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Higher education as a portfolio investment: students' choices about studying, term time employment, leisure, and loans

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Recent UK changes in the number of students entering higher education, and in the nature of financial support, highlight the complexity of students' choices about human capital investments. Today's students have to focus not on the relatively narrow issue of how much academic effort to invest, but instead on the more complicated issue of how to invest effort in pursuit of 'employability skills', and how to signal such acquisitions in the context of a highly competitive graduate jobs market. We propose a framework aimed specifically at students' investment decisions, which encompasses corner solutions for both borrowing and employment while studying.

JEL classifications: J22, J24, I23.

1. Introduction

This paper analyses student choices in higher education (HE). It argues that these are more complex than implied by existing literature, and develops a new framework. We motivate the discussion by considering recent changes in the UK's HE system, which is reasonably representative of HE in the developed world in most respects.[1]

Over the past two decades life for UK HE students has become much more complex. Twenty years ago most were financially supported by a mixture of non-repayable state grants and parental transfers, and whilst many supplemented this with vacation earnings, term-time work was relatively unusual. Thus, for most

[1] An exception is the nature and extent of student financial support, where there is sufficient diversity among countries that no one country can reasonably be regarded as fully representative. The OECD provides a good empirical survey of HE systems across the developed world (OECD 2009).

HE students their only important decision was the amount of effort to invest in studying, and thereby in the quality of degree obtained, indicated by degree classification. A reasonable quality of degree in turn, provided near-automatic entry to a graduate level job.

This simple world has largely disappeared because of two fundamental changes: in sector size (a rise in the proportion of 18–20 year olds in HE from 6% in 1960 (Blanden and Machin, 2004) to 34% in 2006 (DIUS, 2009)); and in funding (the replacement of most student grants by state-backed student loans repayable from future earnings, as well as the introduction of tuition fees in 1998). Increased sector size means that a degree is no longer a semi-automatic passport to a 'graduate job', because the much increased number of graduates makes it difficult for employers to identify the strongest candidates (Brown and Hesketh, 2004). Consequently, many students seek to acquire additional forms of human capital, e.g., work experience, to try to distinguish themselves from other candidates The UK Confederation of British Industry provides a substantial survey of employer and student attitudes which indicates that both groups now pay at least as much, perhaps more, attention to 'employability skills' (e.g., self-management, team working, business awareness, and problem-solving) as to the quality of degree result (CBI, 2009; see also Purcell *et al.*, 2002; Dickerson and Green, 2004). Hence today's students face a more complex investment decision than their predecessors: they have to invest time and effort in a portfolio of activities, not just academic study; and they also need to consider how best to signal such acquisitions in a highly competitive graduate jobs market.

The switch from grants to repayable loans has also complicated students' lives. Today's students have to make more complex financial decisions than their predecessors, involving choices between loans and term-time working as alternative sources of income. Loans impose future repayment costs; term-time work may reduce degree quality if it eats too much into study time (Jones and Sloane, 2005; Purcell *et al.*, 2005; Van Dyke *et al*, 2005). A TUC report found a 50% increase between 1996 and 2006 in the number of UK students working during term time (TUC, 2006). Overall, there is a much increased dependence of students on income from paid employment and loans, particularly among lower social class students (Callender and Kemp, 2000; Callender and Wilkinson, 2003; Van Dyke *et al.*, 2005; Finch *et al.*, 2006). The average annual loan taken by students increased from £1,530 in 1998 to £3,330 in 2006,[2] with average student debt expected to rise as a result of the Browne review[3] and the introduction of higher variable fees of up to £9,000.

Although the consequences of changes in sector size and in funding could be viewed separately, our contention is that they are better viewed together, since they are interconnected. This is well illustrated by term time employment, which

[2] See http://www.slc.co.uk/statistics.aspx.

[3] See http://www.bis.gov.uk/assets/biscore/corporate/docs/s/10-1208-securing-sustainable-higher-education-browne-report.pdf.

(i) provides a potential alternative to student loans as a source of income for (perhaps) debt-averse students; might also be a means both (ii) of gaining additional forms of human capital, and (iii) of signalling specific skills to a future graduate recruiter; but (iv) takes time away from studying and from leisure. Our contention is that all these choices—size of loan, and allocations of time to each of studying, employment, and leisure—should be analysed in a unified framework, since each affects and is affected by all the others. The next section reviews the literature on this issue, and outlines findings from a recent survey of student choices which suggest particular patterns of choice that require explanation. This motivates the development in Sections 3 and 4 of a unified framework for analysing students' investment decisions. Section 5 concludes.

2. Student choices in higher education: previous literature

The massive human capital literature has largely ignored the sorts of student investment choices, even when dealing specifically with UK graduates (e.g., Faggian and McCann, 2006), introduced in the previous section. We could find only three relevant conceptual papers: Oettinger (2005), Kalenkoski and Pabilonia (2010), and Neill (2006).[4] Whilst each provides useful insights, they also ignore at least one core element. Thus, Oettinger ignores student loans and other borrowing, and also assumes that future employment prospects depend only on academic qualifications; Kalenkoski and Pabilonia likewise ignore borrowing, and also leisure; and Neill assumes that each student has to choose between either additional studying (over and above a fixed minimum) or paid employment: in her model students cannot choose combinations of some discretionary studying and some paid employment (see Neill, 2006, p.13, footnote 19). Overall, therefore, none of these studies adequately captures the interlocking nature of HE students' decisions about borrowing, paid employment, studying, employability, and leisure.[5] Although the issues are conceptually similar to other human capital choices, we believe that there is sufficient distinctiveness to justify a specific framework.

This view is strengthened by the patterns of student choice revealed in a recent survey of students in the UK. Jewell (2008) undertook a survey at a university based in the southeast of England, which asked about family background, financial circumstances and student employment (predominately that undertaken during term

[4] Although all three papers are conceptual rather than empirical, the Oettinger and Kalenkoski/Pabilonia models are oriented towards the HE set-up in the USA, and the Neill model towards HE in Canada. The recent UK trends outlined in Section 1 have moved the UK set-up nearer to those in USA and Canada. Neill's work builds on earlier work by Keane (2002) and Keane and Wolpin (2001); she uses an augmented version of the Keane-Wolpin model.

[5] Although there is little such work set in the HE context, there is a large empirical literature on the consequences of high school students' ('school' students in UK terminology) part-time working for their post-school earning capacity, e.g., Ruhm (1997), Light (2001), and Hotz et al. (2002). To date no empirical consensus has emerged. Ruhm and Light both provide summaries.

Table 1 Student loans and term-time employment: patterns of choice

Intensity of term-time employment	Level of borrowing: No loan	Less than maximum loan	Maximum loan	Total
Zero	36	25	259	320
Low intensity	5	6	60	71
Medium intensity	3	5	59	67
High intensity	5	3	58	66
Very high intensity	16	8	74	98
Total	65	47	510	622

Source: Jewell, 2008.
Notes: Each individual cell shows the number of students in the sample choosing the indicated combination of employment intensity and borrowing level.

time). Survey data from 622 British domiciled students graduating in 2006 and 2007 were matched to their university personal record and information about their labour market outcomes six months after graduation.[6] A companion paper (Jewell *et al*., 2010), which provides an econometric analysis of the consequences of students' employment choices on degree and labour market outcomes, analyses patterns of student loan take-up and student term-time employment. For this purpose the amount of term time employment was divided into five categories: zero, low, medium, high, and very high intensity.[7] Table 1 summarizes the dataset.

In interpreting Table 1 we note that both loans and term-time employment normally have upper limits. In the UK, for example, the Student Loans Company sets a maximum loan; this is Table 1's 'maximum loan'.[8] As regards employment, many UK HE institutions publish advisory limits on term-time working. Even if some students consider disregarding such advice, practical limits are still likely to result from studying constraints. We discuss this further in the next section; for now we simply assume that a limit exists, that it is common to all students, and that it is equal to the highest of the five term-time employment levels (very high intensity).

[6] This information was obtained from their Destinations of Leavers in Higher Education (DLHE) survey responses. The DLHE is undertaken by all universities to gather information on graduate destinations; for more information see http://www.hesa.ac.uk/index.php/component/option,com_collns/task,show_collns/targetYear,any/Itemid,231/targetStream,3/.

