“Sustainable Agricultural Production, Income and Eco-Labelling: What Can Be Learned from a Neo-Ricardian Approach?”

Kari Heerman (US Trade Representative’s Office)  
Kari.E.Heerman@ustr.eop.gov

Ian Sheldon (Ohio State University)  
sheldon.1@osu.edu

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Abstract

In this paper we explore modelling trade in sustainably produced agricultural products with eco-labelling using a modern neo-Ricardian framework. Based on this approach, expressions are derived for the share of products importers purchase from specific exporters for low-cost unsustainable and high-cost sustainable production technologies, assuming consumers have non-homothetic preferences. The consumer and sustainability gains from eco-labelling are also analyzed, along with a discussion and comparison of the effects of mutual recognition versus harmonization of countries’ eco-labelling regimes.

Keywords: trade, trade costs, eco-labelling, income trade liberalization

JEL Codes: F11, F14, F15, Q17, Q56
Introduction

Over recent decades, there has been a significant increase in demand for provision of a range of attributes in food products (Sexton, 2013). These attributes, which are typically interpreted as representing higher-quality products, reflect a spectrum of consumer preferences for food product characteristics that, *inter alia*, cover: food safety (pesticide residues), ethical production concerns (animal welfare), the right-to-know about (genetic modification), location of (geographic indicators), and sustainability (environmental impact) of food production methods. With respect to the latter attribute, an OECD survey indicated that there has been very rapid growth in total eco-labelling since the 1970s, and a significant portion of this growth has been in eco-labels relating to food and agricultural products, as well as textile and forest products, the labels covering characteristics such as waste, energy, natural resources, climate change, biodiversity, and chemical control (Gruére, 2013). In the context of the current paper, while water footprint labelling of agricultural and food production is not currently the norm, there is some empirical evidence for consumers in high-income countries being willing to pay a price premium for water-saving food production methods (Grebitus, Steiner, and Veeman, 2016; Krovetz, Taylor, and Villas-Boas, 2017; Pomarici *et al.*, 2018), as compared to consumers in low-income countries (Okpiaifo *et al.*, 2020).

Considerable progress has been made by both mainstream and agricultural economists in incorporating vertical product differentiation into trade models, along with associated empirical work. Articles in this area include, *inter alia*: Amiti and Khandelwal (2013), Baldwin and Harrigan (2011), Crinò and Epifani (2012), Curzi and Olper (2012), Curzi, Raimondi, and Olper (2015), Eum, Sheldon, and Thompson (2021), Gaigné and Larue (2016), Hallack and Schott

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1 The latter study analyzed preferences for sustainable rice practices in Nigeria, the results indicating consumers placed very little weight on efficient water use as compared to food safety.
(2011), Kugler and Verhoogen (2012), Olper, Curzi, and Pacca (2014), Sutton (2007), and Verhoogen (2008). However, much of this literature assumes away the real possibility that vertical product differentiation often suffers from a key information failure: consumers are unable to verify claimed environmental or other benefits of how a product was produced both before and after consumption, such claims being termed as *credence* attributes of the product (Darby and Karni, 1973). Eco-labelling in conjunction with a mechanism for certification of environmental and sustainability claims is regarded as key in resolving any information asymmetry due to products having credence attributes (Roe and Sheldon, 2007; Roe and Sheldon, 2009; Sheldon, 2017). In addition, in an international setting, the choice by trading partners between mutual recognition and harmonization of their eco-labelling standards becomes an important policy issue (Sheldon and Roe, 2009; Swinnen, 2016; 2017).

Many models of the impact of trade on environmental quality typically assume that under certain conditions, negative environmental externalities will be generated with increased international integration, particularly between developed (the North) and developing countries (the South) (Copeland and Taylor, 1994; 1995; 2004), with subsequent empirical analysis focusing on the impact of trade on the environment, (Antweiler, Copeland, and Taylor, 2001; Frankel and Rose, 2005). Other analysis, drawing on the heterogeneous-firms approach of Melitz (2003), shows that if productive firms are also more environmentally efficient, trade liberalization may generate environmental benefits (Kreickemeier and Richter, 2014; Forslid, Okubo and Ulltweit-Moe, 2015; Cherniwchan, Copeland, and Taylor, 2017), a result that has found some empirical support (Cui, Lapan and Moschini, 2016; Holladay, 2016; Cherniwchan, 2017).

Trade in products with credence attributes may also generate environmental/sustainability gains. If production generates such benefits, this should be explicitly incorporated into trade
analysis, and the associated evaluation of trade liberalization and any international harmonization of eco-labelling standards. In this paper a neo-Ricardian-type trade model is developed, drawing on the seminal analysis of Eaton and Kortum (2002; 2012), as well as, *inter alia*, Alvarez and Lucas (2007), Fieler (2011), Costinot, Donaldson, and Komunjer (2012), and Levchenko and Zhang (2014). This class of model has already been applied to evaluating trade and trade liberalization in the agricultural sector (Reimer and Li, 2010; Costinot and Donaldson, 2012; Heerman, Arita and Gopinath, 2015; Heerman and Sheldon, 2018; Heerman, 2020; Sotelo, 2020), as well as analysis of virtual water use and trade (Reimer, 2014), and evaluation of the impact of climate change on comparative advantage in the agricultural sector (Costinot, Donaldson and Smith, 2016; Gouel and Laborde, 2017).

Eaton and Kortum (2002) assume that comparative advantage is a function of a random productivity variable that is independently distributed across products in the sector. Specifically, no two countries are more likely to compete against each other exporting the same products than any other country. Extensions of Eaton and Kortum (2002) to multisector analysis by, *inter alia*, Burstein and Vogel (2010), Chor (2010), Costinot *et al.* (2012), Shikher (2011, 2012), Caliendo and Parro (2015), Tombe (2015), and Kerr (2017) implicitly recognize the limitation of this assumption, allowing average productivity, and in some cases the dispersion of productivity to vary across sectors, generating non-random patterns of trade specialization across sectors and sub-sectors.

However, these models still maintain the assumption of random heterogeneity within each sector or sub-sector. In addition, there is a practical limitation to a multi sub-sector approach within agriculture (Heerman, 2020): the researcher must be able to define sub-sectors of like products such that specialization of a country within that sub-sector can be assumed to be randomly
determined *ex ante*. For example, Reimer and Li (2010) focused on trade in crop agriculture, a well-defined sub-sector, but this still ignores the fact that agricultural product-specific trade policies may be enough to distort any underlying forces of comparative advantage for crops that are substitutes in production.

Therefore, one departure in this paper is the introduction of systematic heterogeneity into the agricultural sector. Specifically, the likelihood a country has a comparative advantage in a set of products depends not only on a randomly drawn technological productivity-augmenting parameter, but also a set of country and product-specific characteristics including land and climate. Fally and Sayre (2018) also allow heterogeneous natural resource productivity in commodities to influence comparative advantage. In contrast to the current approach, which explicitly links a product’s productivity in a common factor, *viz.*, a composite input consisting of land, labor, and water, to country and product characteristics, in their model each commodity is produced with a specific natural resource.

A second departure in this paper is to allow for non-homothetic preferences in demand. While modern trade models differ in their specifications of the supply-side of an economy, they are all based on a common demand structure (Costinot and Rodríguez-Clare, 2014), i.e., a constant elasticity of substitution (CES) utility function. Importantly, CES preferences imply that income elasticities do not vary across products and equal one, i.e., homotheticity is assumed. In the case of consumer demand for say, agricultural products that embody water-saving production methods, this may be an unreasonable assumption, especially if income per capita has the potential to affect trade.

Of course, this idea has a long pedigree, going back to Linder (1961) who argued firms located in high per capita income countries that are spatially close, will have a comparative advantage in
producing high-quality products, with trade volumes being larger across countries with similar income levels. A small number of existing trade models do account for non-homothetic preferences, including Flam and Helpman (1987), Lewis et al. (2020), Markusen (1986), Matsuyama (2000), and Stokey (1991). In this paper, Fieeler’s (2011) constant relative income elasticity approach to incorporating non-homothetic preferences is adopted, allowing for average per capita income to have an impact on trade. However, as noted later in this paper, this approach is somewhat restrictive, the ratio of income elasticities for two products being constant across income levels.

In what follows, sustainable water use in agricultural production and its relationship to trade is discussed in section 1, followed by description of a simple illustrative model in section 2, highlighting the way in which eco-friendly products and labelling can be incorporated into a trade model. A more detailed model is derived in section 3 along with a solution methodology. Analysis of the consumer and environmental gains from trade and eco-labelling, and the effects of alternative policies towards eco-labelling are presented in section 4. Finally in section 5, the paper is summarized along with conclusions about the future direction of this type of research.

1. Trade and Sustainable Water Use in Agricultural Production

In the past half century, competition for scarce water resources has intensified with population growth and changes in diet (Molden, 2007). Although populations in many countries and regions perceive water as an abundant natural resource, fresh water is limited, over-consumption having the potential to increase water stress, thereby posing a threat to humans, ecosystems, and biodiversity (Rost et al., 2008; Vorösmarty et al., 2010; Weinzettel and Pfister, 2019). In a recent article, Balasubramanya and Stifel (2020) also emphasize the need to understand the connection...
between water, agriculture, and poverty, given increasing demand for scarce water resources and increased climatic variability.

