-reducing, are mostly compensated for with adjustments in supply within nine months (Bozic et al. 2012). These structural features combined can make terminal log-price variance at first rise quickly with time-to-maturity followed by convergence to unconditional variance level. In the context of option-pricing model, terminal log-price variance at first rises faster than the square root of time, but stays nearly constant for time-to-maturity horizons longer than 6 months. As a consequence, implied volatilities should increase for the 2nd through 6th nearby contracts, and decrease for contracts with longer time to maturity, rendering the structure of implied volatilities concave.

The second question to address is the apparent flatness of observed average implied volatilities. For the 1st and the 2nd nearby contracts implied volatilities are usually lower due to the fact that the Class III contract cash settles to USDA's Announced Class III price which is formula based using historical commodity plant-level sales prices. The prevalent practice of buying options as 'packs' rather than individual contracts may partially explain the absence of any substantial curvature in implied volatility term structure. Given the continuous nature of milk production, it is very rare that a producer will initiate an option position for just one month. Options are usually traded as bundles covering a minimum of three, and often more months. A "pack" of options is defined by a common strike, and quoted with a single price. When prices are recorded for official purposes, they are most likely split for individual months by assuming an average implied volatility for the entire period covered by the pack.

Because of the relative thinness of the Class III options market, and the need for market coordination for multi-month offers, most options for Class III milk futures are still traded in the pit, with only a handful of market makers. Given the highly regulated U.S. fluid milk market it would not be surprising if floor traders used simplifying heuristics like setting a near flat implied volatility term structure off which to form bid and asks. In conclusion, the apparent option mispricing may be due to the peculiarities of market microstructure.

Impact of Futures Price and Implied Volatility Biases on LGM-Dairy Premiums

In the previous section we performed formal tests of unbiasedness in futures prices and implied volatilities. In this section we use Monte Carlo experiments to examine the consequences of our results for LGM-Dairy premiums. To facilitate our analysis of the impact of price and volatility biases on LGM-Premium calculations, we define 18 representative policy profiles. All policy profiles assume milk marketings of 9,000 cwt. per month, an amount expected from a farm with 500 milking cows with annual milk per cow yield of 21,600 lbs. Benchmarking profiles differ in three dimensions:

(i) *Amount of feed declared per cwt. of milk*. We consider two opposing extremes – minimum and maximum program allowable feed levels. These extremes capture well the two distinct production systems: farms that grow all their feed, and drylot farming systems where all feed are purchased on the market. In addition, we examine the scenario where the LGM-Dairy default feed amounts per cwt of milk are utilized (RMA, 2005)

(ii) *Chosen deductible level*. We consider an insurance under high risk aversion ($0.00 deductible) and usage of LGM-Dairy as a catastrophe insurance that does not cover shallow losses ($1.10 deductible)

(iii) *Risk management strategy utilized*.
Four different risk management strategies are considered in our analysis. In each strategy, we assume the representative farmer purchases an LGM-Dairy contract every month and implements one of four alternative insurance strategies:

1) ***Flat-10***: 1/10 of expected milk marketings are insured for each of the ten insurable months.
2) ***Up Front***: 1/3 of expected milk marketings are insured for the 1st, 2nd and 3rd insurable months
3) ***Middle of the Road***: 1/3 of expected milk marketings are insured for the 4th, 5th and 6th insurable months.
4) ***Looking Ahead***: 1/3 of expected milk marketings are insured for the 8th, 9th, and 10th insurable months.

In this analysis we assume LGM-Dairy contracts were purchased in each of the 12 months of 2011. Regardless of the strategy adopted, as long as it is persistently followed, eventually 100% of the expected milk marketings will be insured under the portfolio of LGM-Dairy contracts. These strategies are chosen to be consistent with our analysis recently undertaken in Bozic et al. (2012).