[7] These categories were based on hours and number of weeks worked, see Jewell (2008) for more information.

[8] In principle students might be able to obtain additional borrowing from other sources, but any such lenders are themselves likely to limit what they will lend. Our dataset has no information on non-SLC borrowing. The idea of borrowing constraints is familiar from the literature on intertemporal household saving; cf., Zeldes 1989, Deaton 1991.

With this interpretation Table 1 suggests that both term-time employment decisions and student loans decisions are bi-modal, with most choices at the extremes of the distributions. Looking first at student loans, the modal choice, made by 82% of the sample, is to borrow the maximum amount. There is a second, albeit much smaller, modal point at zero loans, chosen by 10% of students. Only 8% choose an intermediate point. The same pattern applies to term-time working. The modal choice, made by 51% of students, is zero working; again there is a second modal point at the opposite extreme of the distribution, with 16% of students choosing 'very high work intensity'. In between these two extremes, 11% of students are located at each of three intermediate points. This suggests that the incentive structure systematically pushes HE students towards corner solutions for both loans and employment, with only a minority choosing interior solutions. Whilst economic analysis readily accommodates corner solutions, it routinely focuses on interior solutions and rarely develops models in which the modal choice for even one, let alone two, variables is at a corner.[9] From this perspective the seemingly corner-dominated pattern of student choices merits further analysis.[10]

3. A model of students' investment of time and effort

3.1 The model

We use a two-period framework. The first period covers time at university (typically three years in the UK); the second spans a similar length of time and covers the early years of subsequent working life.[11] We assume the following utility

[9] As noted in the previous footnote, the literature on intertemporal household saving routinely incorporates borrowing constraints, especially in the early stages of the life cycle, and this causes younger individuals to locate at borrowing corners. This literature has never explored the possibility of employment constraints, however, presumably because it typically focuses on individuals in full time employment rather than on HE students.

[10] The joint distribution of loan and work choices in the Jewell (2008) dataset is also worthy of note, since the subset of students who choose the highest level of term time working ('very high intensity') includes the lowest proportion of people students who choose the maximum loan (75%). This is hard to explain if the only or primary motive for working is to fund current consumption (since then one would expect those who need to work the most for this purpose to be the keenest to take out loans). It is of course possible that the Jewell dataset, derived as it is from a single UK university, is unrepresentative of student behaviour more widely within the UK or internationally. Data from the UK Student Loan Company show that in 2005 79% of eligible UK students took out a loan, compared with 89% in the Jewell dataset, suggesting that the latter is not substantially out of line, at least so far as the decision about whether or not to take a loan is concerned. It is much harder to assess its representativeness as regards the amounts and pattern of term-time working, because so far as we are aware there are no detailed national or international data about this, so the only source of information is surveys at individual universities, and even here information is usually limited because few such surveys include details of total hours worked by individual students per week, month, and year. More detailed information about UK student loans is available from the Student Loan Company: see http://www.slc.co.uk/statistics.aspx.

[11] We define the second period in this way because the Jewell (2008) dataset's use of the DLHE data (cf., footnote 6) means that its measures of post-HE employment and wage outcomes focus only on short-run outcomes rather than on lifetime earnings. Extending our model to cover lifetime earnings

function for an individual student i:

$$U(i) = \log[C_1(i)] + V(i)\log[L(i)] + D(i)\log[C_2(i)] \tag{1}$$

$C_1(i)$ is consumption whilst at university, $C_2(i)$ is consumption in early working life, $L(i)$ is leisure time whilst at university, $V(i)$ is the weight attached to leisure relative to consumption, and $D(i)$ is the rate of time discounting, with $V(i) > 0$ and $0 < D(i) \leqslant 1$. Leisure is a choice variable in period 1 but in period 2 we assume that all students work for a fixed amount of time.[12]

We specify budget constraints as follows:

$$\begin{aligned} C_1(i) &= WH_1(i) + T(i) + B(i) \\ L(i) &= 1 - H_1(i) - f(i)S(i) \\ C_2(i) &= W_2(i)H_2 - RB(i) \\ B(i) &\leqslant B_{\max}; \quad 0 \leqslant H_1(i) \leqslant H_{\max} < 1 \end{aligned} \tag{2}$$

We assume a fixed total amount of time in period 1, normalized at unity, of which a fraction $H_1(i)$ is spent in paid term-time employment at an exogenously fixed wage W, and a fraction $S(i) > 0$ is spent studying. H_{max} reflects our previously mentioned assumption of an upper limit on the amount of term-time employment which is the same for all students. We discuss this before considering the other details of the budget constraints. The issues are well captured by the advice which Imperial College currently gives its students:

> The College recommends that full-time students do not take up part-time work during term-time. If this is unavoidable we advise students to work no more than 10–15 hours per week, which should be principally at weekends and not within the normal working hours of the College. Working in excess of these hours could impact adversely on a student's studies or health . . . The College does not condone situations where a student's employment would cause them to miss teaching or other departmental activity during the College working day, or submit coursework after the specified deadline . . . The College's examination boards will not normally consider as mitigating circumstances any negative impact that part-time work during term-time may have had on a student's performance in examinations or in other assessed work. (Imperial College, Senate papers, November 2011).

If Imperial's students follow its advice they will all have the same value for $H_{\max}$, determined by Imperial's 15-hour upper limit. Of course, some students might consider ignoring it, even though the risks (both to degree quality and to health) are clearly spelled out. In practice many, and probably most students, will struggle

would not in itself require any substantial re-specification. However, many of today's students will eventually themselves have children who enter HE, and will thus need to make their own decisions about the size of parental transfer. Extending our model to longer periods would thus logically require us to model such decisions using an overlapping generations framework. This would evidently complicate the analysis and shift it away from our intended focus on intra-HE decision making.

[12] Excluding leisure from period 2 is partly for simplicity but also reflects data considerations: there is no information about students' post-HE leisure choices in the DLHE dataset (cf., footnotes 6 and 11) nor so far as we are aware in any other available dataset.

to exceed the 15-hour limit by a significant amount even if they wish to do so, because the lumpy and inflexible nature of academic programmes and part-time jobs severely constrains daytime employability. Most academic programmes have commitments scattered throughout the working week, leaving few if any sizeable chunks of free time during the working day. This will make it difficult if not impossible to find weekday daytime jobs whose requirements match student availability and which are also financially worthwhile if there are significant fixed costs of travel between campus and workplace. Weekday evening or night-time jobs do not pose the same problems, but are still subject to the health and academic performance risks emphasized by Imperial College. Peak load academic commitments (coursework deadlines, exam revision) exacerbate the difficulties: if students cannot commit to working every week for a significant length of time their employability is further reduced.

Overall, it seems plausible that few if any full-time students can work over a sustained period for the equivalent of more than weekend daytime hours plus occasional weekday evening hours, suggesting an effective limit not much above Imperial's 15-hour guidance. Some students in some circumstances might find ways to work significantly longer hours, but normally only for limited periods of time (e.g., during a non-exam term, or during one year of a three-year degree programme). Bearing in mind that in our framework H_{max} is appropriately interpreted as a maximum limit which is averaged over an entire degree programme rather than as a limit which applies to each individual week or term, assuming a common limit does not seem an unreasonable generalization.

Returning to the budget constraints (2), the parameter $f(i)$ measures the individual's attitude to studying. If she views it as pure work, offering no intrinsic enjoyment, then $f(i) = 1$ and $L(i) = [1\text{-}H_1(i)\text{-}S(i)]$. If instead she views it as a mix of work and enjoyment, then $f(i) < 1$ and $L(i) = [1\text{-}H_1(i)\text{-}f(i)S(i)]$. We assume that studying is never seen as pure leisure; thus $0 < f(i) \leqslant 1$. We also assume no intrinsic enjoyment from term-time employment.

First period consumption $C_1(i)$ equals income from paid term-time work, $WH_1(i)$, plus or minus net financial transfers $T(i)$, which equal parental financial support less tuition fees, plus borrowing $B(i)$.[13] Borrowing is subject to the constraint $B(i) \leqslant B_{max}$. We allow that some students might save (i.e., choose $B(i) < 0$).[14] In the second period each former student works a fixed amount of

[13] We assume that vacation earnings support vacation rather than term-time living expenses.