Scientists focused on planetary boundaries, have defined a “safe operating space” for the world based on biophysical processes that affect the stability of the planet, the processes including global freshwater use, along with others such as climate change and the rate of loss of biodiversity (Steffen, et al., 2015). The argument is that, while the planet’s environment has been quite stable for over 10,000 years during the so-called Holocene, human activities have been the main cause of environmental change since the industrial revolution, an era termed the Anthropocene (Rockström et al., 2009). The risk is that human activity will push the planet beyond the stable state of the Holocene, hence the need for planetary boundaries. In the case of freshwater, Steffen et al. (2015) propose boundaries on both the amount of annual global water consumption (4,000 km$^3$yr$^{-1}$), as well as water withdrawal from specific river basins based on average monthly river flow.$^{2,3}$

The analysis presented in this paper focuses on the interaction between international agricultural trade and water scarcity in a world of where there is incomplete information on the part of consumers about the environmental impact of agricultural products. In this context, it is important to understand the major links between agricultural production and freshwater consumption, and how those links have been analyzed. The literature breaks down into three main areas of analysis (Liu et al., 2014). First, studies focusing on the “water footprint” of agricultural production for both domestic consumption and export (Mekonnen and Hoekstra, 2011; Hoekstra

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$^2$ Water consumed is the amount of water removed for use and not returned to its source, whereas water withdrawal is defined as freshwater taken from ground or surface water sources, either permanently or temporarily, and conveyed to a place of use.

$^3$ As Irwin, Gopkalakrishnan, and Randall (2016) note in their review of the literature on the economics of weak vs. strong sustainability, the approach of setting planetary boundaries is based on advocating a “safety-first” approach, without any assessment of the trade-offs.
and Mekonnen, 2012), and the related concept of virtual water trade (Allan, 1997; Ansink, 2010; Reimer, 2012; 2014; Delin et al., 2012; Konar et al., 2013). Second, a focus on water-use in agriculture, and the set of factors that could either exacerbate or mitigate future availability for production (Gerten et al., 2011; Rosegrant and Cai, 2002), the emphasis being on the use of irrigation by agriculture and its impact on river basins (Vörösmarty et al., 2010), and groundwater depletion (Aeschbach-Hertig and Gleeson, 2012; Richey et al., 2015). Third, studies that evaluate the impact of water scarcity due to irrigation stress, on global agricultural trade and economic welfare (Liu et al., 2014; Reimer, 2014).

Defining the water footprint concept is an important starting point for thinking about international agricultural trade and water scarcity. The concept, originally due to Hoekstra and Chapagain (2003), is defined as the water volume per unit of product, which is equal to the sum of: the green water footprint (rainwater consumed), the blue water footprint (volume of surface and groundwater consumed), and the grey water footprint (volume of freshwater required to assimilate pollutant loads) (Mekonnen and Hoekstra, 2011). The main source of green water is rain falling on the earth’s surface (110,000 km$^3$), with 56% being evapotranspired by various landscape uses such as forestry, and 4.5% being evapotranspired by rainfed agriculture.\textsuperscript{4} Globally, about 39% of rain (43,500 km$^3$) contributes to blue water sources, with blue water withdrawals accounting for 9% of blue water sources (3,800 km$^3$), with 70% of withdrawals going to irrigation (2,700 km$^3$).\textsuperscript{5} Total evapotranspiration by irrigated agriculture is about 2% of rain (2,200 km$^3$), of which 30% is directly from green water, and the remainder from blue water (Molden, 1997).

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\textsuperscript{4} Evapotranspiration is the sum of evaporation from the land surface plus transpiration from plants.

\textsuperscript{5} Water withdrawal is defined as defined as freshwater taken from ground or surface water sources, either permanently or temporarily, and conveyed to a place of use.
Over the period 1996-2005, the annual global water footprint of crop production was 7,404 Gm³, broken down as 78% green, 12% blue, and 10% grey respectively (Mekonnen and Hoekstra, 2011), with wheat (1087 Gm³), rice (992 Gm³), and corn (770 Gm³) having the largest annual footprints. Wheat (204 Gm³) and rice (202 Gm³) had the largest annual blue water footprint, accounting for 45% of the global blue water footprint, and wheat (123 Gm³), corn (122 Gm³) and rice (111 Gm³) had the largest annual grey water footprint due to nitrogen fertilizer use, accounting for 56% of the global grey water footprint (Mekonnen and Hoekstra, 2011). In terms of the water footprint of crop production at the country level, India (1047 Gm³, 231.4 Gm³), China (967 Gm³, 118.9 Gm³), and the United States (826 Gm³, 95.9 Gm³) had the largest annual total and annual blue water consumption, respectively (Mekonnen and Hoekstra, 2011).

Beyond looking at the blue water footprint of agricultural production, it is critical to evaluate the extent to which water security is at risk with respect to rivers, the chief source of renewable water supply for humans and ecosystems. Vörösmarty et al. (2010) report that 80% of the world’s population live in areas where either water security or threat to biodiversity exceeds the 75th percentile, with regions of intensive agriculture and dense settlement showing a high threat of incident, specifically the United States, Europe, and large parts of central Asia, the Middle East, the Indian sub-continent, and eastern China. There are many factors contributing to both water security and biodiversity threat, with cropland being the dominant stressor, nutrient and pesticide loads being key pollution sources.

In addition, given that 90% of water consumption is for irrigation, and about 40% of irrigation water is derived from groundwater, water security also depends on the extent of groundwater depletion (Aeschbach-Hertig and Gleeson, 2012). Groundwater depletion occurs when water output from an aquifer exceeds input, driven by both hydrological and economic factors. Rates of
groundwater extraction are mostly linked to irrigated agriculture, and the fact that it is a common pool resource, subject to the “tragedy of the commons” (Aeschbach-Hertig and Gleeson, 2012). Groundwater depletion not only lowers water tables, reducing discharge to streams and wetlands affecting ecosystems, but also results in land subsidence, and induced groundwater flow leading to salinization from saltwater intrusion, and spread of other pollutants. The empirical evidence suggest that global groundwater depletion has accelerated since the mid-19th century, contributing to the rate of sea-level rise, the rate of depletion being projected to accelerate (Wada, 2010; 2012). The largest rates of groundwater depletion are currently in northern India, Bangladesh, and parts of Pakistan and Nepal, covering the Indo-Gangetic Plain. India currently pumps twice as much groundwater as China and the United States, where the North China Plain, the High Plains, and the California Central Valley all represent over-exploited aquifers (Aeschbach-Hertig and Gleeson, 2012).

Given the water footprint of agriculture, how does this relate to international trade? Allan (1997) coined the term “virtual water” to describe trade in the import of products with a large water footprint. In their analysis of agriculture’s water footprint, Hoekstra and Mekonnen (2012) also calculated international virtual water flows over the period 1996-2005. Again, this is dominated by trade in agricultural products which accounted for annual virtual water trade of 1,597 Gm³, accounting for 91% of the total, the top exporters being the United States, Pakistan, India, Australia, Uzbekistan, China, and Turkey being the largest virtual blue water exporters, accounting for 49% of the total (Hoekstra and Mekonnen, 2012).

While the virtual water concept has been open to much debate (Allan, 2003; Ansink, 2010; Merrett, 2003), Reimer (2012) shows clearly that it is no more than a restatement of Vanek’s (1968) extension of the Hecksher-Ohlin model where the analysis operates in terms of the factor
content of trade. The implication is that trade in water-intensive products could alleviate water scarcity in say the Middle East. There is some empirical support for this argument: for example, using a gravity model based on a Ricardian-type structure, Reimer (2014) finds that trade could help countries suffering from an irrigation shock, while Liu et al. (2014) using an extended version of the GTAP computable general equilibrium model, find that irrigation shortfalls would increase agricultural trade and change its geographical pattern.

However, as pointed out by Weinzettel and Pfister (2019), the role of international agricultural trade in mitigating water scarcity is empirically ambiguous, their analysis showing that arid regions such as the Middle East, Mexico, Portugal, and Spain do benefit from imports of water-intensive products, but their exports also embody their own scarce domestic water. In addition, their empirical results show that developed countries also tend to be importers of scarce water-intensive products from developing countries, even though they typically have no water scarcity problems of their own.

This latter result is perhaps not surprising given that water typically has a zero price, thereby ignoring both its scarcity value and any externalities generated in its use for agricultural production (Hoekstra and Mekonnen, 2012). In this context, the jumping off point of the current paper is as follows: suppose water used in agricultural production is not formally priced, but farmers are willing to incur the cost of applying environmental services to conserving water, but the latter is not observed by consumers willing to pay for those services through higher product prices. Specifically, what are the potential benefits of trade in and certified eco-labelling of products that are environmentally friendly in terms of scarce water resources?
2. An Illustrative Model

The basic story of the current paper is as follows: opening to trade, offers consumers access to products produced using two types of production process by each of their trading partners: the least cost unsustainable \((U)\) and high cost sustainable \((S)\) technologies. Eco-labels solving the credence attribute problem, allow consumers that are willing to pay a higher price for sustainable products to identify them and thus allow producers to be compensated for higher costs of production. Eco-labels increase the net sustainability gains from trade by expanding the market share of products produced with a more environmentally sustainable technology.

To illustrate, a simple two-country model can be described. Following Dornbusch, Fischer, and Samuelson (1977), the world is comprised of two countries, \(i = 1,2\), with many representative buyers and a continuum of agricultural and products, \(j \in [0,1]\). Each country has a unique technology for producing each product in the continuum. All product markets are perfectly competitive. Unit costs of production for each country and product, \(a_i(j)c_i(j)\), are plotted in figure 1, where \(a_i(j)\) and \(c_i(j)\) are the unit input requirement and input cost for product \(j\) in country \(i\). Products are organized in order of increasing unit costs for Country 1 producers and decreasing unit costs for Country 2 producers. In autarky, both countries produce every agricultural product in the continuum that has a less-than-infinite unit cost of production, \(a_i(j)c_i(j) < \infty, i = 1,2, \) and \(j \in [0,1]\).

