Voluntary Standards and International Trade:
A Heterogeneous Firms Approach

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Abstract

We present a model of export participation and adoption of a voluntary standard in the heterogeneous firms and trade framework. Firms produce credence goods, the quality of which can be signaled by adopting a costly voluntary standard. Firms must simultaneously choose whether or not to adopt the standard and whether or not to enter export markets. Heterogeneity in firm behavior is driven by differences in productivity, which indexes the effective cost of each strategy. Comparative statics are derived relating participation in the voluntary standard to changes in key trade policy variables.

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1. Introduction
Critics of globalization often express concern that the lowering of trade barriers under the auspices of the World Trade Organization (WTO) facilitates a “race to the bottom,” creating incentives for countries to lower regulatory standards in order to increase export competitiveness and attract foreign direct investment (FDI) (e.g., see Gill (1995); Tonelson (2002)). Regulators may be tempted to lower environmental or labor standards in order to minimize production costs in their home countries. While this might maximize local economic growth in the short run, many are uncomfortable with the implied unethical treatment of workers or environmental damage.

Despite widespread popular concern, there exists only mixed evidence to support the existence of a “race to the bottom” from trade liberalization. Few studies have found a link between trade flows and environmental policy (Medalla and Lazaro, 2005). Even where such a link exists, the evidence suggests these “pollution havens” may only exist temporarily (Mani and Wheeler, 2004). The same holds true for trade and labor standards. Dehija and Samy (2008) found that higher labor standards were associated with larger trade flows in a study of EU member states, while Greenhill et al. (2009) found a similar result in a panel of 90 developing countries.

Some scholars have argued that increased openness can raise production standards even in the absence of formal government regulation through the use of voluntary industry standards (see e.g. Vogel (2010); Prakash and Potoski (2006); Kirton and Trebilcock (2004)). Voluntary standards are typically overseen by institutions, often non-governmental organizations (NGOs), which operate in parallel to formal legal institutions. Perhaps the best known example is the International Organization for Standardization.
(ISO), creator of the widely adopted ISO 9001 and ISO 14001 standards. While such standards lack the enforcement power of formal legal institutions, they are designed to offer market-based incentives for firms to raise their production standards. Voluntary standards resolve an information asymmetry problem similar to the “market for lemons” described by Akerlof (1970): consumers are willing to pay more for ethically or sustainably produced goods, but they cannot independently observe firms’ production processes. Firms have an incentive to falsely advertise they employ high labor or environmental standards, and if consumers recognize this incentive, they will no longer be willing to offer a premium. Voluntary standards solve this problem by allowing firms to credibly signal their underlying production processes.

An important question is whether or not the proliferation of voluntary standards has helped to avert the “race to the bottom” following trade liberalization. The literature on voluntary standards and international trade has produced a fairly consistent and highly suggestive set of results, but the empirical work has proceeded without a strong theoretical underpinning. The work presented here fills that gap by building on the heterogeneous firms and trade (HFT) framework developed in Melitz (2003). Subsequent work has modified the original framework to allow for vertical differentiation without losing the tractability of the original HFT. Johnson (2010), Baldwin and Harrigan (2011) and Kugler and Verhoogen (2012) modified the HFT framework to incorporate vertical differentiation by allowing quality to enter the utility function as a demand-shifter. Holding price constant, high-quality goods receive a larger budget share than low-quality goods.²

²The specification of consumer preferences adopted here and in Melitz (2003), Johnson (2010) and Baldwin and Harrigan (2011) ensure positive demand for every variety, regardless of its quality.
Our work is most closely related to Podhorsky (2010, 2013). Podhorsky (2010) first adapted the HFT framework for the provision of goods with unobservable quality in a closed economy. Firms market “high-quality” goods to consumers by participating in a voluntary certification program. Podhorsky (2013) extended this model to accommodate frictionless trade between two countries. By assuming zero trade costs, Podhorsky (2013) eliminates the endogenous exporting decision that distinguished the original HFT model, and also makes it impossible to explore the relationship between liberalization and participation in the voluntary certification program. A related study by Sheldon and Roe (2009) modeled trade in credence goods in the presence of a voluntary certification program in an oligopoly setting. They find that market integration results in increased provision of quality in the presence of a third-party certifier by ensuring high-quality goods are produced even if regulators set sub-optimal legal standards.

In the following sections, we present a model in the HFT framework incorporating participation in a credible voluntary standard (or certification) along with fixed export market entry costs and positive transportation costs. Firms make their export and certification decisions simultaneously, so the model yields predictions concerning the relationship between liberalization and the adoption of voluntary standards. Modeling this relationship for the provision of credence goods makes these results applicable to debates over trade liberalization and product safety, sustainability and labor practices.