[14] As already noted, countries differ widely in their arrangements for tuition fees and student financial support. In the UK, for example, no home students paid tuition fees for undergraduate degrees pre-1998; since then a succession of changes has introduced and progressively increased fees for most students, together with selected exemptions, e.g., for students from low-income households. Each university chooses its own fee but subject to a maximum amount determined by the UK government. Home students are entitled to pay their fees upfront if they wish; any unpaid fees are then paid by the SLC, and added to the student's SLC loan. Every home student is also entitled to borrow from the SLC a specified amount towards basic living costs. Thus B_{max} exceeds tuition fees for all home UK students and is adjusted in line with changes in the maximum permitted fee.

time H_2 for wage $W_2(i)$. Second period consumption $C_2(i)$ equals the resulting labour income, $W_2(i)H_2$, less repayments of student loans, $RB(i)$. R incorporates both interest charges and capital repayments, and is exogenous to any individual student.[15] The repayment regime for UK students is relatively generous, so since our second period covers only the first few post-HE years, R will be small for most students, and potentially zero for some.

The second period wage $W_2(i)$ equals the first period wage, W, augmented by human capital acquired at university. Following Section 1's discussion, we distinguish between intellectual ('hard') skills acquired by studying and signalled by the degree class; and personal attributes ('soft skills') e.g., ambition, propensity to work hard, etc., which are both strengthened and signalled by term-time working. We assume that hard skills and degree quality are increasing in period 1 study time $S(i)$. As for soft skills, it is tempting to argue equivalently that these are increasing in period 1 paid employment time $H_1(i)$. In practice things may not be so simple: some types, or amounts, of such work might send stronger signals than others. For example, working in a bar for five hours per week might not impress a prospective employer, but doing so for fifteen hours per week whilst simultaneously studying might signal a capacity for hard work, good time management, etc. To allow for such possibilities we model the second period wage as follows:

$$W_2(i) = W\left\{1 + a(i)S(i) + b(i)\max\left[0, H_1(i) - \hat{H}\right]\right\} \tag{3}$$

$a(i) > 0$ is the rate of return (RoR) to investment of time in studying, and $b(i) > 0$ is the RoR to investment of time in term-time employment. Equation (3) says that this latter effect only operates if the student exceeds a threshold level of work, $\hat{H}$. Other specifications would also be defensible,[16] though (3) is not particularly restrictive. Both $a(i)$ and $b(i)$ are person-specific. We assume that $a(i) > b(i)$ for all i. We also define $[a(i)/f(i)]$ as the utility-adjusted RoR to studying: if $f(i) < 1$ (i.e., studying is enjoyable as well as an investment) then the utility-adjusted RoR is higher than if it is seen as simply another form of work ($f(i) = 1$).

[15] This is true in the sense that the repayment rules are exogenously given. However, because the rules include a lower earnings threshold below which no repayments are required, the proportionate rate of repayment varies according to students' post-HE earnings levels. We ignore this complication. For simplicity we also ignore the possibility of borrowing from other sources, which would normally be at interest rates different from the student loan rate.

[16] For example, unpaid volunteering might send a stronger signal than paid work; a low-paid job with good training opportunities might be better than a higher-paid but routine job; there might be multiplicative effects between hard and soft skills as well as the additive effects assumed in (3). A full analysis of all the possibilities would unduly lengthen the paper, but might be a useful topic for future work.

The model takes as given the prior decision to enter HE. An earlier version considered this participation decision, but a proper treatment would unduly lengthen the paper and is tangential to its main aims.

3.2 Optimal choices

Differentiating (1) subject to (2) and (3) gives the first order conditions:

$$\frac{\partial U(i)}{\partial H_1(i)} = \left(\frac{W}{C_1(i)}\right) - \left(\frac{V(i)}{L(i)}\right) + G(i)\left(\frac{D(i)b(i)WH_2}{C_2(i)}\right) \geqslant or \prec 0 \tag{4}$$

$$\frac{\partial U(i)}{\partial S(i)} = -\left(\frac{V(i)f(i)}{L(i)}\right) + \left(\frac{D(i)a(i)WH_2}{C_2(i)}\right) = 0 \tag{5}$$

$$\frac{\partial U(i)}{\partial B(i)} = \left(\frac{1}{C_1(i)}\right) - \left(\frac{RD(i)}{C_2(i)}\right) \geqslant 0 \tag{6}$$

$$G(i) = \begin{cases} 0 \; for \; H_1(i) \leqslant \hat{H} \\ 1 \; for \; H_1(i) > \hat{H} \end{cases} \tag{7}$$

We assume that the model always generates an interior solution for *S(i)* (i.e., (5) always holds with equality), whence substituting for *V(i)/L(i)* in (4) using (5) gives:

$$\left(\frac{\partial U(i)}{\partial H_1(i)}\right)\Bigg|_{\partial U(i)/\partial S(i)=0} = \left(\frac{1}{C_1(i)}\right) - \left(\frac{D(i)H_2}{C_2(i)}\right)\left(\frac{a(i)}{f(i)} - G(i)b(i)\right) \geqslant \; or \; \prec 0 \tag{8}$$

(8) and (6) together define optimal choices $H_1(i)^*$ and $B(i)^*$ conditional on the optimal choice $S(i)^*$.[17] To analyse their implications it is convenient to define:

$$\begin{aligned} \tilde{R}(i)_a &\equiv \left(\frac{a(i)H_2}{f(i)}\right) \; for \; H_1(i) \leqslant \hat{H} \\ \tilde{R}(i)_b &\equiv H_2\left(\frac{a(i)}{f(i)} - b(i)\right) \; for \; H_1(i) \succ \hat{H} \end{aligned} \tag{9}$$

Assumptions already made ensure $\tilde{R}(i)_a \succ \tilde{R}(i)_b \succ 0$. The $\tilde{R}(i)$ variables measure the utility-adjusted net RoR to future earnings from investing a unit of time in studying when employment is $(\tilde{R}(i)_a)$ or above $(\tilde{R}(i)_b)$ the threshold $\hat{H}$: the net RoR to studying is lower when $H(i) \succ \hat{H}$ because there is then also a positive, albeit lower, RoR from employment (hence the opportunity cost of studying is greater). Both $\tilde{R}(i)_a$ and $\tilde{R}(i)_b$ are exogenous to *i*, since both are composites of exogenous parameters reflecting individual ability (*a(i)* and *b(i)*), and preferences (*f(i)*).[18]

[17] Throughout, we use an asterisk to denote an optimal choice.

[18] They are also influenced by second period labour supply H_2 which by assumption is common to all individuals.

Using (9) we can rewrite (6) and (8):

$$\left.\left(\frac{\partial U(i)}{\partial B(i)}\right)\right|_{\partial U(i)/\partial S(i)=0} \geqslant 0 \ as \left(\frac{C_2(i)}{D(i)C_1(i)}\right) \geqslant R$$
$$\left.\left(\frac{\partial U(i)}{\partial H_1(i)}\right)\right|_{\partial U(i)/\partial S(i)=0} \geqslant or \prec 0 \ as \left(\frac{C_2(i)}{D(i)C_1(i)}\right) \geqslant or \prec \tilde{R}(i)_a \left[\frac{\tilde{R}(i)_b}{\tilde{R}(i)_a}\right]^{G(i)} \tag{10}$$

(10) indicates that (6) and (8) cannot generally both hold with equality: this would require $R=\tilde{R}(i)$,[19] but both R and $\tilde{R}(i)$ are exogenous to individual i. Hence $R=\tilde{R}(i)$ occurs only by coincidence, and in what follows we ignore this possibility and focus on outcomes in which one or both of (6) and (8) is an inequality, so that at least one, and potentially both, of $B(i)^*$ and $H_1(i)^*$ are at corners.

This at once indicates a key difference between our framework in which borrowing, studying, and paid employment are all decision variables, and a more standard approach, exemplified by the papers reviewed in Section 2, in which one or more of these are ignored. In the standard approach corner solutions can result from particular parameter values (e.g., a student with a relatively high utility value of leisure is relatively likely to choose a zero employment corner). Since parameter values typically differ across individuals, the standard approach normally generates a range of outcomes, some at corners and others at internal equilibria. Thus, corner outcomes are not guaranteed but depend on particular parameter combinations. By contrast, in our framework corner outcomes are an intrinsic property of the model, since only by coincidence will any individual have a particular combination of parameter values such that both equations in (10) hold with equality.