After opening to trade, consumers purchase each product from the country that has the lowest unit cost of production. Before trade, Country 1 consumers purchase products from 0 to \(j_1\), and Country 2 consumers purchase products from 1 to \(j_2\). After trade, consumers purchase all products along the continuum. Country 1 specializes in products 0 to \(\hat{j}\), while Country 2 specializes in
products from $\tilde{j}$ to 1, i.e., the so-called “chain of comparative advantage” is broken at that product $\tilde{j}$ in the continuum where the unit costs of production are the same across countries, $a_1(\tilde{j})c_1(\tilde{j}) = a_2(\tilde{j})c_2(\tilde{j})$.\(^6\)\(^7\) Advances in technology are easily captured in this framework: suppose country 1 is subject to a uniform reduction in its unit input requirement across the continuum, $d \log a_1(j) < 0, j \in [0, 1]$, its unit cost schedule rotating down, Country 1 now specializing in producing more products in the continuum, $\tilde{j}$ to $\tilde{j}'$.

In the context of this paper, what matters from the standpoint of sustainability is that the products in which a country has the lowest unit production costs require fewer inputs per unit of output. Therefore, if it is assumed that fewer inputs imply a smaller environmental impact, there is an environmental gain $C_1$ from trade when products $\tilde{j}$ to $j_i$ in the continuum are no longer produced by Country 1, or $C_1'$ after technological change has occurred in Country 1. Of course, these gains could be offset by the environmental impact of importing the relevant products from Country 2, but this potential source of environmental loss is abstracted from in the current paper.

The sustainability benefits from trade can be further emphasized by assuming that rather than moving from autarky to free trade, each economy imposes a continuum of distorting trade barriers such as tariffs. Introducing an initial set of *ad valorem* tariffs $t_i(j)$ into the simple model, has the effect of rotating up each country’s unit cost function – see figure 2. In the tariff-ridden equilibrium, Country 1 produces products 0 to $\tilde{j}_1$, exporting in the range 0 to $\tilde{j}_2$, while Country 2

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\(^6\) As noted by Eaton and Kortum (2012), which country produces $\tilde{j}$ is irrelevant as it is an infinitesimal fraction of the total number of products in the continuum.

\(^7\) In Dornbusch, Fischer and Samuelson (1977), the equilibrium is expressed as, $a_2(\tilde{j})/a_1(\tilde{j}) = c_1(\tilde{j})/c_2(\tilde{j})$, relative input costs being relative wages. With a continuum of products, the ratio of unit labor requirements is a smooth downward-sloping function $A(\tilde{j})$ characterizing the chain of comparative advantage. Equilibrium is where $A(\tilde{j})$ intersects a continuous upward-sloping relative wage function $\omega$, the larger the share of products produced by Country 1 driving up the relative wage.
produces products in the range \(1, \tilde{j}_2\), exporting in the range \(1, \tilde{j}_1\). In other words, with tariffs in place, there is a set of non-traded products in the range \(\tilde{j}_1, \tilde{j}_2\) where it is more efficient for each country to produce in order to meet its domestic consumption requirements, but which also results in a loss of environmental benefits \(C'_i\) and \(C'_j\) respectively.\(^8\)

Extending the analysis, suppose producers have access to a second set of more sustainable \((S)\) technologies. Unit costs of production for these products are higher because the inputs required are more costly. One subset of consumers is always willing to pay a higher price for \(S\) products, while another subset will purchase \(S\) products, if the difference in price between the \(S\) product and the product produced with the less sustainable technology \(U\) is less than \(w\). In this case there is an environmental gain from trade that is a function of the share of consumers that always purchase sustainably produced products, \(\chi^S\), and those that purchase some sustainably produced products \(\chi^B\), and \(w, f(\chi^S, \chi^B, w)\).

In terms of figure 3, after opening to trade, Country 1 will, in addition to producing products from 0 to \(\tilde{j}^U\) with \(U\) technology, specialize in products from \(\tilde{j}^U\) to \(\tilde{j}^S\), that are produced with \(S\) technology. If \(S\) products are credence products, certified eco-labels allow consumers who are willing to pay a higher price for such products to identify them and allow producers to obtain compensation for using the \(S\) technology. Thus, eco-labelling provides Country 1 with the ability to capitalize on a comparative advantage in \(S\) products from \(\tilde{j}^U\) to \(\tilde{j}^S\).

In the very simple two-country world depicted in this section, trade gives consumers access to the lowest cost and most sustainable technologies in each of their trading partners. The role of

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\(^8\) Dornbusch, Fischer and Samelson (1977) originally introduced the idea of non-traded products in the presence of iceberg transport costs. Environmental standards have been shown to have a similar effect in a Ricardian model (Sheldon, 2012).
an eco-label in this model is to allow consumers to exercise their preference for the $S$ technology over the $U$ technology and allow producers to be compensated with a higher price. This story can be easily extended to a many-country world using the probabilistic approach originally developed in Eaton and Kortum (2002). The key is to model unit costs for each product in each country as a random variable. Then, rather than conceptually organizing products on the continuum in order of unit costs, they can be organized according to any other classification system that is convenient, e.g., by something like an HS code with an infinite number of digits.

3. Full Model

Consumption

There are $I$ countries engaged in international trade of agricultural products, importers indexed by $n$ and exporters by $i$. In each country, agricultural products are divided into two types based on production technology, $k \in \{U, S\}$, where $U$ and $S$ denote unsustainable and sustainable technologies, respectively. Within each technology type, there is a continuum of agricultural products indexed by $j^k \in [0,1]$. All consumers choose quantities of agricultural products $j^k, \{q_i(j^k)\}_{j^k \in [0,1]}$ to maximize the same utility function:

$$
\frac{\sigma^U}{\sigma^U - 1} \left( \int_0^1 q_i^U (j^U) \frac{\sigma^U - 1}{\sigma^U} dj^U \right) + \frac{\sigma^S}{\sigma^S - 1} \left( \frac{1}{\omega_i^S} \int_0^1 q_i^S (j^S) \frac{\sigma^S - 1}{\sigma^S} dj^S \right),
$$

where $\omega_i > 0$ is a weight representing the value, consumers place on sustainable production methods, and $\sigma^k > 1$ for $k \in \{U, S\}$.\(^9\) Importantly, sustainable agricultural products are assumed to have credence characteristics where expert sellers know more about the type of product the

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\(^9\)Unlike, for example, Copeland and Taylor (1994), the possibility of sub-optimal use of water resources in agricultural production entering the utility function as a local public bad is ignored.
consumer needs than consumers themselves. Specifically, an expert seller can identify the product quality fitting a customer’s need best by performing a “diagnosis”. The expert seller then either provides the right “treatment” and charges for it, or they exploit the informational asymmetry by defrauding the consumer (Dulleck and Kerschbamer, 2006). Essentially, there are two informational asymmetries here: the “diagnosis” – reducing the water footprint of agricultural production matters for sustainability; and the “treatment” – the agricultural product sold as sustainable does have a lower water footprint. Here the “diagnosis” is assumed correct, but there is potential for consumers to be defrauded over “treatment”, i.e., there is a risk they are sold an unsustainable product at a high price. To resolve the latter problem, it is assumed that there is a credible certification and eco-labelling mechanism in place.

In the typical neo-Ricardian trade model, the parameter $\sigma^k$ is defined as the elasticity of substitution, but here it also relates to the income elasticity of demand. From the consumer maximization problem, total expenditure on S products relative to U products is:

$$\frac{X^S_i}{X^U_i} = \lambda^S_{\sigma^U - \sigma^S} \left( \frac{\omega_i P^S_{i(1-\sigma^S)}}{P^U_{i(1-\sigma^U)}} \right),$$

where $P^k_i$ is the CES price index of products $k \in \{U, S\}$, and $\lambda_i > 0$ is the Lagrange multiplier, which is strictly decreasing in consumer income from composite input ownership (see Appendix A). In terms of substitution, relative expenditure on sustainable products increases with the weight $\omega_i$ placed on them by consumers and decreases in their price $P^S_i$. The impact of changes in income is a function of the term $\lambda^S_{\sigma^U - \sigma^S}$: suppose $\sigma^S > \sigma^U$, relative expenditure on sustainable products is decreasing in $\lambda_i$, and therefore increasing in consumer income. Note that the income elasticity of
demand for product \( k \in \{U,S\} \) is
\[
\nu / X_t^k \, dX_t^k / d\nu = (-\sigma^k \nu d\lambda_i / d\nu),
\]
where \( \nu \) is consumer income derived from ownership of a composite input.

Therefore, following Fieler (2011), at all levels of income and prices, the income elasticity of demand for the sustainable product relative to that for the unsustainable product is given by \( \sigma^S / \sigma^U \), i.e., consumers with higher incomes, concentrate their expenditure on sustainable products. An important \textit{caveat} should be noted here: as consumer incomes increase, their relative consumption of sustainable products increases, but their consumption of both types of product increases in absolute terms, i.e., the way non-homotheticity is introduced does not allow for substitution over time from unsustainable to sustainable products.

\textit{Production}

Producers have access to two technologies for each product: an unsustainable \( U \) technology and a sustainable \( S \) technology, where the technology to produce \( q_t^k(j^k), \, k = U, S \), uses a composite input \( \Upsilon_i \), which consists of a combination of land, labor and water:

\[
q_t^U(j^U) = z_i(j^U) \left[ \left( a_i(j^U) \Upsilon_i \right)^{\alpha} \right] \]

\[
q_t^S(j) = z_i(j^S) \left[ \left( a_i(j^S) \Upsilon_i \right)^{\alpha} H_i^{1-\alpha} \right],
\]

where \( z_i(j^k) \) is a productivity-enhancing random variable specific to product \( j^k \) in country \( i \), and \( a_i(j^k) \) is country \( i \), product \( j^k \)-specific productivity of the composite input \( \Upsilon_i \). The \( S \) technology also requires employment of environmental services \( H_i \), focused on reducing its blue and grey water footprint, i.e., drought-resistant production technology, less-intensive irrigation, and control of nutrient runoff. The relationship between the output of each crop and its total water footprint
$W_i^k(j)$ is denoted by $W_i^k(j) = \Lambda^k q_i^k(j)$, where $\Lambda^U > \Lambda^S$, i.e., sustainable production generates a lower water footprint.\textsuperscript{10}

Following Eaton and Kortum (2002), it is assumed that $z_i(j^k)$ is distributed independently following a Type II extreme value or Fréchet distribution:

$$F^k_i(z) = \exp\left\{-T_i z^{-\theta^k}\right\}$$

where $T_i > 0$, and $\theta^k > 1$. The parameter $T_i$ describes average agricultural productivity in country $i$, with higher values of $T_i$ implying that a country is more productive on average. This parameter can be thought of as capturing a country’s absolute advantage, and because it does not depend on $k$, it implies that a country is efficient at utilizing both unsustainable and sustainable technologies.