To examine the impact of unbiasedness of soybean meal futures prices, assume that the true relationship between futures prices and expected terminal prices is

$$\left(1-r_j^f\right) f_t = E_t\left(p_T\right) \tag{19}$$

where index $j = 1,...,8$ represents the nearby-ranking of the contract, and r_j^f represents the risk premium. From (19) it follows that $r_j^f = E_t\left(PPE_{t,j}\right)$. Given the results from the previous section, the important scenario to analyze is the one in which soybean meal futures prices are downward biased.

In the scenario examined here, risk premiums for soybean meal prices are set at an observed sample average prediction percentage error, calculated separately for each time-to-maturity horizon, as shown in Table 3. With 6th nearby soybean contracts being the highest time-to-maturity horizon we tested in previous section, we set the risk premium for 7th and 8th nearby contracts at the level observed for the 6th nearby contracts.

In our next test, we examine the effect of potential volatility biases in Class III milk prices. In absence of biases and under assumed lognormality of terminal prices, from (2) it follows that $Var_t\left(\ln p_{t+\tau}\right) = \sigma^2\tau$. Allowing for volatility biases, and indexing nearby-ranking of Class III milk contracts with $j = 3,...,12$, the relationship between implied volatility coefficient and expected terminal log-price variance becomes:

$$\sigma_{t,j} = \left(1 + r_j^\sigma\right)\sqrt{\frac{1}{\tau}Var_t\left(\ln p_{t,j}\right)} \tag{20}$$

where r_j^σ denotes volatility bias. Although we continue to use the letter r, we deliberately avoid referring to these biases as risk premiums, as the direction of the observed bias is downward, and is likely due to market microstructure effects rather than extra reward needed for option sellers.

To obtain volatility biases we can use in the simulation, we first calculated *calibrated volatility coefficients*, defined as the lowest volatility coefficients consistent with the observed root mean square prediction errors, as described in the previous section in the context of Figure 3. We then calculate the average options-based implied volatility coefficients. The ratio of average implied volatility to calibrated volatility is taken as $1 + r_j^\sigma$ in equation (20). Calibrated volatility coefficients, average implied volatilities, as well as volatility biases r_j^σ are presented in Table 3. We use Class III milk volatility bias coefficients r_j^σ in the LGM-Dairy rating method sensitivity analysis to adjust implied volatility coefficients $\sigma_{i,j}$:

$$\tilde{\sigma}_{t,j} = \frac{1}{1 + r_j^\sigma}\sigma_{t,j} \tag{21}$$

Bias-adjusted volatility coefficients $\tilde{\sigma}_{t,j}$ are then used in calculating LGM-Dairy premiums.

[Insert Table 3 about Here]

Results of the Monte Carlo Experiments

Table 5 is used to summarize the results of the above described simulations. Columns (1) through (3) show the representative insurance policy profiles in terms of risk management strategy chosen, level of deductible, and the amount of feed declared. In column (4) we provide average monthly insurance premiums using the official rating method currently utilized to price LGM-Dairy policies. Column (5) is used to present the premiums under the assumption of downward bias in soybean meal prices. We find that even for the policies that use maximum feed amounts and buy insurance for the distant months in which bias is most pronounced, accounting for the bias in the rating method would render insurance policy premiums less than 2% higher.

[Insert Table 4 about Here]

Results of the simulation in which Class III milk implied volatility biases are accounted for in the rating method are presented in column (6). LGM-Dairy premiums are found to be very sensitive to volatility biases, and inflating the volatility coefficients to account for these biases increased premiums up to 21%, with the highest increases observed for policy profiles utilizing middle-of-the-road risk management strategies and high deductible.

Conclusions

In this article, we have developed a parametric bootstrap method that can be used to test for presence of bias in futures prices and implied volatilities in deferred contracts with overlapping time to maturity horizons. We applied our method to evaluate actuarial assumptions of the Livestock Gross Margin Insurance for Dairy Cattle. We find milk prices to be unbiased, but test results suggest the term structure of volatility is underestimating the risk in medium-run window covering 5 to 10 months to maturity. We propose that concave curvature of the term structure for the data-consistent implied volatility coefficients arises from inelastic supply and demand for milk, while flat term structure of options-based implied volatilities is due to peculiarities of Class III milk options market microstructure, i.e. particular CME options settlement procedures and heuristics used by floor traders. Excessive reliance on simple option pricing models that assume terminal price variance increases linearly with time-to-maturity may be a cause of option mispricing. Although we find that observed root mean square percentage prediction errors are consistent with volatility coefficients that are at least 3-5 percentage points higher, without an in-depth analysis of bid-ask spreads it is not possible to say if our results imply a profit opportunity.