Since this is an unusual property in economic models, we briefly discuss its intuition in the context of intertemporal models in general. Consider the simplest possible two-period model in which the objective function is as in (1) but with leisure initially ignored (i.e., $V=0$ in (1)). Assume that the individual has a fixed endowment E in each period and has access to a single financial instrument F_1 which allows saving or borrowing at interest rate R_1. Thus consumption in periods 1 and 2 is $C_1=(E+F_1)$ and $C_2=(E—R_1F_1)$, where a positive (negative) value of F_1 denotes borrowing (saving) in period 1. If neither saving nor borrowing is constrained an optimizing individual will always locate at an interior optimum $F_1{}^*$ at which the ratio of present to discounted future marginal utility equals R_1. If instead there are borrowing and/or saving constraints, then whether the individual locates at an interior or corner outcome depends on parameter values, which determine whether or not a constraint makes the interior optimum unattainable.

Now modify this framework by introducing a second financial instrument F_2 with interest rate $R_2 \neq R_1$. Consumption is now $C_1=(E+F_1+F_2)$ and $C_2=(E-R_1F_1$

[19] Where there is no ambiguity, we use $R(i)$ to denote whichever of $R(i)_a$ or $R(i)_b$ is appropriate.

$-R_2F_2$). An interior optimum for both choice variables is impossible: to satisfy the first order conditions for both F_1 and F_2 with equality would require that the ratio of present and discounted future marginal utilities is equated to both R_1 and R_2, but this is impossible. For concreteness, suppose $R_2 > R_1$. Then, if there are no borrowing or saving constraints the individual will seek to save unlimited amounts of F_2 and borrow unlimited amounts of F_1, subject only to maintaining the ratio C_2/C_1 within the range $R_1D < C_2/C_1 < R_2D$. Thus, the intertemporal optimizing process becomes destabilizing instead of stabilizing: optimization drives the values of F_1 and F_2 ever further apart rather than towards stable interior values. If instead there are borrowing and/or saving constraints, it is easy to see that this 'driving apart' process will then make corner solutions an intrinsic part of the model: at least one and potentially both of F_1 and F_2 will always be driven to a corner. (Parameter values still play a role: they determine whether both instruments, or only one, end at a corner.)

Markets will not normally allow two risk-free financial instruments to coexist with unequal rates of return, so the previous paragraph's setup is unrealistic. Our model, however, is analytically equivalent but more realistic. We have three instruments—term-time employment, study time, and borrowing/saving—each of which can alter future relative to current consumption; and unlike in the case of different financial instruments, there is no obvious mechanism to equate their rates of return. Hence the 'driving apart' mechanism operates in our model to push the choice variables towards corner solutions. Our model also differs from that in the previous paragraph because we include period 1 leisure as an additional choice variable. This additional degree of freedom allows the model to accommodate two choice variables, rather than only one, within the standard interior optimizing framework. It is straightforward to check that if any one of the three choice variables is exogenously fixed (as in the papers reviewed in Section 2), the model then generates either corner or interior solutions for the two remaining variables depending on parameter values, just as in the standard framework. Once we model the full range of student choice variables, however, the 'driving apart' mechanism comes into play, just as in the previous paragraph. As noted in the previous section, modal choices by students do appear to gravitate towards corners; hence, our framework may help to provide a better understanding of the pattern of student choice.

Using (10) we can distinguish seven possible cases, summarized in Table 2, which also specifies three possible configurations of R and $\tilde{R}(i)$. There is no *a priori* reason to assume that any of these configurations is more general than the others, and in what follows we assume that each represents a proportion of the student population.[20] We define Type 1 individuals as those for whom $R \prec \tilde{R}(i)_b$, Type 2 as those for whom $\tilde{R}_b(i) \prec R \prec \tilde{R}_a(i)$, and Type 3 as those for whom $\tilde{R}(i)_a \prec R$.

[20] This assumption can be justified by noting that each configuration excludes at least one combination of $B(i)^*$ and $H_1(i)^*$ which is present in Table 1's data. For example for any individual for whom $\tilde{R}(i)_a \prec R$ (the final column in Table 2) only Cases 6 and 7 are possible, both of which involve

Table 2 Feasible combinations of first order conditions for HE students

Case	First order conditions	B^*	H^*	$R \prec \tilde{R}_b$ (type 1 individuals)	$\tilde{R}_b \prec R \prec \tilde{R}_a$ (type 2 individuals)	$\tilde{R}_a \prec R$ (type 3 individuals)
1	$U_B > U_H > 0$	$B^* = B_{max}$	$H^* = H_{max}$	√		
2	$U_B > U_H = 0$	$B^* = B_{max}$	$\hat{H} \prec H* \prec H_{\max}$	√		
3	$U_B > U_H = 0$	$B^* = B_{max}$	$0 \prec H* \prec \hat{H}$	√	√	
4	$U_B > 0 > U_H$	$B^* = B_{max}$	$H^* = 0$	√	√	
5	$U_B = 0 > U_H$	$B^* < B_{max}$	$H^* = 0$	√	√	
6	$U_H > U_B > 0$	$B^* = B_{max}$	$H^* = H_{max}$		√	√
7	$\mathrm{U_H} > \mathrm{U_B} = 0$	$\mathrm{B}^* < \mathrm{B_{max}}$	$\mathrm{H}^* = \mathrm{H_{max}}$		√	√

Notes: All '*i*' terms have been omitted for notational simplicity. $\mathrm{U_B}$ denotes $\partial U(i)/\partial B(i)$ evaluated at $\partial U(i)/\partial S(i) = 0$. $\mathrm{U_H}$ denotes $\partial U(i)/\partial H_1(i)$ evaluated at $\partial U(i)/\partial S(i) = 0$. The presence (absence) of √ indicates that the combination of the case described in the relevant row and the configuration of R and $\tilde{R}$ indicated in the relevant column is possible (impossible).

The intuition underpinning Table 2 can be seen by noting that (6) balances the increase in current utility of an increase in borrowing (and thus in current consumption) against the discounted decrease in future utility resulting from the need to repay the loan. The higher the repayment rate R *ceteris paribus* the less attractive is borrowing. Similarly, (8) balances the increase in current utility of a shift of time away from studying towards employment (and resulting increase in current income and consumption) against the discounted decrease in future utility resulting from the shift towards lower-return soft skills, and consequent fall in future earnings. The higher is $\tilde{R}(i)$, the higher the opportunity cost of reducing high-return investment in hard skills. Combining all these points, an individual i who wants to increase current consumption can choose between a shift of time from studying towards employment, and an increase in borrowing. In each case there is an adverse effect on future consumption. If $\tilde{R}(i)$ is bigger (smaller) than R, the adverse impact of the shift from studying to employment is bigger (smaller) than that of extra borrowing. Thus, if R is low relative to $\tilde{R}(i)$ then *ceteris paribus* we would expect solutions tilted towards borrowing and away from term-time employment, and *vice versa*. Table 2 reflects this intuition. Thus, the configuration $R < \tilde{R}(i)_b$ (Type 1 individuals) is consistent with five cases which are all tilted towards borrowing: in four, $B = B_{max}$ and in the fifth (Case 5) although $B(i) < B_{max}$, this is accompanied by a zero employment ($H_1(i) = 0$) corner solution. By contrast, the configuration $R > \tilde{R}(i)_a$ (Type 3 individuals) is consistent with two cases, both involving $H(i) = H_{max}$.

a corner solution for first period employment at its maximum possible value ($H_1(i)^* = H_{max}$). Since we saw in the previous section that many individuals choose outcomes with much lower employment (including many choosing zero employment) this at once indicates that the configuration $\tilde{R}(i)_a \prec R$ cannot apply to all individuals. Similarly, each of the other two configurations of R and $\tilde{R}(i)$ rules out at least one $B(i)^*$, $H_1(i)^*$ combination that is represented in Table 1.