The parameter $\theta^k$ describes the dispersion of agricultural productivity, which is common to all countries, but may differ across the two technology types, i.e., $\theta^k$ serves two functions in the trade model. First, a smaller value of $\theta^k$ results in greater dispersion of agricultural productivity and hence greater price dispersion, which in turn results in a greater volume of trade, i.e., trade will be more intense in products where the value of $\theta^k$ is small, i.e., comparative advantage exerts a strong force. Second, variability in agricultural productivity across countries, also affects comparative advantage across types of product.

Finally, the parameter $a_i(j^k)$ reflects the suitability of exporter $i$’s natural environment for production of product $j^k$, and it is assumed to be a continuous parametric density that is a deterministic function of exporter $i$’s agroecological characteristics and product $j^k$’s agroecological production requirements (Heerman, Arita and Gopinath, 2015; Heerman, 2020). Incorporating

\textsuperscript{10} This is clearly a gross simplification – see Mekonnen and Hoekstra (2011) for a technical discussion of how to characterize the water footprint of any specific crop.
this parameter ensures that countries with similar agroecological characteristics such as land, access to water, and climate will systematically specialize in the same agricultural products and therefore compete head-to-head in the international market (Heerman and Sheldon, 2018). This approach has similarities to the literature that has explicitly incorporated land heterogeneity into trade models, e.g., Costinot and Donaldson (2012; 2016), Costinot, Donaldson and Smith (2016), Gouel and Laborde (2018), Gouel (2020), Fally and Sayre (2019), and Sotelo (2020).

Perfectly competitive producers set domestic prices equal to unit costs of production (see Appendix A). Producers in country $i$ can export to market $n$ but face an iceberg transport cost $\tau_{ni} > 1(\tau_{nn} = 1)$ to do so, and triangle inequality holds, i.e., for any three countries, $i$, $l$, and $n$, $\tau_{ni} \leq \tau_{nl} \tau_{li}$. Producers must also pay an additional cost $\zeta_{ni}$ above their domestic eco-labelling costs to meet country $n$ eco-labelling requirements. In the absence of a credible method for certification, $\zeta_{ni}$ will be infinitely high. Of course, for unsustainable products, $\zeta_{ni} = 1$. Therefore, exporter $i$'s price offers in market $n$ are:

$$p^U_{ni}(j^U) = \frac{\tilde{a}_i(j^U)c^U_i(j^U)\tau_{ni}}{z_i(j^U)}$$

$$p^S_{ni}(j^S) = \frac{\tilde{a}_i(j^S)c^S_i(j^S)\tau_{ni}\zeta_{ni}}{z_i(j^S)}$$

where $\tilde{a}_i(j^S) = a_i(j^S)^\alpha$, and $c^k_i$ is the cost of a product $j$ input bundle in sector $k = U, S$. For cost-minimizing producers,

$$c^U_i(j^U) = \nu_i$$

$$c^S_i(j^S) = \kappa \nu_i^\alpha h_i^{1-\alpha}$$

where $\kappa$ is a constant, $\nu_i$ is the unit cost of the composite input, which consists of land rent, wages, and any charges levied for water use, and $h_i$ is the unit cost of environmental services. In this model, the only source of variation across products comes from random agricultural productivity differences $z_i(j^k)$ and systematic input-productivity differences. Thus, it is assumed that
producers in country \( i \) have identical efficiency of producing \( j \) with the \( S \) technology relative to \( U \) technology for all products.\(^{11}\)

Returning to the dispersion parameter \( \theta^k \) from the Fréchet distribution, the price of producing delivering product \( j^k \) from country \( i \) to country \( n \), relative to the price of producing and delivering it in \( n \) is:

\[
\frac{p_{ni}(j^k)}{p_{mn}(j^k)} = \frac{z_n(j^k) \bar{a}_i(j^k)c_i^k(j^k)\tau_{mi}z_{mi}}{z_i(j^k) \bar{a}_n(j^k)c_n^k(j^k)},
\]

and taking the expectation over \( j^k \):

\[
\frac{E(p_{ni}(j^k))}{E(p_{mn}(j^k))} = \left( \frac{T_i}{T_n} \right)^{-1/\theta^k} \frac{E(a_i(j^k))c_i^k(j^k)\tau_{mi}z_{mi}}{E(a_n(j^k))c_n^k(j^k)}.
\]

The expected price of products from exporting country \( i \) relative to the price of products in importing country \( n \) is a function of two factors: the ratio of their average agricultural productivities \( T_i / T_n \), and the ratio of their unit costs of production and delivery, given relative expected values of \( a_i(j^k) \). As \( \theta^k \) increases, the first term approaches 1, and relative unit costs of production and delivery dominate in determining costs of production, and hence relative prices.

In her analysis, Fieler (2011) argues low-income (high-income) countries will specialize in products with a high (low) value of \( \theta^k \), given low wages (high average productivities). However, this will not necessarily be the case for agricultural products, i.e., even if a high-income country has high average agricultural productivity \( T_i \), its potential for exporting product \( j^k \) will be conditioned on it having the necessary agroecological characteristics such as soil type and climate to produce it in the first place – for example, coffee beans and pineapples cannot be grown in the

\(^{11}\) The amount of environmental input could also be modeled as product specific, i.e., \( H_i(j^k) \) to accommodate different patterns of specialization across products under the \( S \) technology.
Mid-West despite the US having high average agricultural productivity. In other words, while high-income countries are very likely to specialize earlier in a range of sustainable products, any general argument for Vernon’s (1966) product cycle theory being applied to sustainable agricultural production technologies is unlikely to stand up empirically. This will prove important later when considering the effect of income on trade in sustainable products when evaluating eco-labelling.

**Trade and Prices**

Consumers in market \( n \) purchase each \( U \) and \( S \) product from the exporter with the lowest price offer. The price actually paid for product \( j \) in market \( n \) is therefore:

\[
p_n^k(j^k) = \min_i \{ p_{ni}^k(j^k) \}, \quad k = U, S.
\]

To understand what this implies for the distribution of prices in importing country \( n \), the productivity distribution (1) is made use of to first derive the distribution of price offers from exporter \( i \) in market \( n \) for the sustainable technology, \( G_m^S(p) \):

\[
\Pr(p_m^S(j^S) \leq p) = \Pr\left( \frac{\tilde{a}_i(j^S)c_i^S(j^S)\tau_m\xi_m}{z_i(j^S)} \leq p \right)
= 1 - \Pr\left( z_i(j^S) \leq \frac{\tilde{a}_i(j^S)c_i^S(j^S)\tau_m\xi_m}{p} \right)
= 1 - F_{z_i}\left( z_i(j^S) \leq \frac{\tilde{a}_i(j^S)c_i^S(j^S)\tau_m\xi_m}{p} \right)
= 1 - \exp\left\{ -T_i\left( \tilde{a}_i(j^S)c_i^S(j^S)\tau_m\xi_m \right)^{-\theta} p^\theta \right\} \equiv G_m^S(p).
\]

Equation (3) implies \( p_m^S(j^S) \leq p \), unless all countries’ price offers are greater than \( p \). Given the density of \( \tilde{a}(j^S) = [\tilde{a}_1(j^S), ..., \tilde{a}_n(j^S)] \):
\[ \Pr\left( p_n^S(j^S) > p \right) = \Pr\left( p_n^S(j^S) > p \forall l \right) \]
\[ = \prod_{l=1}^{l} \left( 1 - G_m^S(p(j^S)) \right) \]
\[ = \prod_{l=1}^{l} \exp \left\{ -T_l \left( \bar{a}_i(j^S) c_i^S(j^S) \tau_{nl} \zeta_n \right)^{-\theta^S} p^{\theta^S} \right\} \]
\[ = \exp \left\{ -\sum_{l=1}^{l} T_l \left( \bar{a}_i(j^S) c_i^S(j^S) \tau_{nl} \zeta_n \right)^{-\theta^S} p^{\theta^S} \right\}. \]

Therefore:

\[ \Pr\left( p_n^S(j^S) \leq p \forall l \right) = 1 - \exp \left\{ -\sum_{l=1}^{l} T_l \left( \bar{a}_i(j^S) c_i^S(j^S) \tau_{nl} \zeta_n \right)^{-\theta^S} p^{\theta^S} \right\}. \]

Notice that this expression is the same for all exporters. That is, the distribution of prices for products purchased from every exporter by importer \( n \) is identical. Since \( z_i(j^S) \), and \( \bar{a}_i(j^S) \) follow independent distributions in each country, the distribution of prices for products purchased in market \( n \) is the integral of (5) over the density of \( \bar{a} = [\bar{a}_1, ..., \bar{a}_l] \):

\[ \Pr\left( p_n^S(j^S) \leq p \right) = 1 - \exp \left\{ -\Phi_n^S(j^S) p^{\theta^S} \right\} dF_{\bar{a}_n}(\bar{a}) \equiv G_n^S(p), \]

where \( \Phi_n^S(j^S) = \sum_{l=1}^{l} T_l \left( \bar{a}_i(j^S) c_i^S(j^S) \tau_{nl} \zeta_n \right)^{-\theta^S} \), and \( dF_{\bar{a}_n}(\bar{a}) \) is the density of \( \bar{a} \) over agricultural products consumed in market \( n \).

