“Strong” Ricardian Theorem

Given two goods, \( j = 1, 2 \), and one factor labor, in equilibrium, the wage rate \( w \) will equal marginal revenue product:

\[
\begin{align*}
    p_1 \alpha &= w \quad \text{(1)} \\
    p_2 \beta &= w \quad \text{(2)}
\end{align*}
\]

where \( \alpha \) and \( \beta \) are the relevant marginal products of labor in producing goods 1 and 2.

Dividing (1) and (2) through by \( \alpha \) and \( \beta \) respectively:

\[
\begin{align*}
    p_1 &= 1/\alpha . w = a_1 . w \quad \text{(3)} \\
    p_2 &= 1/\beta . w = a_2 . w \quad \text{(4)}
\end{align*}
\]

where \( a_1 \) and \( a_2 \) can be interpreted as the labor required to produce one unit of each good.

Suppose there are two countries, home = \( h \), and foreign = \( f \).
For h to have a lower relative price of good 1 under autarky compared to f, and hence a comparative advantage in producing good 1, the following must hold:

\[ \frac{a_1^h}{a_2^h} < \frac{a_1^f}{a_2^f} \text{ or } \frac{a_1^f}{a_1^h} > \frac{a_2^f}{a_2^h} \]  \hspace{1cm} (5)

This is the “strong” Ricardian theorem, where the pattern of trade is determined exclusively by comparative factor productivities (Bhagwati, 1964)

With more than two goods, this is reduced to the weaker proposition that goods can be ranked in a “chain” of decreasing labor costs:

\[ \frac{a_1^f}{a_1^h} > \frac{a_2^f}{a_2^h} > \cdots < \frac{a_n^f}{a_n^h} \]  \hspace{1cm} (6)

The fundamental question is how can the “chain” be broken, i.e. which country produces which goods? Demand conditions have to be introduced (Dornbusch, Fisher and Samuelson, 1977)
“Breaking” the Chain

Assume labor endowments are $L^h$ and $L^f$ respectively.

There is a continuum of goods $z$ that can be produced, spread over the interval $[0,1]$; relative costs of production are:

$$a^h(z)w^h \leq a^f(z)w^f$$  \hspace{1cm} (7)

where $a$ is the labor requirement for producing a specific good in the continuum $z$.

Re-arranging (7):

$$\frac{w^h}{w^f} \leq \frac{a^f(z)}{a^h(z)} = A(z)$$  \hspace{1cm} (8)

$A(z)$ is illustrated in Figure 1 as the downward-sloping function - it is the continuous analogue of the “chain” in (6); essentially, as $h$’s relative wage falls, it produces a larger range of goods.

The “chain” is broken by introducing demand into the model via the *trade balance*. 
Figure 1: International Equilibrium

\[ w^h / w^f \]

\[ (w^h / w^f) \]

\[ 0 \leq z \leq 1 \]
Let \( v(\tilde{z}) \) be the fraction of income spent on goods produced in \( h \), and \( 1-v(\tilde{z}) \) be fraction of income spent on goods produced in \( f \)

Demand for imports by \( h \) depends on its income \( w^hL^h \), and export demand for \( h \’s \) goods depends on \( f \’s \) income \( w^fL^f \)

The home country trade balance is given by:

\[
v(\tilde{z}).(w^fL^f) = (1-v(\tilde{z})).(w^hL^h) \tag{9}\]

which on re-arranging is:

\[
w^h/w^f = v(\tilde{z})/(1-v(\tilde{z})).(L^f/L^h) = B \tag{10}\]

This the upward-sloping function in Figure 1; an increase in the range of goods produced in \( h \), given constant relative wages generates a trade imbalance, equilibrium is restored by an increase in home relative wages

Overall equilibrium is where \( A(z) \) and \( B \) intersect, giving the relative wage \((w^h/w^f)\) and the range of goods produced in the two countries, i.e. \([0,\tilde{z}] \) in \( h \) and \([\tilde{z},1] \) in \( f \)
A simple comparative static is to allow for \( f \)'s productivity to uniformly improve, i.e. \( a_f \) falls, hence, the \( A(z) \) function shifts to the left in Figure 2.

With no change in relative wages, \( h \) produces the range \([0,z']\), it runs a trade deficit, and \( f \) runs a trade surplus, but relative wages fall to \((w^h/w_f)'\), offsetting \( h \)'s relative productivity deterioration, \( h \) producing the range \([0,z']\).

This highlights an important insight of the Ricardian model; despite the decline in \( h \)'s relative productivity, it still participates in and gains from trade, consumers benefitting from the increase in \( f \)'s productivity via terms-of-trade effects - also, \( h \) still has a *comparative advantage* in some goods.

Also possible to allow \( f \)'s endowment of labor, \( L_f \), to increase, which rotates the \( B \) function upwards in Figure 3.

With no change in relative wages, \( f \) has a trade deficit, while \( h \) has a trade surplus, but relative wages rise to \((w^h/w_f)'\) as the wage in \( f \) declines, increasing its comparative advantage in producing more of the range of goods.
Figure 2: Productivity Change
Figure 3: Change in Labor Endowment

$\left( \frac{w^h}{w^f} \right)$

$\left( \frac{w^h}{w^f} \right)'$

$\left( \frac{w^h}{w^f} \right)$

$A(z)$

$B'$

$B$
“Most discussion of U.S. competitiveness misstates the problem, focusing on the trade deficit and on fears that an economy whose productivity lags that of its rivals will face economic disaster. In fact strong automatic forces ensure that the U.S. economy will remain in business and indeed roughly balance its trade even if its productivity performance is dismal.” (Krugman, 1991, *Science*)