EFFICIENCY IN EXCHANGE

Professor Ian Sheldon
To gain an initial understanding of efficiency, focus on an economy of pure exchange. Think of this economy as a desert island with the following characteristics:

- 2 consumers, A and B
- 2 goods, 1 (fruit) and 2 (fish)
- A’s consumption bundle is: 
  \[ X_A = (x^1_A, x^2_A) \]
- B’s consumption bundle is: 
  \[ X_B = (x^1_B, x^2_B) \]
There are fixed amounts of fish and fruit available, each consumer starting off with an initial endowment, $\omega$, of each good:

$\omega^1_A + \omega^1_B = W^1$

$\omega^2_A + \omega^2_B = W^2$

where $W^1$ and $W^2$ are total amounts of the two goods available

The exchange economy is assumed competitive, each consumer taking prices as given

Consumers A and B have well-behaved preferences
REVIEW

What are well-behaved preferences?

Consumer A’s utility function can be written as:

\[ U_A = f(x^1_A, x^2_A) \]

Essentially, the *utility function* of consumer A, can be represented by the standard *indifference map*

Efficiency in exchange depends critically on this
CONSUMPTION BUNDLES

Fish

x^2_A

X''_A (consumption bundle 2)

x^1_A

X'_A (consumption bundle 1)

Fruit
Consumer A has to *rank* consumption bundle 1 ($X'_A$) with consumption bundle 2 ($X''_A$)

- $X'_A > X''_A$  $\implies$ consumer A *strictly prefers* bundle 1 to bundle 2

- $X'_A \sim X''_A$  $\implies$ consumer A is *indifferent* between bundles 1 and 2

- $X'_A \geq X''_A$  $\implies$ consumer A *weakly prefers* bundle 1 to bundle 2

Weak preference means consumer A either prefers bundle 1 over bundle 2 or is indifferent between them.
Preferences are assumed to conform to the following axioms:

- **Completeness** given two bundles, consumers can make a choice, i.e. $X'_A > X''_A$, or $X'_A \geq X''_A$, or $X'_A \sim X''_A$

- **Reflexivity** any bundle is at least as good as itself, i.e. $X'_A \geq X'_A$

- **Transitivity** if $X'_A > X''_A$, and if $X''_A > X'''_A$, then $X'_A > X'''_A$

i.e. if consumer A prefers bundle 1 to bundle 2, and prefers bundle 2 to another bundle 3, then consumer A prefers bundle 1 to bundle 3
INDIFFERENCE CURVE

Weakly Preferred Set

Fish

Fruit

\( X^1_A \)

\( X^2_A \)

\( X'_A \)

\( X''_A \)
Well-behaved preferences have two features:

- more is better, less is worse, i.e. fruit and fish are "goods" not "bads"
- indifference curves are convex to the origin

Convexity relates to the slope of the indifference curve

- slope of an indifference curve at any one point is the marginal rate of substitution of the two goods 1 and 2 - $MRS_{1,2}^A$
- the marginal rate of substitution is given by the slope of a tangent at a specific point

$$\text{slope of curve} = - \frac{\Delta x_2^A}{\Delta x_1^A} = MRS_{1,2}^A$$
MARGINAL RATE OF SUBSTITUTION

Slope at T is:

\[-\frac{\Delta x^2_A}{\Delta x^1_A} = MRS^A_{1,2}\]

Fish

Fruit
the marginal rate of substitution of good 1 for good 2 is defined as the number of units of good 2 that must be given up in exchange for an extra unit of good 1, and keep the consumer on the same indifference curve.

Convexity of the indifference curve means that as one moves down a curve from left to right, its slope decreases (in absolute terms), i.e. the marginal rate of substitution is diminishing (see next figure).

The concept of a diminishing marginal rate of substitution is closely related to the concept of diminishing marginal utility:

\[
\text{MRS}_{1,2}^A = - \frac{\Delta x_2^A}{\Delta x_1^A} = \frac{MU_1^A}{MU_2^A}
\]

where MU is marginal utility; as more of good 1 is substituted for good 2, the marginal utility of good 1 declines, while that for good 2 increases.
At point $a$, the sixth unit of fish can be replaced by $(b-a)$ units of fruit.

At point $b$, the third unit of fish requires $(d-c)$ units of fruit.
Anchovies “bad” to Pepperoni “good”

“worse”

“better”