Equivalent expressions for the distribution of \( U \) products can be obtained by setting \( \zeta = 1 \):

\[ G_n^U(p) = 1 - \exp \left\{ -T_i \left( \bar{a}_i(j^U) c_i^U(j^U) \tau_{mi} \right)^{-\theta^U} p^{\theta^U} \right\}, \]
\[ G_n^U(p) = 1 - \exp \left\{ -\Phi_n^U(j^U) p^{\theta^U} \right\} dF_{\bar{a}_n}(\bar{a}), \]

where \( \Phi_n^U(j^U) = \sum_{i=1}^{l} T_i \left( \bar{a}_i(j^U) c_i^U(j^U) \tau_{nl} \right)^{-\theta^U} \).
The parameters describe how average agricultural productivity, input-specific productivity, input costs, and trade and labelling costs around the world affect prices in each import market. Lowering trade and labelling costs increases \( \Phi_n^k(j^k) \) and thus the average level of production efficiency of importer \( n \) consumption. To the extent that, higher production efficiency reflects a more sustainable environment for production with a smaller negative environmental impact net of any transportation externalities, lower trade costs enable more sustainable consumption with a smaller environmental impact, even if consumption is not reallocated to \( S \) products.

Using the price distributions (4) and (6), the probability that exporter \( i \) offers the lowest price for an \( S \) product in importer \( n \) can be calculated. The probability the lowest price offer comes from exporter \( i \) is the probability that, all its competitors offer higher prices. Let \( p_m^S(j^S) = p^s \). Then:

\[
Pr(p_m^S(j^S) > p^s \forall l \neq i) = \prod_{i=l} \Pr(p_m^S(j^S) > p^s) = \exp\left\{-\sum_{t=i} T_t \left( \tilde{a}_t(j^S)c_t^S(j^S)\tau_{mn}\zeta_{ml} \right)^{-\theta^l} p^{\theta^s} \right\}.
\]

Integrating over all possible realizations of \( p_m^S(j^S) \):

\[
Pr(p_m^S(j) > p_m^S(j) \forall l \neq i) = \int \exp\left\{-\sum_{t=i} T_t \left( \tilde{a}_t(j)c_t^S(j)\tau_{ml}\zeta_{ml} \right)^{-\theta^l} p^{\theta^s} \right\} dG(p(j^S))
\]

\[
= \int \exp\left\{-\sum_{t=i} T_t \left( \tilde{a}_t(j)c_t^S(j)\tau_{ml}\zeta_{ml} \right)^{-\theta^l} p^{\theta^s} \right\} \exp\left\{-T_t \left( \tilde{a}_t(j)c_t^S(j)\tau_{ml}\zeta_{ml} \right)^{-\theta^l} p^{\theta^s} \right\}
\]

\[
= \int \exp\left\{-\Phi_n^S(j^S)p^{\theta^s} \right\} \theta^S T_t \left( \tilde{a}_t(j)c_t^S(j)\tau_{ml}\zeta_{ml} \right)^{-\theta^l} p^{\theta^s-1} dp
\]

\[
= \int \exp\left\{-\Phi_n^S(j^S)p^{\theta^s} \right\} \theta^S T_t \left( \tilde{a}_t(j)c_t^S(j)\tau_{ml}\zeta_{ml} \right)^{-\theta^l} p^{\theta^s-1} dp.
\]

Multiply by \( \frac{\Phi_n^S(j^S)}{\Phi_n^S(j^S)} = 1 \):
\[
T_i \left( \tilde{a}_i(j^s) c_i^s(j^s) \tau_{ni} \zeta_{ni} \right)^{-\theta^s} \int \Phi_n^S \exp \left\{ -\Phi_n^S(j^s) p^{\theta^s} \right\} p^{-1} dp.
\]

The expression under the integral is \(dG_n^S(p)\) and is thus equal to 1. Therefore:

\[
Pr(p_n^S(j^s) > p_l^S(j^s)) \forall l \neq i = \frac{T_i \left( \tilde{a}_i(j^s) c_i^s(j^s) \tau_{ni} \zeta_{ni} \right)^{-\theta^s}}{\Phi_n^S(j^s)} \equiv \pi_{ni}^S.
\]

The unconditional probability that exporter \(i\) offers the lowest price in \(n\) is then,

\[
\pi_{ni}^S = \int T_i \left( \tilde{a}_i(j^s) c_i^s(j^s) \tau_{ni} \zeta_{ni} \right)^{-\theta^s} dF_{\alpha}^S(\tilde{a}).
\]

Since there is a continuum of \(S\) products, and by invoking the law of large numbers, this is also the fraction of products that consumers in importer \(n\) purchase from exporter \(i\). By setting \(\zeta_{ni} = 1\), the fraction of \(U\) product purchased in market \(n\) from exporter \(i\) can similarly be defined:

\[
\pi_{ni}^U = \frac{T_i \left( \tilde{a}_i(j^u) c_i^u(j^u) \tau_{ni} \right)^{-\theta^u}}{\Phi_n^U(j^u)}.
\]

Equations (7) and (8) indicate that country \(n\)’s probability of importing a product \(k\) from country \(i\) increases with: higher average agricultural productivity in \(i(T_i)\), higher input-specific productivity in \(i(a_i(j^k))\), lower input prices in \(i(\nu_i, h_i)\), lower bilateral trade costs \(\tau_{ni}\), and lower eco-labelling costs \(\zeta_{ni}^U\) for \(S\) products. Equations (7) and (8) are also gravity-like relationships between market share on the one hand, and exporter characteristics and bilateral trade costs on the other, i.e., they are the weighted sum of \(\pi_{ni}^U(j^k)\), where the weights reflect the importance of each product in \(n\) consumption.
Expenditure and Consumption

Equations (7) and (8) are also the share of importer $n$ expenditure on $S$ and $U$ products, respectively. To see this, notice that country $n$’s average expenditure per product does not vary by source. The average price per type $k$ product is the mean of the price distribution:

$$\int pdG^k_n(p).$$

Since $G_n(p)$ is also the distribution of prices for products purchased from country $i$, this is also the average price for a product purchased from exporter $i$. Total expenditure on type $k$ products can therefore be written:

$$X^k_n = \int_0^1 Q^k(j^k) dj^k \int_0^\infty pdG^k_n(p).$$

Since $\bar{\pi}^k_{ni}$ is the share of products purchased from country $i$:

$$X^k_{ni} = \pi^k_{ni} \int_0^1 Q^k(j^k) dj^k \int_0^\infty pdG^k_n(p).$$

Therefore:

$$\frac{X^k_{ni}}{X^k_n} = \frac{\pi^k_{ni} \int_0^1 Q^k(j^k) dj^k \int_0^\infty pdG^k_n(p)}{\int_0^1 Q^k(j^k) dj^k \int_0^\infty pdG^k_n(p)} = \bar{\pi}^k_{ni},$$

Consumers have preferences over agricultural products produced with each technology. $S$ products have credence attributes – but the production method does not alter a product’s intrinsic characteristics as they are perceived by consumers, and $S$ products have higher costs of production and thus higher prices. Therefore, consumers only choose $S$ products if they are credibly labelled as such, i.e., without such a label, consumers will not be willing to pay a higher price for $S$ products, only $U$ products being consumed. With eco-labelling, consumers choose quantities of each type of product to maximize their utility function, and as noted earlier, this implies total expenditure on $S$ products relative to $U$ products is:
\[
\frac{X_i^S}{X_i^U} = \lambda^{\sigma^U - \sigma^S} \left( \frac{\omega_i P_i^{s^U - a^U}}{P_i^{t^U - \sigma^U}} \right),
\]

where \( P_i^k \) is a CES price index for products produced with type \( k \) technology (see Appendix C):

\[
P_i^k = \gamma \left( \int \Phi_k(j^k) \frac{\theta^k}{\theta^k - \sigma^k} dF_{\alpha k}(\bar{\alpha}) \right)^{-1/\sigma^k} \quad k = U, S.
\]

where \( \gamma = \Gamma \left( \frac{\theta^k + 1 - \sigma^k}{\theta^k} \right)^{-1/\sigma^k} \), \( \Gamma \) is the Gamma function used to express a definite integral (Johnson and Kotz, 1970), and we must have \( \theta^k > (\sigma^k - 1) \).

**Equilibrium**

Given parameters capturing average agricultural productivity levels \( (T_i) \), input-specific productivity \( (a, (j)) \), bilateral trade and labelling costs \( (\tau_{ni}, \zeta_{ni}) \), the weight attached to \( S \) \( (\omega_i) \), and the Lagrange multiplier \( (\lambda_i) \), the equilibrium consist of composite input prices \( (\nu_i) \), the value of environmental services \( (h_i H_i) \), bilateral expenditure shares \( (\pi_{ni}^U, \pi_{ni}^S) \), an allocation of consumer expenditure \( (X_i^U, X_i^S) \), and the composite input \( (\Upsilon_i^U, \Upsilon_i^S) \) across \( U \) and \( S \) products such that the composite input market clears and trade is balanced.

To solve the model for equilibrium Levchenko and Zhang (2014) are followed. From the consumer’s problem (Appendix A), total demand for \( S \) and \( U \) products in country \( n \) are, respectively:

\[
X_i^S = \lambda_i^{\sigma^S} \omega_i P_i^{s^U - \sigma^S},
\]

\[
X_i^U = \lambda_i^{\sigma^U} P_i^{t^U - \sigma^U}.
\]
Total exports of type $k$ products are $EX_n^k = \sum_{i=1}^n \pi_m^k x_i^k$ and total imports are $IM_n^k = \sum_{i=1}^n \pi_m^k x_n^k$.