The cases in Table 2 vary in the relative weight placed on current as against future consumption. Looking first at the configuration $R \prec \tilde{R}(i)_b$, Cases 1–5 which are consistent with this configuration are ranked in descending order of the relative weight on current consumption. Thus, Case 1 involves maximum possible first period consumption: both borrowing and employment are at maximum levels. Cases 2–4 all still involve $B(i) = B_{max}$, but progressively lower employment and thus, first period consumption. Finally, Case 5 involves zero employment and less than maximum borrowing. Now consider the configuration $R \succ \tilde{R}(i)_a$. Cases 6 and 7 are consistent with this, and are likewise ranked in descending order of emphasis on current consumption (both involve maximum employment, but $B(i) = B_{max}$ in Case 6 whereas $B(i) < B_{max}$ in Case 7.)

4. Borrowing and employment choices

In this section we analyse borrowing and employment choices. We focus on the intuition, and provide algebraic details in the Appendix. We first solve (4)–(7) for optimal borrowing $B(i)^*$, term-time employment $H_1(i)^*$, and study time $S(i)^*$, for each of the seven cases in Table 2. Table A1 in the Appendix summarizes the results. Substituting optimal choices from Table A1 into the budget constraints in (2) gives optimal first and second period consumption, $C_1(i)^*$ and $C_2(i)^*$, and leisure $L(i)^*$, again for all seven cases. Appendix Table A2 reports the results. We then use Tables A1 and A2 to analyse borrowing and employment choices. We focus on pair-wise comparisons between cases in which one element of choice is common whilst the other differs (e.g., both involve $H = H_{max}$, but $B = B_{max}$ in one case whilst $B < B_{max}$ in the other).

4.1 Borrowing choices

We first consider borrowing choices. Section 1 of the Appendix provides a detailed analysis which can be summarized as follows:

Proposition 1 The probability that any individual student borrows the maximum permitted amount is decreasing in (i) the size of parental transfer, (ii) the weight which she puts on the future relative to the present, (iii) the weight which she puts on leisure relative to consumption, and (iv) the rate of future loan repayment; is increasing in her rate of return to human capital from studying; and is also increasing in the level of tuition fees (provided that a rise in fees does not trigger a corresponding rise in the maximum available loan). In addition, her probability of borrowing the maximum amount is increasing in her rate of return to human capital from term-time employment if she works the maximum permitted amount, but not if she undertakes zero term-time employment. Also, if she borrows less than the permitted maximum her size of loan is decreased (and her probability of not taking a loan at all is increased) by the same set of influences that decrease her probability of taking a maximum loan.

All these predictions make intuitive sense: a large parental transfer reduces the pressure to take a loan in order to finance current spending; a high weight on the future (i.e., a low time discount rate) discourages borrowing because future repayments are a greater cause for concern; a high weight on leisure relative to spending reduces the incentive to borrow in order to fund spending; a high repayment rate increases the repayment burden of any given amount of current borrowing and thus discourages borrowing; and an increase in tuition fees increases the need for a loan. As for the rates of return on studying ($a(i)/f(i)$) and term-time employment ($b(i)$), both affect current borrowing incentives via (6), which is a standard Euler equation that equates current and discounted future marginal utilities of consumption. High rates of return to current human capital investment raise future income and therefore future consumption, and thus reduce the associated future marginal utility. To maintain intertemporal balance requires correspondingly high current consumption (and low current marginal utility), and this increases the incentive to borrow in order to finance this consumption. This applies to the rate of return to studying in all circumstances; by contrast, it applies to the rate of return to term-time employment only if the latter is above the signalling threshold $\hat{H}$.

4.2 Term-time employment choices

Turning now to term-time employment choices, we focus on Type 1 individuals, who from Table 2 have five feasible cases and four possible employment levels: $H_1(i)^* = 0$, $0 \prec H_1(i)* \prec \hat{H}$, $\hat{H} \prec H_1(i)* \prec H_{\max}$, and $H_1(i)^* = H_{max}$, We label these four levels zero employment, low employment, high employment, and maximum employment outcomes respectively.[21] Each individual chooses the level yielding the highest utility. Figure 1 illustrates four possible forms for the partial relationship between utility and term-time employment, each defining a different optimal employment choice: corner solutions at $H_1^* = 0$ (Fig. 1a) or at $H_1^* = H_{max}$ (Fig. 1d), and interior optima at low (Fig. 1b) or high (Fig. 1c) employment.

Section 2 of the Appendix provides a detailed analysis which can be summarized as follows:

Proposition 2 For a borrowing-constrained student the optimal term-time employment level is decreasing in (i) the parental transfer; (ii) the utility weight on leisure; (iii) the utility weight on the future; (iv) the utility-adjusted rate of return to human capital from studying; (v) the rate of loan repayment; and (vi) the size of the maximum available loan; and is increasing in (a) the level of tuition fees (provided that the latter does not itself influence the maximum available loan);

[21] Whereas Table 2 has four employment categories, Table 1 has five, corresponding to the definitions in the Jewell (2008) dataset from which it is derived. We can think of Table 1's 'medium intensity' category as being subsumed into either the 'low intensity' or the 'high intensity' category for the purposes of Table 2.

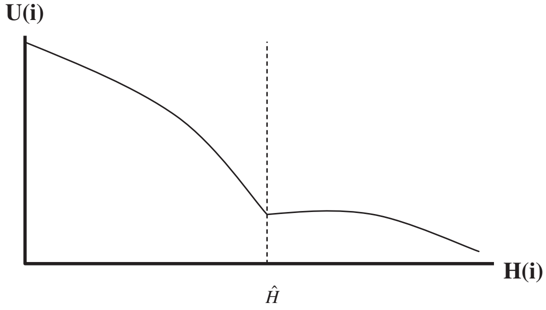

Fig. 1a Zero employment corner solution.

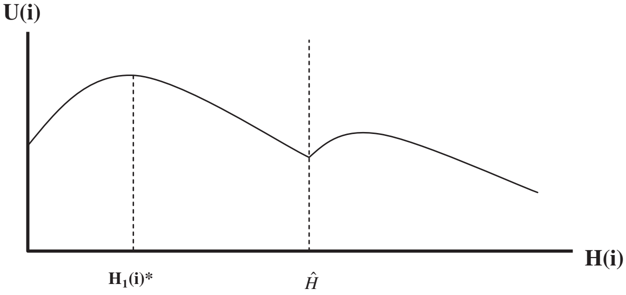

Fig. 1b Low employment interior equilibrium.

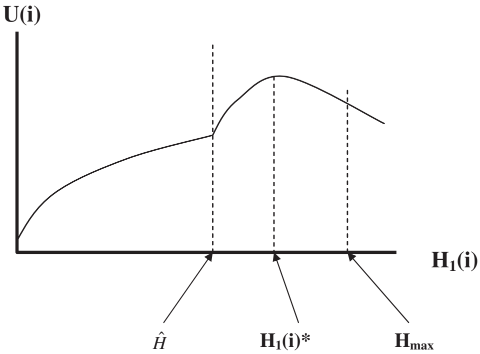

Fig. 1c High employment interior equilibrium.

and (b) the rate of return to human capital from term-time employment at employment levels which are high enough to send a signal about soft skills to future employers.

All these results are intuitive. A high parental transfer reduces the need for paid employment, and high tuition fees increase the need *ceteris paribus*; a high weight on leisure reduces the desire to earn and spend; a high weight on the future

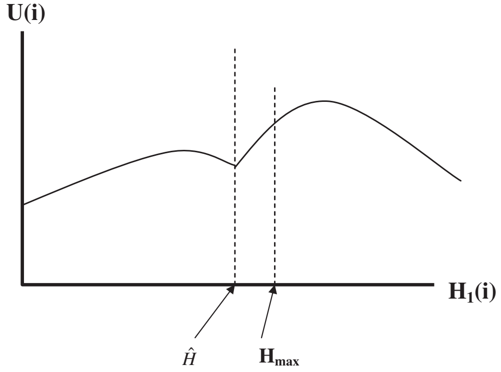

Fig. 1d Very high employment corner solution.

incentivizes studying (whose pay-off is entirely in the future) relative to earning (whose pay-off is partly in the present); a high rate of loan repayment (and therefore, *ceteris paribus*, lower future spending) likewise incentivizes studying over work in order to raise future income; and a high B_{max} increases current resources and reduces the need to earn. All these influences act to reduce term-time employment; conversely, a high rate of return to future human capital from term-time employment provides a direct incentive to work more.