The remainder of the paper is outlined as follows: the modeling environment and model equilibrium are described in section two. Comparative statics for three policy-relevant parameters are presented in section three. Concluding remarks and directions for future work are presented in section four.
2. Model Framework

2.1 Consumption

Consumers in each country maximize a utility function characterized by a constant elasticity of substitution \((\sigma > 1)\) among each of the \(\omega \in \Omega\) varieties available in their home market.

Consumers solve:

\[
\max_{x_{i}(\omega)} U = \left( \int_{\omega \in \Omega_i} \left( \frac{1}{\sigma} \lambda(q_{\omega})^{\frac{1}{\sigma}} x(\omega) \right)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}}
\]

subject to

\[
\int_{\omega \in \Omega_i} p(\omega) x(\omega) \leq E
\]

The quantity of variety \(\omega\) consumed in country \(i\) is \(x_i(\omega)\). The unit price of variety \(\omega\) in country \(i\) is \(p_i(\omega)\). Total expenditure in the country is \(E_i = w_i L_i\), where \(w_i\) is the wage rate in country \(i\), and \(L_i\) is the total labor supply in \(i\). The term \(\lambda(q_{\omega})\) captures the effect of vertical differentiation on consumer behavior. It acts as a demand shifter, allocating larger budget shares to varieties with higher quality \((q_{\omega})\). For simplicity, assume \(\lambda(q_{\omega}) = q_{\omega}^\gamma\) and \(\gamma \geq 0\).

The consumer maximization problem yields the following demand function:

\[
x_i(\omega) = p_i(\omega)^{-\sigma} \lambda(q_{\omega}) \frac{E_i}{\tilde{P}_i^{\frac{\sigma}{\sigma-1}}}
\]

where \(\tilde{P}\) is the quality-adjusted CES price index:

\[
\tilde{P}_i \equiv \left( \int_{\omega \in \Omega_i} \lambda(q_{\omega}) \cdot p_i(\omega)^{1-\sigma} d\omega \right)^{\frac{1}{1-\sigma}}
\]

Following Podhorsky (2010), this model assumes that consumers derive more utility from higher quality varieties, but cannot observe the quality of the variety themselves. Consumers are aware firms can participate in a credible voluntary standard.
that will certify whether they meet the (exogenously determined) minimum quality standard: \( q_\omega \geq q_H \). Consumers therefore perceive the quality of each variety (\( \omega \)) as:

\[
q_\omega = \begin{cases} 
q_H & \text{if certified} \\
q_L & \text{otherwise}
\end{cases}
\]

The sum of attributes observable by the consumer can be thought of as \( q_L \). Since there are no returns to investments in product quality above \( q_H \) or between \( q_H \) and \( q_L \), this specification of consumer preferences turns the firm’s choice of optimal quality into a binary decision determined exactly by the firm’s optimal certification strategy.

2.2 Production

As in Melitz (2003), firms are monopolistically competitive and heterogeneous in terms of their underlying productivity, here represented by the parameter \( \theta \). Following Melitz (2003), assume \( \theta \) follows a Pareto distribution with the cumulative distribution function

\[
G(\theta) = 1 - \left( \frac{\theta}{\theta_0} \right)^{-s},
\]

where \( \theta_0 \) is the lower bound on the support of \( G(\theta) \) and \( s > 1 \) is the scale parameter. Firms must sink an entry cost, \( F_E \), expressed in labor units, to enter the differentiated products sector. Firms do not know their productivity level before entering the industry. Following entry, each firm will maximize operating profit by choosing an optimal price and quality as a function of their productivity. Firms solve:

\[
\max_{p_j(\omega), q_\omega} \pi_j(\omega) = p_j(\omega) x_j(\omega) - w_i c(q_\omega) x_i(\omega)
\]