Balanced trade requires:

$$\sum_{k=S,U} EX_n^k = \sum_{k=S,U} IM_n^k.$$

Product market clearing implies:

$$Y_i^S = \sum_{i=1}^l \pi_m^S X_n^S = \sum_{i=1}^l \pi_m^S \lambda_n^S \omega_n S_n^{s-s}$$

$$Y_i^U = \sum_{i=1}^l \pi_m^U X_n^U = \sum_{i=1}^l \pi_m^U \lambda_n^U P_n^{u-j}. $$

First order conditions from the producer’s problem (Appendix D) imply optimal composite input allocation:

$$Y_i^S = \frac{\nu_i Y_i^S}{\alpha},$$

$$Y_i^U = \frac{\nu_i Y_i^U}{\alpha},$$

and composite input market clearing implies $Y_i = Y_i^S + Y_i^U$. Finally, the value of environmental services is obtained from the $S$ producer’s problem (see Appendix D):

$$h_i H_i = \frac{(1-\alpha)}{\alpha} \nu_i Y_i^S.$$

### 4. Trade and Eco-Labelling

**Consumer Gains**

Consumer gains from the introduction of eco-labels on imported products arise to the extent that they lower the price of $S$ products. Introducing eco-labels can be represented as a decrease in $\zeta_{ni}$.
from infinity. Without eco-labelling and trade, the price of $S$ products is fully determined by domestic production costs:

$$
P_{n}^{S} = \gamma \left\{ \int \left( T_{n} \left( \bar{a}_{n} (j^{S}) c_{n}^{S} (j^{S}) \right) \right)^{-\sigma^{S}} \frac{\sigma^{S}-1}{\theta^{S}} dF_{m} (\bar{a}) \right\} \frac{1}{1-\sigma^{S}}.
$$

Introducing eco-labels for imported $S$ products provides consumers access to the lower prices associated with products for which its trading partners have a comparative advantage and $P_{n}^{S}$ becomes:

$$
P_{n}^{S} = \gamma \left\{ \int \left( T_{n} \left( \bar{a}_{n} (j^{S}) c_{n}^{S} (j^{S}) \right) \right)^{-\sigma^{S}} + \left( \sum_{i} T_{i} \left( \bar{a}_{i} (j^{S}) c_{i}^{S} (j^{S}) \tau_{n} \zeta_{i} \right) \right)^{-\sigma^{S}} \frac{\sigma^{S}-1}{\theta^{S}} dF_{m} (\bar{a}) \right\} \frac{1}{1-\sigma^{S}}.
$$

Holding the price of the composite input and environmental services constant, this is an unambiguous decline in $P_{n}^{S}$. Moreover, since $\zeta_{i}$ does not affect $P_{n}^{U}$, it is also a decline in the price of $S$ products relative to $U$ products. From equation (9), this implies an increase in the share of expenditure allocated to $S$ products. To see this, we rewrite (9) as:

$$
\frac{X_{i}^{S}}{X_{i}} = \lambda_{i} \left( \rho_{i}^{U} \right)^{1-\sigma^{U}} \left( 1 + \rho_{i} \left( \rho_{i}^{S} / \rho_{i}^{U} \right)^{1-\sigma^{S}} \right),
$$

where $X_{i} = X_{i}^{U} + X_{i}^{S}$ is total expenditure, and assuming for simplicity, $\sigma^{S} = \sigma^{U}$:

$$
\frac{\partial \left( X_{i}^{S} / X_{i} \right)}{\partial \left( P_{i}^{S} / P_{i}^{U} \right)} = (1-\sigma) \times \frac{\rho_{i}}{1 + \rho_{i} \left( \rho_{i}^{S} / \rho_{i}^{U} \right)^{1-\sigma}} \times \left( 1 - \frac{X_{i}^{S}}{X_{i}} \right),
$$

which is negative if $\sigma > 1$, i.e., falling prices of $S$ products increases their share in total expenditure. In general equilibrium, $S$ products will increase their share of country $i$ consumers’ budgets to the
extent that their price falls more than $U$ prices, which will depend on the distribution of adjustments to composite input costs, as well as trade and labelling costs around the world.

With a fully parameterized model, consumer gains from eco-labelling can be estimated by approximating the utility obtained with and without imported $S$ products, holding expenditure fixed. Consumer welfare is defined as:

$$CW_i = \left( \frac{X_i^{U'}}{P_i^{U'}} + \frac{1}{\omega^{p^U}} X_i^{S'} \right) - \left( \frac{X_i^{U'}}{P_i^{U'}} + \frac{1}{\omega^{p^U}} X_i^{S'} \right),$$

where $\left( \frac{X_i^{U'}}{P_i^{U'}} + \frac{1}{\omega^{p^U}} X_i^{S'} \right)$ is weighted real expenditure on agricultural products with the introduction of eco-labels on imported products, and total expenditure is constrained to equal expenditure without eco-labels on imports:

$$\frac{X_i^{U'}}{P_i^{U'}} + \frac{X_i^{S'}}{P_i^{S'}} = \frac{X_i^{U'}}{P_i^{U'}} + \frac{X_i^{S'}}{P_i^{S'}}.$$

The magnitude of consumer gains from the introduction of eco-labelling with international trade will vary across countries depending on the value consumers place on environmentally friendly/sustainable production as well as trade and labelling costs.

**Sustainability Gains**

The magnitude of sustainability gains from eco-labelling depends crucially on the objective of the eco-label. Only cases in which the eco-label signifies that a specific production process was followed are considered here. The sustainability benefits of using a given $S$ production process relative to the $U$ production process are rarely if ever easy to quantify on a large scale. The interaction of natural resources and agricultural inputs for the purpose of producing crops or animal products always has some environmental impact. Calculating the negative environmental cost
associated with that interaction at a given place and time requires very specific criteria and may yet be difficult or impossible to measure. Moreover, just as an identical input bundle will not produce the same output in Canada as it will in Spain, so the environmental impact from an identical production process will vary depending on the characteristics of the natural resource base across countries over time.

Since the sustainability gains from a given production technology cannot be directly calculated without much more information than is generally available, as in Larson (2003) the sustainability gains from eco-labelling and trade $EW_i$ are measured as a function of the increase in the share of the composite input allocated to $S$ production:

$$EW_i = f\left(\frac{Y^S_i - Y^{S'}_i}{Y^S_i}\right).$$

To see how introducing trade in eco-labelled products provides sustainability gains in the exporting country, equation (11) is used to show that the optimal allocation of the composite input implies:

$$\frac{Y^S_i}{Y^U_i} = \frac{\sum_n \pi^S_n X^S_n}{\sum_n \pi^U_m (X_n - X^S_n)}.$$

The numerator in equation (12) is the value of country $i$’s total production of $S$ products – exports plus domestic production. As $\zeta_m$ falls from infinity, $\pi^S_m$ rises under general circumstances. Therefore, the numerator of (12) increases with the introduction of eco-labels. The denominator is the total value of $U$ products production. Again, from Section 4, $(X_n - X^S_n)$ is expected to decrease with the introduction of eco-labelling on imports under general conditions. Given that markets must clear, it can therefore be expected that the introduction of eco-labels with trade will
increase the share of the composite input allocated to \( S \) products, thereby providing a sustainability gain in terms of a lower water footprint.

**Alternative Eco-Labelling Policies**

While measuring the absolute level of sustainability gains from eco-labels and trade is complicated by the challenge of objectively measuring the relative environmental impact of two production processes, our model offers valuable insights into the relative gains from eco-labelling and trade under various policies. Here the impact of two eco-labelling policy scenarios is examined when two separate economies choose to integrate through a regional trade agreement: mutual recognition and regulatory harmonization.

Mutual recognition of eco-labels among countries implies that products meeting domestically sufficient criteria for an eco-label may be sold with that eco-label in an import market without meeting additional criteria or providing additional proof that domestically sufficient criteria have been met. For example, since 2012, the European Union and United States have had a mutual recognition agreement for organic products: products meeting the criteria for an organic label in the United States may be exported and labelled as organic in the European Union.

Under a policy of mutual recognition, labelling costs in the export market are identical to domestically sufficient labelling costs. That is, \( \zeta_{ni} = \zeta_{mn} = \zeta_{n} = \zeta_{m} = 1 \), and mutual recognition lowers labelling costs in foreign markets from \( \zeta_{ni}, \zeta_{mi} > 1 \). From equation (7) it is clear that, holding all prices and all other countries’ labelling costs constant, lowering \( \zeta_{ni} \) increases \( \pi_{ni}^{S} \) – an increase in bilateral trade in eco-labelled products.

The extent to which mutual recognition provides consumer and environmental/sustainability gains will depend on its effects on the prices of \( S \) and \( U \) products. Holding the prices of the composite input and environmental services constant, mutual recognition unambiguously lowers
both, the absolute price of $S$ products as well as the price of $S$ relative to $U$ products. To see this, suppose there is mutual recognition of eco-labels in countries $n$ and $i$. Then, the price of $S$ products in market $n$ becomes:

$$P_n^S = \gamma \left\{ T_n \left( \tilde{a}_n (j^S)c_n^S (j^S) \right)^{-\sigma^S} + T_i \left( \tilde{a}_i (j^S)c_i^S (j^S)\tau_i \right)^{-\sigma^S} \right\}^{1/1-\sigma^S}. $$

The first term in parentheses is the contribution of domestic prices, which is unchanged under mutual recognition. The second term is country $i$’s contribution, which has increased from $T_i \left( \tilde{a}_i (j^S)c_i^S (j^S)\tau_i \right)^{-\sigma^S}$, lowering $P_n^S$. The contribution of all other countries is unchanged. Since $P_n^U$ is not a function of $\zeta_n$, it is unchanged. Therefore, from equation (9), with $\sigma > 1$ the share of expenditure allocated to $S$ rises with mutual recognition, also implies an increase in consumer welfare in both countries. In addition, an increase in expenditure on $S$ products and increased bilateral trade in $S$ products generates environmental gains in both countries as the expanded export opportunity increases the share of the composite input allocated to $S$ production.