Although LGM-Dairy is a government-sponsored agricultural risk insurance product, with transparent rating method and explicitly designed to be actuarially fair, large underwriter's gains over the past 4 years have cast doubt on the soundness and robustness of the official rating method. Based on undertaken tests and simulation experiments, our conclusion is that assumptions regarding marginal distributions of milk and feed prices do not produce insurance premiums that could be considered excessive. On the contrary, biases in Class III milk implied volatilities may suggest LGM-Dairy premiums are too low. Monte Carlo experiments suggest accounting for biases in implied volatility coefficients could increase the price of LGM-Dairy insurance between 3% and 21%, depending on the risk management strategy chosen. LGM-Dairy premium overcharges, if they exist, are to be found not in assumptions regarding marginal distributions, but in choice and parameterization of copula tying marginal distributions together, a point we explore in depth in Bozic et al. (2012)

Despite observed biases, we are not ready to recommend any changes to the rating method, as far as marginal distributions are concerned. The apparent biases may likely be a characteristic of coming-of-age issues in relatively young milk options market. As volume, liquidity and market maker sophistication grow we would expect the terms structure of implied volatilities to start better reflecting the nature of the dairy markets.

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Table 1. Descriptive Statistics for Data Used in Parametric Bootstrap

Commodity/Nearby	Num. Obs.	Time to Maturity (Calendar Days)				Futures ($)				Implied Volatility			
Class III Milk		Mean	S.D.	Min	Max	Mean	S.D.	Min	Max	Mean	S.D.	Min	Max
3	139	66	3	63	72	13.61	2.79	9.33	20.67	0.195	0.048	0.070	0.374
4	139	96	3	91	100	13.67	2.54	9.77	20.49	0.209	0.043	0.105	0.369
5	139	127	3	119	134	13.68	2.37	9.80	20.56	0.210	0.040	0.112	0.348
6	139	157	4	154	163	13.69	2.24	9.77	20.16	0.207	0.038	0.115	0.314
7	139	188	3	182	191	13.72	2.17	9.83	20.00	0.204	0.034	0.114	0.299
8	139	218	3	217	225	13.70	2.11	9.85	19.95	0.201	0.032	0.114	0.298
9	139	249	4	245	254	13.70	2.07	9.85	19.65	0.198	0.030	0.115	0.300
10	139	279	3	273	287	13.69	2.03	10.50	19.39	0.196	0.029	0.115	0.309
11	139	309	3	307	317	13.65	2.02	10.61	19.45	0.194	0.029	0.116	0.298
Corn													
1	93	57	3	49	63	3.34	1.54	1.96	7.68	0.301	0.091	0.151	0.500
2	93	130	15	112	154	3.41	1.51	2.08	7.87	0.298	0.081	0.171	0.473
3	93	203	23	175	245	3.46	1.46	2.17	8.04	0.292	0.074	0.170	0.465
4	93	277	23	238	302	3.49	1.42	2.24	8.11	0.285	0.066	0.196	0.461
5	93	350	15	329	365	3.53	1.42	2.31	8.14	0.274	0.060	0.189	0.432
Soybean Meal													
1	59	26	3	21	29	232.65	74.38	146.70	424.70	0.273	0.077	0.151	0.464
2	59	72	15	56	92	230.62	73.11	144.10	421.70	0.269	0.073	0.156	0.449
3	59	117	21	84	149	228.99	72.02	142.30	412.00	0.264	0.069	0.155	0.431
4	59	163	26	112	212	227.30	70.75	143.80	409.70	0.256	0.066	0.149	0.410
5	59	208	26	175	245	225.25	69.29	142.80	411.70	0.249	0.063	0.154	0.400
6	59	254	26	210	304	223.80	68.42	143.10	413.70	0.247	0.063	0.154	0.403

Note: For time to maturity and implied volatility, minimum and maximum values are reported. For futures, mean and standard deviations are listed, and for mean prediction error and root mean prediction error only means are listed.