The preceding discussion focuses on borrowing-constrained individuals. We briefly consider the choice between Cases 5 and 7. In both cases $B^* < B_{max}$, and in both cases employment is at a corner: $H_1{}^* = 0$ in Case 5, and $H_1{}^* = H_{max}$ in Case 7. We proceed as in the Appendix, Section 2, by comparing the highest attainable utility level in each case. Using the solutions for $C_1(i)^*$, $C_2(i)^*$, and $L(i)^*$ in Table A2 it can be checked that utility in Case 5 is higher (lower) than in Case 7 if $C_1(i)^*$ is higher (lower) in Case 5 than in Case 7; and it is easily established that this applies if $X(i)$ (defined in Table A3) is positive (negative). Using Table A3's definition of $X(i)$:

Proposition 3 The probability that a borrowing-unconstrained student will prefer maximum to zero employment is increasing in the loan repayment rate and the rate of return to human capital from term-time employment, and decreasing in the utility-adjusted rate of return to human capital from studying.

Again these results are intuitive: a high loan repayment rate incentivizes employment and income earning in order to reduce the required size of loan; a high $b(i)$ directly incentivizes employment; and a high $[a(i)/f(i)]$ incentivizes studying and therefore disincentivizes alternative uses of time, including employment.

5. Conclusions

Higher education students face a pattern of incentives and constraints which appears to push them systematically towards corner solutions rather than interior solutions for both their allocations of time and their levels of indebtedness. We have formulated a framework which is consistent with this, and have used it to derive results about students' choices.

In conclusion we mention several implications and extensions of the analysis. At a theoretical level, we have made various simplifications (e.g., all lending and borrowing is at the same rate; the human capital eq. (3) is entirely additive; etc.) and it would be useful to analyse to what extent relaxing these and other assumptions would affect the results. At an empirical level, the paper develops many testable predictions, and some of the underpinning assumptions (e.g., that the rate of return on soft skills is always less than that on hard skills) are also testable. Finally, the paper also identifies, but does not analyse, various public policy implications which would merit further work. For example, an important issue both for public policy and for universities is whether term-time working is a good or a bad thing. The present paper analyses this only from the viewpoint of an individual student, but its framework could be used to develop an analysis of the economy-wide social welfare implications; and it also points towards policy levers that could be activated (either at governmental level or by individual universities) either to encourage term-time working (e.g., making it part of the formal curriculum), or to discourage it. In this and other ways, the paper's framework could be used as a starting point for analysing the many important policy issues that stem from the expanding size and changing nature of the higher education sector in the UK and elsewhere.

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Appendix

1. Borrowing choices

Table 2 indicates that $B(i) = B_{max}$ in five of the seven cases, and $B(i) < B_{max}$ in Cases 5 and 7. Hence we need to identify in what circumstances individuals will prefer Cases 5 or 7 to any of the $B = B_{max}$ options. We begin with the choice between Cases 6 and 7. For Case 6 to be optimal requires both $\partial U(i)/\partial H_1(i) \succ 0$ in (4) and $\partial U(i)/\partial B(i) \succ 0$ in (6) at the optimal values of $C_1(i)^*$, $C_2(i)^*$, and $L(i)^*$ given in the relevant row of Table A2.[22] Substituting these optimal values into (4) and (6) allows the inequality conditions to be stated in terms of the size of the net financial transfer $T(i)$:

$$\frac{\partial U(i)}{\partial H_1(i)} \succ 0 \; if \; T(i) \prec \frac{A(i) - WH_2\left[\left(a(i) - f(i)b(i)\right)H_{\max} + f(i)b(i)\hat{H}\right]}{H_2\left(a(i) - f(i)b(i)\right)(D(i) + V(i))} - (WH_{\max} + B_{\max}) \equiv T(i)_1 \tag{A1}$$

$$\frac{\partial U(i)}{\partial B(i)} \succ 0 \; if \; T(i) \prec \frac{A(i) - WH_2\left[\left(a(i) - f(i)b(i)\right)H_{\max} + f(i)b(i)\hat{H}\right]}{f(i)R(D(i) + V(i))} - (WH_{\max} + B_{\max}) \equiv T(i)_6 \tag{A2}$$

$$A(i) = WH_2\left(a(i) + f(i)\right) - f(i)RB_{\max} \tag{A3}$$

Using (A1)–(A3) and the definition of $\tilde{R}(i)_b$ in (9) indicates that

$$T(i)_1 \geqslant or \prec T(i)_6 \; as \; R \geqslant or \prec \tilde{R}(i)_b \tag{A4}$$

Noting from Table 2 that the choice between Cases 6 and 7 is only feasible for Types 2 and 3, for both of whom $R \succ \tilde{R}(i)_b$, it follows that $T(i) < T(i)_6$ is the relevant condition for Case 6 to be preferred to Case 7. By implication Case 7 will be preferred if $T(i) > T(i)_6$. This can be checked using Table A1's specification of Case 7's optimal value of $B(i)^*$: manipulating this expression indicates that the requirement for it to be less than B_{max} (as required in Case 7) is that $T(i) > T(i)_6$. Thus the choice between Cases 6 and 7 turns on the value of the net transfer $T(i)$ relative to the critical value $T(i)_6$. This and subsequent results are summarized in Table A3.

Equivalent considerations govern the choice between Cases 4 and 5, which is feasible for Types 1 and 2 but not for Type 3. In both cases $H_1(i)^* = 0$, with $B(i) < B_{max}$ in Case 5 and $B(i) = B_{max}$ in Case 4. To derive the condition governing this choice we follow the same methodology as in (A1–A3): Case 4 requires $\partial U(i)/\partial B(i) \succ 0$ in (6) at the optimal consumption and leisure values (specified

[22] As explained earlier, $C_1(i)^*$, $C_2(i)^*$ and $L(i)^*$ are obtained by substituting into the budget constraints (2) the optimal values of $H_1(i)^*$, $L(i)^*$ and $S(i)^*$ obtained from (4)–(7). These optimal values incorporate whichever corner solution[s] is/are appropriate to the case being considered. Thus, for Case 6 they explicitly build in both the stated inequalities (and similarly for all the other cases discussed below.)

Table A1 Optimal solutions for studying time, employment, and borrowing

Case	$B(i)*$	$S(i)*$	$H_1(i)*$
1	$B_{\max}$	$Q_2[DQ_1 - V(Q_5 - RB_{\max})]$	$H_{\max}$
2	$B_{\max}$	$Q_3Q_{11}^{-1}[WH_2Q_7 + Q_9 + H_2Q_6Q_0]$	$afQ_3Q_{11}^{-1}[Q_4 - bWH_2\hat{H}$ $- H_2(D+V)(a/f - b)Q_0]$
3	$B_{\max}$	$Q_3[WH_2Q_8 + R(1+V)B_{\max} + (a/f)DH_2Q_0]$	$Q_3[fQ_4 - aH_2Q_0(D+V)]$
4	$B_{\max}$	$Q_2[WH_2(1+Q_8) + VRB_{\max}]$	0
5	$(fW\tilde{R}_a/R)Q_3[Q_4 + RB_{\max}$ $- (D+V)RT]$	$Q_3\{WH_2(Q_8 + (a/f) + 1] - VRT\}$	0
6	$B_{\max}$	$Q_2[DQ_1 - V(Q_5 + RB_{\max})]$	$H_{\max}$
7	$(fW\tilde{R}_a/R)Q_3[Q_1 + Q_5$ $- R(D+V)Q_{10}]$	$Q_3[(1+D)Q_1 - V(Q_5 + RQ_{10})]$	$H_{\max}$

Key (applies also to Table A2):

$Q_0 = (T + B_{\max})$; $Q_1 = WH_2(a/f)(1 - H_{\max})$; $Q_2 = [aWH_2(D+V)]^{-1}$; $Q_3 = [aWH_2(1+D+V)]^{-1}$;
$Q_4 = WH_2[1 + (a/f)] - RB_{\max}$;
$Q_5 = WH_2[1 + b(H_{\max} - \hat{H})]$; $Q_6 = (aD + fbV)[(a/f) - b]$; $Q_7 = (aD - fV)[(a/f) - b] - a(1+b)$;
$Q_8 = (a/f)D - (1+V)$;
$Q_9 = (bWH_2\hat{H} + RB_{\max})[a + V(a - fb)]$; $Q_{10} = (T + WH_{\max})$; $Q_{11} = (a - fb)$.