Under a policy of regulatory harmonization, eco-labelling criteria are standardized across countries. Thus, as with mutual recognition, labelling costs are constant in the domestic and foreign market. However, the cost of meeting agreed labelling criteria may differ from the cost of meeting domestically sufficient criteria. Let $\zeta_n'$ be the costs of meeting mutually agreed criteria, which are greater than the domestically sufficient criteria. Now:
\[ P_n^S = \gamma \left( \int \left( T_n \left( \tilde{a}_n (j^S) c_n^S (j^S) \zeta_n' \right)^{-\sigma^S} + T_i \left( \tilde{a}_i (j^S) c_i^S (j^S) \tau_i \zeta_i' \right)^{-\sigma^S} \right)^{\sigma^S - 1} \theta^S \right)^{-\frac{1}{\theta^S}} dF_{ia}(\tilde{a}) \].

The first term in parentheses is again the contribution of domestic prices, which have risen from \( \int T_n \left( \tilde{a}_n (j^S) c_n^S (j^S) \right)^{-\sigma^S} \), increasing \( P_n^S \). The second term reflects country \( i \)'s contribution, which may be larger or smaller depending on whether \( \zeta_i' \) is larger or smaller than \( \zeta_n \). In this case the price of \( S \) products may rise or fall, depending on the magnitudes of \( \zeta_n \) and \( \zeta_i' \). If the prices of \( S \) products rise, their share in total expenditure will fall, reducing consumer welfare.

If mutually agreed criteria are more costly and not accompanied by sufficiently larger environmental benefits, such an effort may reduce environmental/sustainability benefits. To see this, observe that with \( \zeta_n' > 1 \) domestic market share falls under mutual recognition:

\[ \Pi_{mn}^S = \frac{\int T_n \left( \tilde{a}_n (j^S) c_n^S (j^S) \zeta_n' \right)^{-\sigma^S}}{\Phi_N^S} < \frac{\int T_i \left( \tilde{a}_i (j^S) c_i^S (j^S) \right)^{-\sigma^S}}{\Phi_N^S}. \]

If this is manifest as a decline in production for the domestic market that is not offset by an increase in exports to market \( i \), equation (12) implies that the share of the composite input allocated to \( S \) production in country \( n \) will decline, resulting in an environmental/sustainability loss.

**Income and Trade in Sustainable Products with Eco-Labelling**

Drawing on equation (9), and the earlier discussion, if it is assumed that \( \sigma^S > \sigma^{ij} \), relative expenditure on sustainable products is increasing in income. From this, and following Fielet (2011), it can be argued country \( n \)'s imports from \( i \) relative to its domestic consumption will be a function of income, i.e., for a high-income country, \( X_{ni} / X_{mn} \approx X_{ni}^S / X_{mn}^S \), and by (7):
Given \( \tau_{nm} = \zeta_{nm} = 1 \), the term in brackets will in general be greater than 1, but as \( \Theta^S \) becomes smaller, the impact of relative costs of sustainable production declines, and country \( n \)'s imports from \( i \) increases, driven by differences in average agricultural productivity. The analytical logic here is that production of and trade in sustainable agricultural products will be more intense among high-income countries when \( \sigma^S > \sigma^U \), and \( \Theta^S < \Theta^U \). This result also suggests that, even if eco-labelling costs are significant, fundamental differences in average agricultural productivity can drive trade.

However, this does not capture exports of, say tropical agricultural products produced in low-income countries, driven by their having the relevant agroecological characteristics, even if their average agricultural productivity is low. This sets up an interesting dynamic: low-income countries produce and trade a range of agricultural products using the unsustainable technology \( U \), but demand for them to switch to production and export of the same range of products using the sustainable technology \( S \) comes from high-income country consumers willing to pay higher prices.

For this to be an equilibrium with the associated environmental benefits, the term in brackets in (13) will have to be less than 1, and the value of \( \Theta^S \) not too low. In other words, producers in low-income countries have sufficiently low unit costs of sustainable production, \( \bar{\alpha}_i (j^S) c_i^S (j^S) \tau_{ni} \zeta_{ni} \), which mitigates any frictions created by transport costs and other barriers to trade \( \tau_{ni} \), as well as the costs of eco-labelling \( \zeta_{ni} \). This all assumes producers in low-income countries have access to the environmental services \( H_i \) necessary for sustainable production, and an ability to certify credence products for sale to consumers in high-income countries.
5. Summary and Conclusions

The key motivation for this paper is that consumers are increasingly demanding food characterized by credence attributes such as sustainable production methods, where eco-labelling and certification of any sustainability claims are critical in resolving the associated informational asymmetry. Importantly, trade in such products may generate sustainability gains. In this context, a Ricardian-type trade model with non-homothetic preferences is used to explore the potential gains to consumers and the environment of increased trade in and eco-labelling of sustainably produced agricultural products, as well as compare the alternatives of mutual recognition versus harmonization of different countries’ eco-labelling regimes.

The analysis presented in the paper generates four key results. First, with eco-labelling, the price of sustainable products falls, generating benefits to consumers. Second, introduction of eco-labelling increases the share of a composite input (land, labor, and water) allocated to sustainable products with associated sustainability benefits. Third, in terms of choice of eco-labelling regime, a policy of mutual recognition will increase trade in eco-labelled products, resulting in gains to both consumers and the environment, while regulatory harmonization may result in an environmental loss, depending on the cost of harmonized eco-labelling relative to any environmental benefits. Fourth, under certain conditions, non-homothetic preferences imply production and trade in sustainable agricultural products will be most intense among high-income countries, except for those instances where location of production in low-income countries is dictated by agroecological characteristics.

Ultimately, the extent which trade in eco-labelled products benefits consumers and the environment is an empirical issue. Therefore, calibrating the current model, and using it to conduct relevant policy simulations will be the next step in this line of research, the biggest challenge being
development of a proper and robust connection between agricultural production, agroecological characteristics, and the water footprint of sustainable vs. unsustainable production.

In terms of the model itself, there are two standout issues that need addressing. First, finding a more appropriate and tractable way of capturing non-homothetic preferences is critical, especially if the model is to be tied more closely to the existing literature on vertical product differentiation, i.e., one that predicts that, with increasing incomes, consumers substitute away from unsustainable products (low-quality) toward sustainable products (high-quality). Second, the model as currently structured lacks an explicit vertical market structure, i.e., it ignores linkages between agricultural producers and downstream retailers/processors, which parties set sustainability standards, and the potential for post-contractual bargaining over any available rents.
References


Figure 1: Comparative Advantage in the Continuum

- Red line: Country 1
- Blue line: Country 2

Mathematical expressions:

\[ a_j(j)c_j(j) \]

Country 1 and Country 2 lines intersect at certain points, indicating comparative advantage in the continuum.
Figure 2: Tariffs and Non-Traded Products in the Continuum
Figure 3: Sustainable Products in the Continuum

\[ a_i(j)c_i(j) \]

\[ \infty \]

\[ 0 \quad j_2 \quad j_u \quad j_s \quad j_1 \quad 1 \]

\[ U_1 \quad U_2 \quad S_1 \quad S_2 \]
Appendices

Appendix A

The ratio of expenditure on each type of product is derived from the consumer’s problem as follows. For a given level of utility $\bar{U}$, consumers choose $q^U_i(j^U)$ and $q^S_i(j^S)$ for each product $j$ by solving:

$$
\min \int_0^1 p^U_n(j^U)q^U_n(j^U)dj^U + p^S_n(j^S)q^S_n(j^S)dj^S
$$

subject to $\bar{U} = \frac{\sigma^U}{\sigma^U - 1} \left( \int_0^1 q^U_i(j^U) \frac{\sigma^U - 1}{\sigma^U} dj^U \right) + \frac{\sigma^S}{\sigma^S - 1} \left( \frac{1}{\omega^S} \int_0^1 q^S_i(j^S) \frac{\sigma^S - 1}{\sigma^S} dj^S \right)
$$

First-order conditions with respect to $q^U_i(l)$ for some product $l$:

$$
p^S_i(l) = \frac{1}{\lambda_i \omega^S} \frac{1}{q^S_i(l)} \frac{1}{\sigma^S}.
$$

Rearranging yields:

$$
q^S_i(l) = \left( \frac{p^S_i(l)}{\lambda_i \omega^S} \right)^{-\sigma^S}
$$

Multiplying both sides by $p^S_i(l)$, total expenditure on product $l$ is:

$$
p^S_i(l)q^S_i = p^S_i(l)^{-\sigma^S} \lambda^S \omega_i
$$

Integrating over all products, total expenditure on $S$ products is:

$$
\int_0^1 p^S_i(j^S)q^S_i(j) dj^S = X^S_i = \lambda_i \sigma^S \omega_i P^S_i^{1-\sigma^S}
$$

Where $P^S_i$ is a price index for $S$ products (see Appendix C).