Table 2. Parametric Bootstrap Results

Commodity/ Nearby	Mean Prediction Error (%)	Root Mean Square Prediction Error (%)	Root Mean Square Standardized Prediction Error (%)	Bootstrap Prediction Error Confidence Interval (%)	Bootstrap Root Mean Square Standardized Prediction Error Confidence Interval (%)	Bootstrap test for Unbiasedness of Futures Prices (p-values)	Bootstrap test for Unbiasedness of Implied Volatilities (p-values)
Class III Milk							
3	-0.72	9.33	1.27	(-2.09, 2.08)	(0.86, 1.14)	0.505	<0.001
4	-0.83	13.62	1.36	(-3.01, 2.96)	(0.84, 1.16)	0.582	<0.001
5	-1.49	16.74	1.42	(-4.17, 3.93)	(0.82, 1.19)	0.485	<0.001
6	-2.12	19.12	1.45	(-4.84, 4.73)	(0.80, 1.20)	0.376	<0.001
7	-2.61	20.95	1.45	(-5.67, 5.42)	(0.79, 1.22)	0.350	<0.001
8	-3.25	22.27	1.43	(-6.60, 6.15)	(0.78, 1.24)	0.325	0.001
9	-3.76	23.13	1.39	(-7.15, 6.85)	(0.76, 1.24)	0.282	0.002
10	-4.16	23.52	1.37	(-7.76, 7.62)	(0.76, 1.26)	0.286	0.008
11	-4.87	24.15	1.35	(-9.42, 8.53)	(0.74, 1.28)	0.283	0.017
Corn							
1	1.10	12.80	1.05	(-3.26, 3.21)	(0.82, 1.18)	0.507	0.515
2	0.82	20.50	1.13	(-6.80, 6.50)	(0.78, 1.22)	0.817	0.242
3	-0.29	26.08	1.12	(-9.99, 9.33)	(0.74, 1.27)	0.916	0.338
4	-1.69	29.69	1.06	(-13.39, 11.82)	(0.72, 1.30)	0.777	0.614
5	-2.66	31.63	1.06	(-15.84, 13.85)	(0.71, 1.31)	0.694	0.613
Soybean Meal							
1	-1.23	8.22	1.08	(-1.59, 1.54)	(0.86, 1.14)	0.128	0.270
2	-3.27	13.03	1.09	(-3.13, 3.08)	(0.84, 1.15)	0.041	0.244
3	-4.95	16.84	1.13	(-5.12, 4.93)	(0.81, 1.19)	0.058	0.173
4	-6.82	20.72	1.19	(-6.87, 6.65)	(0.79, 1.22)	0.051	0.079
5	-8.65	22.53	1.20	(-8.39, 7.78)	(0.77, 1.25)	0.043	0.096
6	-10.53	24.69	1.21	(-10.45, 9.44)	(0.74, 1.28)	0.049	0.118

Table 3. Class III Milk Volatility Biases Used in LGM-D Rating Method Sensitivity Analysis

Nearby	Calibrated Volatility	Average Implied Volatility	Volatility Bias
3rd	0.191	0.195	2.09%
4th	0.226	0.209	-7.52%
5th	0.232	0.210	-9.48%
6th	0.235	0.207	-11.91%
7th	0.232	0.204	-12.07%
8th	0.225	0.201	-10.67%
9th	0.215	0.198	-7.91%
10th	0.206	0.196	-4.85%
11th	0.195	0.194	-0.51%

Note: Volatility bias is here expressed as the ratio of options-based average implied volatility to calibrated (data-consistent) volatilities minus 1. Calibrated volatility is the lowest average volatility coefficient for which null hypothesis would not be rejected. For example, for 6th nearby contract, calibrated volatility is 0.235. Average options-based implied volatility for 6th nearby contract is 0.207, and their ratio is 0.8809. Volatility bias is expressed as -11.91%.