Notes: 1. The entry in each cell denotes the optimal (interior or corner) solution for the variable listed at the top of the relevant column in the circumstances summarized by the case number listed in the leftmost column; these cases are defined in Table 2. 2. Each optimal solution is that chosen by a specific individual *i*; for notational simplicity all the '*i*' terms are omitted from the cells and from the Key definitions.

in Table A2) which are applicable to Case 4, and some manipulation allows this inequality to be re-written as $T(i) < T(i)_4$, where $T(i)_4$ is specified in Table A3. Again proceeding as in the previous paragraph, it can be checked that if this inequality is reversed Case 5 is then preferred instead.

The pair-wise choices between Cases 4 and 5 and between Cases 6 and 7 thus yield equivalent results: in both cases optimal borrowing is at (below) B_{max} if the net financial transfer to student *i*, $T(i)$, (i.e., the parental transfer less tuition fees) is below (above) the relevant critical value. The latter is $T(i)_6$ when term-time working is at its maximum level, i.e., the choice between Cases 6 and 7, and $T(i)_4$ when there is zero term-time working, i.e., the choice between Cases 4 and 5. We now use this to derive a series of probabilistic influences on the optimal borrowing choice. First, an increase in the parental transfer (and thus in $T(i)$ relative to its critical value) unambiguously decreases the probability that student i's optimal borrowing is at B_{max}. Second, an increase in tuition fees (and thus a decrease in $T(i)$) unambiguously increases the probability that optimal borrowing is at B_{max} *provided* that the level of B_{max} is not itself positively related to the level of tuition fees. This proviso is by no means guaranteed, however (e.g., as already noted, the UK Student Loan Company normally increases B_{max} one-for-one with

Table A2 Optimal consumption and leisure choices

Case	$C_1(i)*$	$L(i)*$	$C_2(i)*$
1	$WH_{\max} + Q_0$	$fQ_2V(Q_1 + Q_5 - RB_{\max})$	$(DW\tilde{R}_a/V)L*$
2	$afWQ_3Q_{11}^{-1}\{Q_4 - bWH_2\hat{H} + H_2[(a/f) - b]Q_0\}$	$(VQ_{11}/aW)C_1*$	$D\tilde{R}_bC_1*$
3	$fWQ_3[Q_4 + (a/f)H_2Q_0]$	$(V/W)C_1*$	$D\tilde{R}_aC_1*$
4	Q_0	fVQ_2Q_4	$(DW\tilde{R}_a/V)L*$
5	$(fW\tilde{R}_a/R)Q_3(Q_4 + RQ_0)$	$(VR/W\tilde{R}_a)C_1*$	$(RD)C_1*$
6	$WH_{\max} + Q_0$	$fQ_2V(Q_1 + Q_5 - RB_{\max})$	$(DW\tilde{R}_a/V)L*$
7	$(fW\tilde{R}_a/R)Q_3(Q_1 + Q_5 + RQ_{10})$	$(VR/W\tilde{R}_a)C_1*$	$(RD)C_1*$

Key: see Table A1.
Notes: 1. The individual cell entries have the same interpretation as detailed in note (1) in Table A1. 2. All '*i*' terms have been omitted from individual cells as in Table A1. 3. Where a cell in the L^* or C_2^* column includes the term C_1^* or L^*, this refers to the term defined in the relevant column within the row concerned.

tuition fees increases). Third, the probability that the net transfer $T(i)$ is above its critical value (so that $B < B_{max}$) is decreasing (increasing) in parameters that increase (decrease) whichever critical value, $T(i)_4$ or $T(i)_6$, is relevant. From Table A3 both these critical values are decreasing in $D(i)$, $V(i)$, and $R(i)$, and increasing in $[a(i)/f(i)]$. In addition $T(i)_6$ (but not $T(i)_4$) is increasing in $b(i)$. Further, when $B(i)^* < B_{max}$ (Cases 5 and 7) Table A1 indicates that $B(i)^*$ is decreasing in $T(i)$, $V(i)$, $D(i)$ and $R(i)$ and increasing in $a(i)/f(i)$ and (when employment is at its maximum level, Case 7) is increasing in $b(i)$. For appropriate parameter combinations $B(i)^* < 0$ is possible (i.e., net saving, and a zero student loan). All the foregoing points are combined and summarized in Proposition 1 in Section 4.

2. Term-time employment choices

We first consider Cases 1–4, in all of which $B = B_{max}$; we then consider separately Case 5, where $B^* < B_{max}$. We analyse a series of pair-wise comparisons, beginning with the pair-wise choice between maximum (Case 1) and high (Case 2) employment. The requirements for Case 1 to be preferred to Case 2 are equivalent to those for Case 6 to be preferred to Case 7, i.e., both $\partial U(i)/\partial H_1(i) \succ 0$ in (4) and $\partial U(i)/\partial B(i) \succ 0$ in (6) at the optimal values of $C_1(i)^*$, $C_2(i)^*$ and $L(i)^*$ given in the relevant row of Table A2. Hence conditions (A1)–(A4), derived above for Case 6 to be preferred to Case 7, are also the requirements for Case 1 to be preferred to Case 2. However, whereas the choice between Cases 6 and 7 is only feasible for Types 2 and 3 individuals for whom $R \succ \tilde{R}(i)_b$, the choice between Cases 1 and 2 is only relevant for Type 1 individuals for whom $R \prec \tilde{R}(i)_b$. It follows that whereas $T(i) < T(i)_6$ is the relevant condition for Case 6 to be preferred to Case 7, the relevant condition for Case 1 to be preferred to Case 2 is instead $T(i) < T(i)_1$ (cf., (12)).

Table A3 Parameter value influences on pair-wise choices between outcomes

Choice between cases	Condition	Applicable to type[s]	Definitions
1 and 2	Case 1 $\geqslant$ or $<$ Case 2 when $T(i) \leqslant$ *or* $\succ T(i)_1$	1	$T(i)_1 \equiv \dfrac{A(i) - WH_2\{[a(i) - f(i)b(i)]H_{\max} + f(i)b(i)\hat{H}\}}{H_2[a(i) - f(i)b(i)][D(i) + V(i)]} - (WH_{\max} + B_{\max})$
2 and 3	Case 2 $\geqslant$ or $<$ Case 3 when T(i) $\leqslant$ or $>$ $\mathrm{T(i)}_2$	1	$\mathrm{T(i)}_2$ defined in (13b)
3 and 4	Case 3 $\geqslant$ or $<$ Case 4 when $T(i) \leqslant$ *or* $\succ T(i)_3$	1 and 2	$T(i)_3 \equiv \dfrac{A(i)}{aH_2[D(i) + V(i)]} - B_{\max}$
4 and 5	Case 4 $\geqslant$ or $<$ Case 5 when $T(i) \leqslant$ *or* $\succ T(i)_4$	1 and 2	$T(i)_4 \equiv \dfrac{A(i)}{f(i)R(i)[D(i) + V(i)]} - B_{\max}$
5 and 7	Case 5 $\geqslant$ or $<$ Case 7 when X(i) $\geqslant$ or <0	2	$X(i) = b(i)H_2\hat{H} - H_{\max}[R - \tilde{R}(i)_b]$
6 and 7	Case 6 $\geqslant$ or $<$ Case 7 when $T(i) \leqslant$ *or* $\succ T(i)_6$	2 and 3	$T(i)_6 \equiv \dfrac{A(i) - WH_2\{[a(i) - f(i)b(i)]H_{\max} + f(i)b(i)\hat{H}\}}{f(i)R(i)[D(i) + V(i)]} - (WH_{\max} + B_{\max})$

Key: $A(i) = WH_2[a(i) + f(i)] - f(i)R(i)B_{\max}$.
Note: the cases in the left hand column, and the individual types in column 3, correspond to those defined in Table 2.