Likewise, total expenditure on $U$ products is:

$$
X^U_i = \lambda_i \sigma^S P^U_i^{1-\sigma^S}
$$

Thus, the ratio of expenditure on $S$ to $U$ products is:

$$
\frac{X^U_i}{X^S_i} = \lambda_i \frac{\sigma^S}{\sigma^U} \left( \frac{\omega_i P^S_i^{1-\sigma^S}}{P^U_i^{1-\sigma^S}} \right)
$$
Appendix B

Since perfect competition is assumed, price is equal to unit cost. Producers of the $S$ choose the composite input and environmental inputs to solve:

$$
\min_{\nu_i, h_i} \nu_i \bar{Y}_i + H_i h_i
$$

s.t.  \( q_i^S (j^S) = z_i(j^S) \left[ (a_i(j^S) \bar{Y})^{\alpha} H_i^{1-\alpha} \right] \).

From the constraint:

$$
\bar{Y}_i = \left( \frac{q_i^S (j^S)}{z_i(j^S) a_i(j^S) H_i^{1-\alpha}} \right)^\frac{1}{\alpha}.
$$

Therefore, the problem can be re-written:

$$
\min_{\nu_i} \nu_i \left( \frac{q_i(j^S)}{z_i(j^S)} \right)^\frac{1}{\alpha} a_i(j^S) H_i^{-\alpha} + h_i H_i
$$

The first-order condition is:

$$
\left( \frac{1-\alpha}{\alpha} \right) \nu_i \left( \frac{q_i(j^S)}{z_i(j^S)} \right)^\frac{1}{\alpha} a_i(j^S) H_i^{-1} = \nu_i
$$

Therefore:

$$
H_i^* = \left( \frac{h_i}{\nu_i} \right)^{-\alpha} \left( \frac{\alpha}{1-\alpha} \right)^{-\alpha} \left( \frac{q_i(j^S)}{z_i(j^S)} \right) a_i(j)^{-\alpha}
$$

and:

$$
\bar{Y}_i^* = \left( \frac{h_i}{\nu_i} \right)^{1-\alpha} \left( \frac{\alpha}{1-\alpha} \right)^{-\alpha} \left( \frac{q_i(j^S)}{z_i(j^S)} \right) a_i(j)^{1-\alpha}
$$

The cost of product $j$ is:

$$
e_i^S (j^S) = \kappa \nu_i^\alpha h_i^{(1-\alpha)}
$$

where:  \( \kappa = \left( \frac{1}{1-\alpha} \right) \left( \frac{\alpha}{1-\alpha} \right)^{-\alpha} \).
Therefore, the unit cost function of a product $j$ in sector $S$ is:

$$C^S_i(j^S) = \frac{\tilde{a}_i(j^S)c^S_i(j^S)}{z_i(j^S)},$$

where $\tilde{a}_i(j^S) = a_i(j^S)^\alpha$.

To obtain the unit cost function of product $j$ in sector $U$, set $\alpha = 1$, so that:

$$c^U_i(j^U) = u_i,$$

and:

$$C^U_i(j^U) = \frac{\tilde{a}_i(j^U)c^U_i(j^U)}{z_i(j^U)},$$

where $\tilde{a}_i(j^U) = a_i(j^U)$. 
Appendix C

Here the price indices for each type of product are derived. Buyers aggregate $S$ products with constant elasticity $\sigma^S$. As such, for a given $\bar{Q}^S$ they solve the problem:

$$\min_{\phi^S, (\phi^S)} \int_0^1 p^S_i (j^S) q^S_i (j^S) dj^S$$

subject to:

$$\bar{Q}^S = \int_0^1 q^S_i (j^S) \frac{\sigma^S - 1}{\sigma^S} dj^S$$

First order conditions for product $l$ give:

$$p^S_i = \lambda_i \alpha q^S_i (l) \frac{1}{\sigma^S}$$

The ratio of first order conditions for products $l$ and $k$ is thus:

$$\frac{p^S_l (l)}{p^S_k (k)} = \frac{q^S_l (l)^{\sigma^S - 1}}{q^S_k (k)^{\sigma^S - 1}}$$

Rearranging:

$$q^S_l = q^S_k \left( \frac{p^S_l (l)}{p^S_k (k)} \right)^{-\sigma^S}$$

Placing this in the constraint:

$$\bar{Q}^S = q^S_k \left( \frac{\sigma^S - 1}{\sigma^S} \right) \int_0^1 \left( p^S_i (j^S)^{1-\sigma} \right) dj^S$$

Rearranging:

$$q^S_k = \frac{p^S_k (k)^{-\sigma^S} \bar{Q}^{\sigma^S - 1}}{\left( \frac{\sigma^S - 1}{\sigma^S} \right) \int_0^1 \left( p^S_i (j^S)^{1-\sigma} \right) dj^S}$$

Multiplying by $p^S_k (k)$ gives expenditure on product $k$. Integrating over all $S$ products gives total expenditure on $\bar{Q}^S$ $S$ products:
\[
\int_0^1 p_t^S (j^S) q_t^S (j^S) \, dj^S = \int_0^1 \frac{p_t^S (j^S)^{1-\sigma^S}}{\sigma^S} \, dj^S \, \bar{Q}^{S_{\sigma^S}} = \int_0^1 (p_t^S (j^S) -1 \sigma^S) dj^S \, \bar{Q}^{S_{\sigma^S}}
\]

Let \( \bar{Q}^S = 1 \) and the unit price index becomes:

\[ P_t^S = \int_0^1 (p_t^S (j^S) -1 \sigma^S) dj^S \]  \( \bar{Q}^{1_{\sigma^S}} \) \quad (i)

Now it is shown that \( P_t^S = \gamma \left( \int \Phi_n^S (j^S) F_a^S (\tilde{a}) \right) \) . Equation (i) can be rewritten:

\[ P_t^S = \left( \int_0^{\infty} p_t^{1-\sigma} dG_t^S (p) dp^S \right)^{1/(1-\sigma)} \]  \( \bar{Q}^{1/(1-\sigma)} \) \quad (ii)

By definition: \( dG_t^S (p) = \Phi_t^S (j^S) p^{1-\sigma} \exp \left\{ -\Phi_t^S (j^S) p^{1-\sigma} \right\} \)

Using this in (ii):

\[ P_t^S = \left( \int_0^{\infty} \int_0^1 p_t^{1-\sigma} \Phi_t^S (j^S) p^{1-\sigma} \exp \left\{ -\Phi_t^S (j^S) p^{1-\sigma} \right\} F_a^S (\tilde{a}) dp^S \right)^{1/(1-\sigma)} \]

Let \( x = \Phi_t^S (j^S) p^{1-\sigma} \). Then \( dx = \Phi_t^S (j^S) p^{1-\sigma} \) and \( p = \left( \frac{x}{\Phi_t^S (j^S)} \right)^{1/(1-\sigma)} \Rightarrow p^{1-\sigma} = \left( \frac{x}{\Phi_t^S (j^S)} \right)^{1/(1-\sigma)} \)

Then:

\[ P_t^S = \left( \int_0^{\infty} \int_0^1 \left( \frac{x}{\Phi_t^S (j^S)} \right)^{1/(1-\sigma)} \exp \left\{ -x \right\} dx F_a^S (\tilde{a}) dp^S \right)^{1/(1-\sigma)} \]

Using the definition of the gamma function, \( \Gamma(z) = \int_0^{\infty} t^{z-1} e^t dt \):

\[ P_t^S = \gamma \left( \int \Phi_n^S (j^S) F_a^S (\tilde{a}) \right)^{1/(1-\sigma^S)} \text{ where } \gamma = \Gamma \left( \frac{\theta^S + 1 - \sigma^S}{\theta^S} \right)^{1/(1-\sigma^S)} \]

It is straightforward to show that: \( P_t^U = \gamma \left( \int \Phi_n^U (j^U) F_a^U (\tilde{a}) \right)^{1/(1-\sigma^U)} \)
Appendix D

Resource allocation across product types solves:

$$\max_{h_i, y_i^u, y_i^s} = h H_i + \nu_i y_i^u + \nu_i y_i^s$$

s.t.  

$$y_i = \int z_i(j^u)\left[a_i(j^u) y_i^u\right]dj^u + \int z_i(j^s)\left[a_i(j^s) y_i^s H_i^{1-a}\right]dj^s$$

The Lagrangian is:

$$\ell = h H_i + \nu_i y_i^u + \nu_i y_i^s + \lambda\left[y_i - \int z_i(j^u)\left[a_i(j^u) y_i^u\right]dj^u - \int z_i(j^s)\left[a_i(j^s) y_i^s H_i^{1-a}\right]dj^s\right]$$

First-order conditions:

(D1)  

$$h_i = \frac{\lambda(1-a)}{H_i} y_i^s$$

(D2)  

$$\nu_i = \frac{\lambda}{y_i^u} y_i^u$$

(D3)  

$$\nu_i = \frac{\lambda \alpha}{y_i^s} y_i^s$$

From (D1) and (D3):

(D4)  

$$h_i H_i = \frac{(1-a)}{\alpha} \nu_i y_i^s$$

From (D2) and (D3):

(D5)  

$$\nu_i y_i^u = \frac{1}{\alpha} \frac{y_i^u}{y_i^s} \nu_i y_i^s$$

Using (D5) in the objective function:

$$y_i = \frac{1-a}{\alpha} \nu_i y_i^s + \nu_i y_i^s + \frac{1}{\alpha} \frac{y_i^u}{y_i^s} \nu_i y_i^s = \nu_i y_i^s \left[\frac{1-a}{\alpha} + 1 + \frac{1}{\alpha} \frac{y_i^u}{y_i^s}\right]$$

$$= \nu_i y_i^s \left(\frac{1}{\alpha} + \frac{1}{\alpha} \frac{y_i^c}{y_i^s}\right) = \nu_i y_i^s \left(\frac{1}{\alpha} \left(1 + \frac{1}{\alpha} \frac{y_i^c}{y_i^s}\right)\right)$$

$$= \nu_i y_i^s \frac{y_i}{\alpha y_i^s}$$

$$\Rightarrow \alpha y_i^s = \nu_i y_i^s$$