\( \pi_j(\omega) \) refers to the profit earned in country \( j \) by the firm producing variety \( \omega \) in country \( i \). The firm’s cost function \( c(q_\omega) \) is measured in labor units, paid at wage rate \( w_i \). For simplicity, assume that \( c(q_\omega) = 1 \). When \( j = i \), the profit maximization problem can be solved by substituting (2) into (4) and differentiating with respect to \( p_j(\omega) \). This reveals price is the standard mark-up over marginal cost:
When \( j \neq i \), firms incur the standard “iceberg” transportation costs when they ship their output to the foreign market. The firm must produce \( \tau \) units of output for every unit they sell in the foreign market. The firm therefore solves
\[
\max_{p(\omega), q_\omega} \pi_j(\omega_i) = p_j(\omega_i) x_j(\omega) - w_i c(q_\omega) \tau x_j(\omega)
\]
Substituting (2) into (6) and solving for the profit maximizing price yields:
\[
p_j(\omega_i) = \tau w_i \left( \frac{\sigma}{\sigma - 1} \right) = \tau p_i(\omega_i)
\]
Using (2) and (7) to calculate the revenue firms from country \( i \) earn in each market results in:
\[
p_i(\omega_i) x_i(q_{\omega_i}) = p_i(\omega_i) 1 - \sigma \lambda(q_{\omega_i}) \frac{E_i}{\beta_i - \sigma}
\]
\[
p_j(\omega_i) x_j(q_{\omega_i}) = p_j(\omega_i) 1 - \sigma \lambda(q_{\omega_i}) \frac{E_j}{\beta_j - \sigma}
\]
Substituting (7) into (9) and (2) yields:
\[
p_j(\omega_i) x_j(q_{\omega_i}) = \tau p_i(\omega_i) x_j(q_{\omega_i}) = \{ \tau p_i(\omega_i) \} 1 - \sigma \lambda(q_{\omega_i}) \frac{E_j}{\beta_j - \sigma}
\]
Firm profit in its home market is calculated as:
\[
\pi_i(\omega_i) = p_i(\omega_i) x_i(\omega_i) - w_i x_i(\omega_i)
\]
Substituting from (5) yields:
\[
\pi_i(\omega_i) = p_i(\omega_i) x_i(\omega_i) \left[ 1 - \frac{\sigma - 1}{\sigma} \right] = \frac{p_i(\omega_i) x_i(\omega_i)}{\sigma}
\]
So profits are simply a constant fraction of total revenues. A similar calculation is performed to find the profit a firm earns in a foreign market:
\[
\pi_j(\omega_i) = p_j(\omega_i) x_j(\omega_i) - \tau w_i x_j(\omega_i)
\]
Substituting from (7) yields:
(11) \[ \pi_j(\omega_i) = \frac{p_j(\omega_i) x_j(\omega_i)}{\sigma} \]

Equations (11) and (12) show that firm profit depends on the choice of output quality. The specification of consumer preferences adopted here means that firms must choose either high \((q_H)\) or low \((q_L)\) quality. Following Podhorsky (2010), firms that choose to produce high quality goods must pay a fixed cost (denominated in labor units) to be certified under the voluntary standard. Firms seeking certification incur the following fixed costs:

(12) \[ \pi_j(\omega_i) = \frac{p_j(\omega_i) x_j(\omega_i)}{\sigma} \]

Fixed certification costs are increasing in the strictness of the standard \((q_H - q_L)\), but decreasing in the firm’s productivity \((\theta)\). An identical assumption is made in Podhorsky (2010, 2013). Firms that seek certification under popular voluntary industry standards, such as the ISO family of standards, must make substantial investments in improving their production processes as well as improving information sharing and accountability within the firm (Nishitani (2009)). The notion of productivity implied by (13) can be thought of as the firm’s managerial quality as well as the quality of their production process at the time of certification.

Equations (11) and (12) imply that profits are higher for high-quality firms at every productivity level, while (13) implies that the cost of marketing high-quality goods falls monotonically with productivity. Therefore, there exists a cut-off productivity level \((\theta^*)\) beyond which the cost of producing and certifying high-quality goods is small enough to make \(q_H\) the profit-maximizing level of quality. This cut-off condition is illustrated in Fig. 1. Consider a firm deciding whether or not to sell high-quality output in a given market. If the
firm sells low-quality output, it will earn a payoff equal to $\pi_i(q_L)$. If the firm decides to market high-quality output, it will earn a payoff equal to $\pi_i(q_H) - w_i \delta(\theta)$. Equations (8), (9) and (13) ensure that the payoffs associated with this strategy are non-decreasing and concave in productivity ($\theta$). Firms with $\theta \in [\theta_{min}, \theta^*)$ will choose to sell only low-quality products. Firms with $\theta \in [\theta^*, \infty)$ will pay for certification and sell high-quality goods.

As in Melitz (2003), firms also face a fixed export cost when they enter a foreign market. This can be specified as:

$$F_X(\theta) = \frac{F_X}{\theta}$$

(14)

As with (13), it is assumed fixed export costs are decreasing in productivity. Fixed export costs are also assumed to be independent of quality. If the firm sells output of a given quality only in the domestic market, it will earn a payoff equal to $\pi_i(q_{\omega})$. If the firm decides to sell in both the home and foreign markets, it will earn a payoff equal to $\pi_i(q_{\omega}) + \pi_f(q_{\omega}) - w_i F_X(\theta)$. The result is a cut-off condition similar to the one illustrated for certification.