Table 4. Sensitivity Analysis of Changes in LGM-Dairy Premiums Under Biased Futures Prices and Implied Volatilities

(1) Strategy	(2) Deductible Level ($/cwt)	(3) Feed Declared	(4) RMA Rating Method ($)	(5) Biased SBM Futures Prices ($)	(6) Biased SBM Futures Prices (%)	(7) Biased Milk Implied Volatilities ($)	(8) Biased Milk Implied Volatilities (%)
Flat-10	$0.00	Minimum	8,195	8,197	0.02%	8,757	6.86%
		Default	8,582	8,600	0.21%	9,118	6.25%
		Maximum	11,447	11,576	1.13%	11,843	3.46%
	$1.10	Minimum	3,843	3,844	0.03%	4,317	12.33%
		Default	4,256	4,274	0.42%	4,712	10.71%
		Maximum	7,154	7,276	1.71%	7,499	4.82%
Up Front	$0.00	Minimum	6,819	6,821	0.03%	7,239	6.16%
		Maximum	8,783	8,836	0.60%	9,101	3.62%
	$1.10	Minimum	2,723	2,724	0.04%	3,057	12.27%
		Maximum	4,685	4,727	0.90%	4,946	5.57%
Middle of the Road	$0.00	Minimum	9,743	9,744	0.01%	10,972	12.61%
		Maximum	13,316	13,438	0.92%	14,235	6.90%
	$1.10	Minimum	5,191	5,192	0.02%	6,287	21.11%
		Maximum	8,873	8,992	1.34%	9,686	9.16%
Looking Ahead	$0.00	Minimum	12,017	12,019	0.02%	12,180	1.36%
		Maximum	16,276	16,483	1.27%	16,396	0.74%
	$1.10	Minimum	7,250	7,252	0.03%	7,399	2.06%
		Maximum	11,710	11,919	1.78%	11,819	0.93%