The pair-wise choice between low (interior solution) and zero (corner solution) employment (Cases 3 and 4) uses an equivalent methodology. For zero employment to be preferred we require $\partial U(i)/\partial H_1 \prec 0$ in (8) at the optimal values $C_1(i)^*$ and $C_2(i)^*$ which apply to Case 4, as defined in Table A2. It can easily be checked that this is equivalent to $T(i) > T(i)_3$, where the latter is defined in Table A3; and that if instead $T(i) < T(i)_3$, then low employment is preferred to zero employment.

We now turn to the choice between low and high employment (Cases 2 and 3). Neither involves a corner solution for employment, so the methodology of previous pair-wise choices does not apply. Instead we define $U(i2)^*$ and $U(i3)^*$ as the maximum utility obtainable by choosing high (Case 2) and low (Case 3) employment respectively; and we also define $\Delta U_{2/3}(i) \equiv U(i2)^*$—$U(i3)^*$. Then i prefers high (low) employment if $\Delta U_{2/3}(i)$ is positive (negative). Substituting the values for $C_1(i)^*$, $C_2(i)^*$ and $L(i)^*$ defined in Table A2 for Cases 2 and 3 into the utility function (1), simplifying, taking antilogs and further simplifying gives the following:

$$\Lambda U_{2/3}(i) \geqslant \textit{ or } \prec 0 \textit{ if } T(i) \leqslant \textit{ or } \succ T(i)_2 \tag{A5}$$

$$T(i)_2 \equiv \frac{K(i)\Omega(i)}{H_2[1 - K(i)]} \tag{A6}$$

$$K(i) = \frac{1 - [1 - F(i)]^{P(i)}}{F(i)} \tag{A7}$$

$$\Omega(i) = WH_2(1 - \hat{H}) + [f(i)/a(i)][WH_2 - RB_{\max}] \tag{A8}$$

$$F(i) = \left[\frac{f(i)b(i)}{a(i)}\right] \tag{A9}$$

$$P(i) = [1 + D(i) + V(i)]^{-1} \tag{A10}$$

High (low) employment is preferred if $\Delta U_{2/3}(i)$ in (A5) is positive (negative). The probability that $\Delta U_{2/3}(i)$ is positive is evidently decreasing in $T(i)$ and increasing in $T(i)_2$. $T(i)_2$ in turn is increasing in $K(i)$ and in $\Omega(i)$. We analyse the impact of the parameters $D(i)$, $V(i)$, $[a(i)/f(i)]$, and $b(i)$ which affect $T(i)_2$ via $K(i)$ and/or $\Omega(i)$. We first note that earlier assumptions ensure that $0 < F(i) < 1$, whence $K(i)$ is increasing in $P(i)$. Since $D(i)$ and $V(i)$ affect $T(i)_2$ only via $P(i)$, which they affect negatively, it follows that $T(i)_2$ is decreasing in both $D(i)$ and $V(i)$. It is also straightforward to check that $T(i)_2$ is decreasing in R.

The parameter $[a(i)/f(i)]$ affects $T(i)_2$ via $\Omega(i)$, and also via $K(i)$. The latter effect operates via $F(i)$, and we therefore need to establish the impact of $F(i)$ on $K(i)$. We first note that $K(i) \rightarrow 0$ as $F(i) \rightarrow 0$ and $K(i) \rightarrow 1$ as $F(i) \rightarrow 1$. Differentiating (A7):

$$\frac{\partial K(i)}{\partial F(i)} \geqslant \textit{ or } \prec 0 \textit{ as } \Psi[F(i)] = 1 - F(i) + F(i)P(i) - [1 - F(i)]^{[1-P(i)]} \geqslant \textit{ or } \prec 0 \tag{A11}$$

Differentiating $\Psi[.]$:

$$\frac{\partial\Psi[F(i)]}{\partial F(i)} = [1 - P(i)]\left(\frac{1}{[1 - F(i)]^{P(i)}} - 1\right) \succ 0 \qquad \text{(A12)}$$

Using (A12) and noting that $\Psi[F(i)] \rightarrow 0$ as $F(i) \rightarrow 0$ and that $\Psi[F(i)] \rightarrow P(i)$ $[0 < P(i) < 1]$ as $F(i) \rightarrow 1$, it follows that $\Psi[F(i)] > 0$ for all admissible values of $F(i)$ and hence, from (A11) that $\partial K(i)/\partial F(i) \succ 0$ unambiguously. Noting that both $F(i)$ and $\Omega(i)$ are decreasing in $[a(i)/f(i)]$ we can now sign the impact of the latter upon $T(i)_2$ unambiguously:

$$\frac{dT(i)_2}{d\left(\frac{a(i)}{f(i)}\right)} = \left(\frac{\partial T(i)_2}{\partial K(i)}\right)\left(\frac{\partial K(i)}{\partial F(i)}\right)\left(\frac{\partial F(i)}{\partial\left(\frac{a(i)}{f(i)}\right)}\right) + \left(\frac{\partial T(i)_2}{\partial \Omega(i)}\right)\left(\frac{\partial \Omega(i)}{\partial\left(\frac{a(i)}{f(i)}\right)}\right) \prec 0 \qquad \text{(A13)}$$

Finally, noting that $b(i)$ affects $T(i)_2$ only via $F(i)$ and that $\partial F(i)/\partial b(i) \succ 0$, it follows from the foregoing analysis that $T(i)_2$ is increasing in $b(i)$. Summarizing the analysis of (A11)–(A13), $T(i)_2$ is decreasing in $D(i)$, $V(i)$, R, and $a(i)/f(i)$, and is increasing in $b(i)$. Feeding these results into (A5) in turn generates results concerning the sign of $\Delta U_{2/3}$.

The discussion in this section has so far related to adjacent employment choices (e.g., zero-low) rather than non-adjacent choices (e.g., zero-high). To derive global results we note from Table A3 and (A6)–(A10) that $T(i)_1 < T(i)_3$ unambiguously but that $T(i)_2$ can be larger or smaller than either or both. Using this and earlier results allows us to identify three global patterns of choice for borrowing-constrained individuals:

Pattern A $T(i)_2 < T(i)_1 < T(i)_3$: zero employment is chosen if $T(i) > T(i)_3$; low employment if $T(i)_1 < T(i) < T(i)_3$; either low or maximum employment if $T(i)_2 < T(i) < T(i)_1$; maximum employment if $T(i) < T(i)_2$.
Pattern B $T(i)_1 < T(i)_2 < T(i)_3$: zero employment is chosen if $T(i) > T(i)_3$; low employment if $T(i)_2 < T(i) < T(i)_3$; high employment if $T(i)_1 < T(i) < T(i)_2$; maximum employment if $T(i) < T(i)_1$.
Pattern C $T(i)_1 < T(i)_3 < T(i)_2$: zero employment is chosen if $T(i) > T(i)_2$; either zero or high employment if $T(i)_3 < T(i) < T(i)_2$; high employment if $T(i)_1 < T(i) < T(i)_3$; maximum employment if $T(i) < T(i)_1$.

Pattern B identifies a straightforward inverse relationship between the net financial transfer $T(i)$ and the optimal employment category. Patterns A and C are similar, but in each case one potential employment category is infeasible. All three patterns replicate at a global level the relationships that we have identified from (A5)–(A13) between adjacent employment levels. In particular, the employment category that is optimal for any individual student is inversely related to her net transfer $T(i)$; and the probability that, for a given transfer, she is located in or below any given employment category is positively related to the relevant $T(i)_N$ value ($N = 1,2,3$) which determines its upper boundary point. The $T(i)_N$ values are themselves determined by a series of independent parameters whose impact can be obtained by differentiation of the $T(i)_N$. To sign the impact of $[a(i)/f(i)]$ we assume $(WH_2\text{-}RB_{max})>0$, i.e., student loan repayments are never large enough to exhaust period 2 labour income even if the latter is not increased at all by human capital obtained whilst at university. In differentiating $T(i)_2$ in (A6) we make use of results derived in (A11)–(A13). All the points listed so far in this paragraph relate to discrete employment level categories, but two of these—the low and high employment categories—encompass a range of possible employment levels, and contingent on either category being optimal the precise employment level which is optimal within the relevant range likewise depends on a series of independent parameters whose effects can be determined by differentiating the appropriate expressions for $H_1(i)^*$ in Table (A1). Combining all these points together gives the results summarized in Proposition 2 in Section 4.