Figure 1. Observed vs. Bootstrapped Percentage Prediction Errors

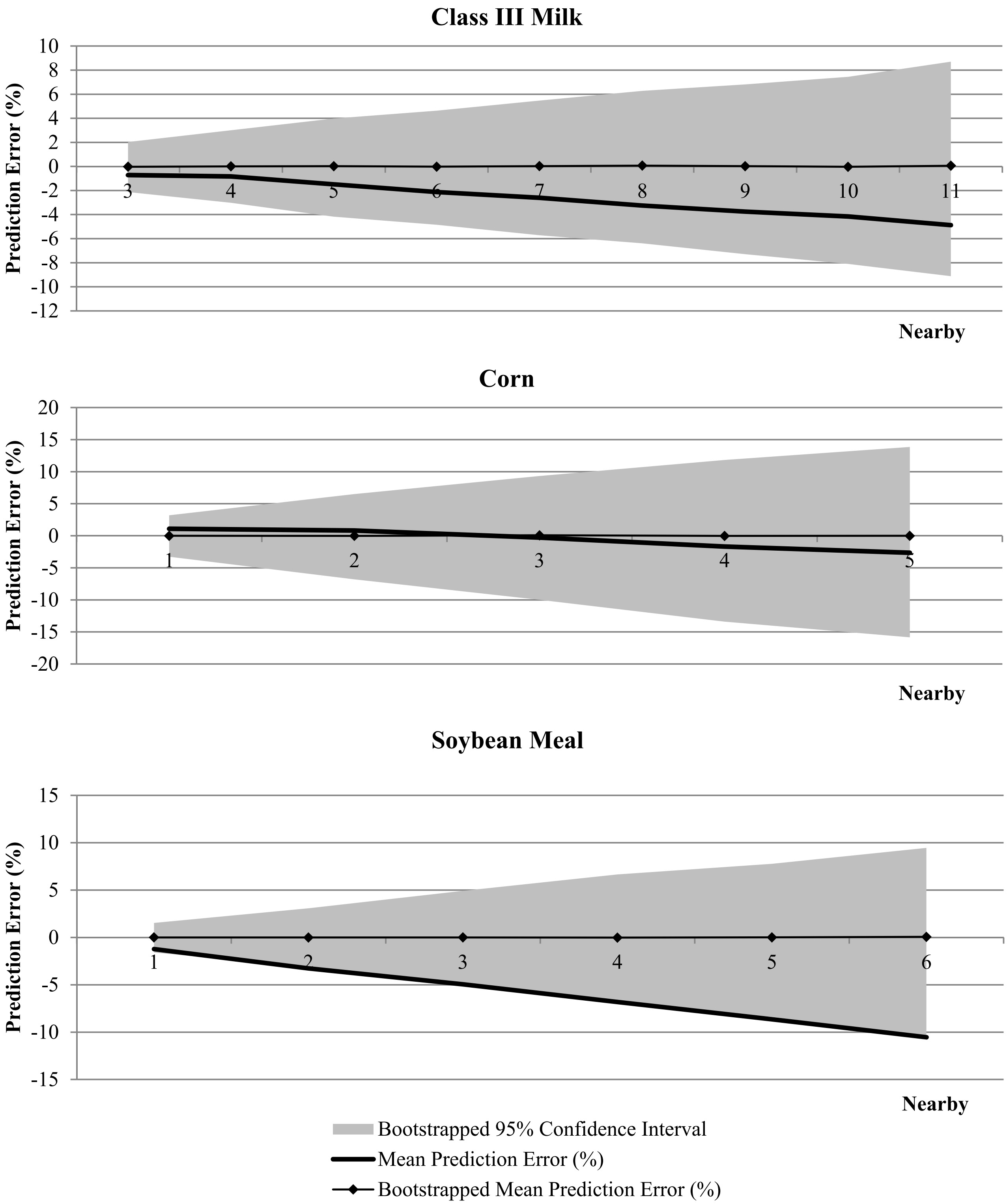


Figure 2. Parametric Bootstrap Tests for Unbiasedness of Implied Volatility

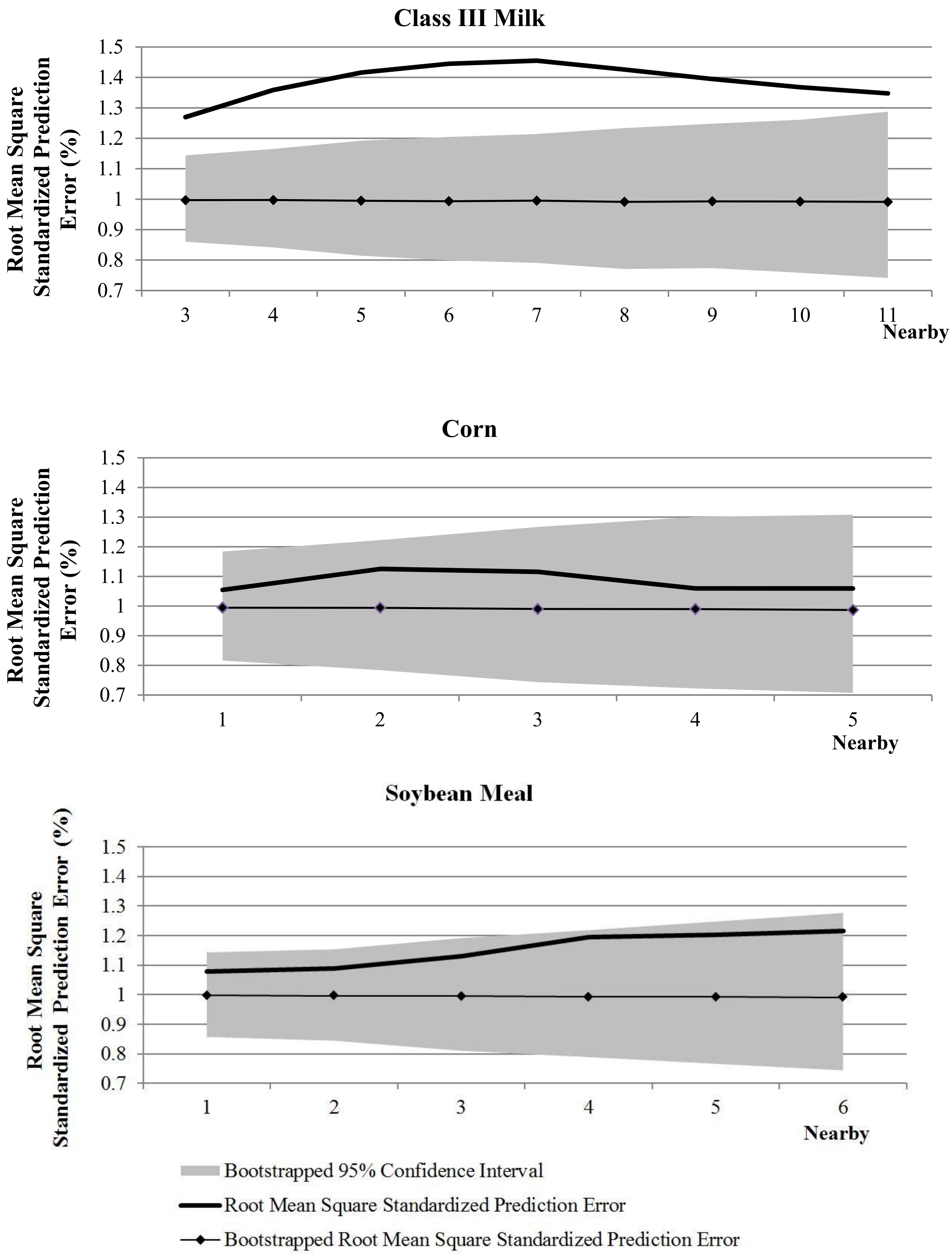
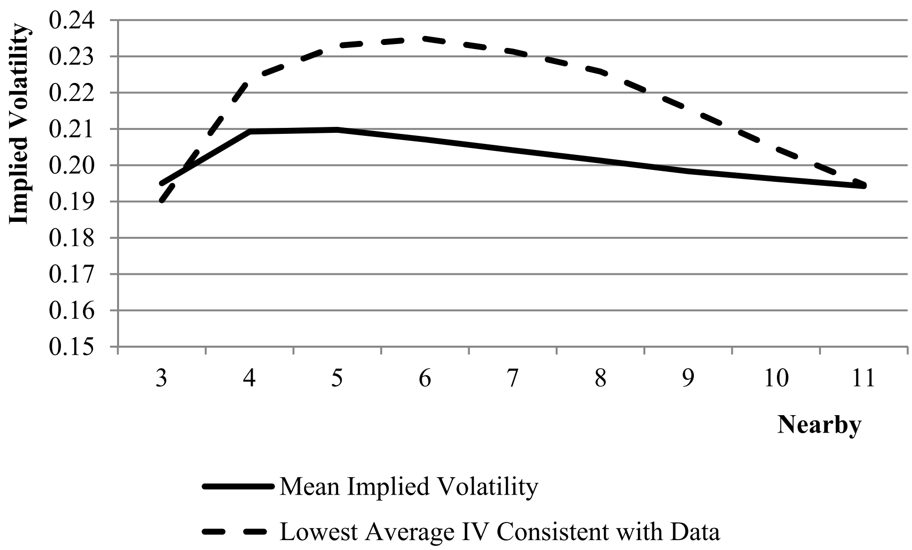


Figure 4. Option-based Implied Volatilities vs. Calibrated Volatilities in Class III milk futures



Note: *Lowest Average IV Consistent with Data* are average calibrated volatilities coefficients obtained such that the upper bound of 95% confidence interval for the bootstrapped root mean square prediction errors matches the mean of observed mean root square percentage prediction error.