The payoffs associated with each strategy are shown in Figure 2. As before, equations (8), (9) and (14) ensure the payoff functions associated with this strategy are non-decreasing and concave in productivity. Firms with $\theta \in [\theta_{min}, \theta^X)$ will choose to serve the domestic market only. Firms with $\theta \in [\theta^X, \infty)$ will sink the fixed export cost and sell output of a given quality ($q_{\omega}$) in both the foreign and domestic markets.

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Melitz (2003) assumes marginal production costs are decreasing in productivity, but this distinction is unimportant. As long as pay-offs are monotonically increasing in productivity and slope at different rates, the assumption made here simply serves to make the model more tractable.
2.3 Characterizing Model Equilibrium

The model structure outlined above implies firms must choose their export and certification strategies simultaneously. Payoffs for each potential strategy for a firm in country $i$ are presented in Table I. The highest productivity firms will always sell high quality goods and export. Call this the \textit{HE} strategy. To see this, note that equations (11) and (12) imply operating profit (gross of any fixed costs) in any given market is always positive. Equations (8) and (9) imply that operating profit is always increasing in output quality. From the definition of $G(\theta)$, the support of $G(\theta)$ is such that $\theta \in [\theta, \infty)$. As $\theta$ approaches infinity, $F_x(\theta)$ and $\delta(\theta)$ go to zero. As fixed costs go to zero, firms will always maximize profit by selling high-quality output in as many markets as possible. Similarly, $F_x(\theta)$ and $\delta(\theta)$ go to infinity as $\theta$ approaches $\underline{\theta}$ for small values of $\underline{\theta}$. These firms will maximize profits by minimizing fixed costs, selling low quality output and not exporting. Call this the \textit{LN} strategy.

Placing some reasonable restrictions on certain model parameters, it is possible for a subset of firms to adopt the strategy in either the lower-left or upper-right hand corners of Table I. However, if one of these intermediate strategies is chosen, it will necessarily dominate the other over the relevant range of $\theta$ (see parts 1 and 2 in the appendix).

Assume some firms sell only low-quality goods, but sell them at home and abroad. Call this the \textit{LE} strategy. Some subset of firms at higher levels of productivity will be able to cover the cost of certification using revenues derived from selling high-quality goods only in the

\footnote{For simplicity, it is assumed firms cannot sell different quality output in different markets.}
home country. Call this the $HN$ strategy. Since export costs are already sunk, any firm that could earn positive profit from the $HN$ strategy would maximize profits by also selling them abroad, or by adopting the $HE$ strategy. Therefore, no firms would choose to adopt $HN$ in equilibrium.

Conversely, assume some firms do adopt the $HN$ strategy in equilibrium. Some subset of firms at higher levels of productivity would be able to cover fixed export costs by selling even low quality goods abroad. Since certification costs are already sunk, these same firms would maximize profits by selling high quality goods in the foreign market, or by adopting the $HE$ strategy. Therefore, no firms would choose to adopt the $LE$ strategy in this characterization of the equilibrium. In the analysis that follows, we ignore the case where firms adopt $HN$ in equilibrium, and focus on the case where firms select $LN, LE, or HE$ depending on their level of productivity.$^5$

2.4 Determining Model Equilibrium

The definition of the model equilibrium can be derived using three pieces of information. First, the payoff matrix can be used to define the productivity cut-offs separating each strategy.

Call $\theta^A$ the productivity satisfying:

$$\pi_i(q_L) + \pi_j(q_L) - w_i F_x(\theta^A) = \pi_i(q_L)$$

or,

$$\pi_j(q_L) = w_i F_x(\theta^A)$$

$^5$ The observed correlation between exporting and participation in voluntary industry standards in the empirical literature suggests this is the most relevant case.
This expression defines the firm that is indifferent between selling in the domestic market and sinking $F_x(\theta)$ to sell output in both the foreign and domestic markets, given it will only be selling low-quality output.

Call $\theta^B$ the productivity satisfying:

$$\pi_i(q_L) + \pi_j(q_L) - w_i F_x(\theta^B) = \pi_i(q_H) + \pi_j(q_H) - w_i \delta(\theta^B) - w_i F_x(\theta^B)$$

or,

$$(16) \quad [\pi_i(q_H) - \pi_i(q_L)] + [\pi_j(q_H) - \pi_j(q_L)] = w_i \delta(\theta^B)$$

This expression defines the firm that is indifferent between selling low-quality and sinking $\delta(\theta)$ to sell high-quality goods, given it will sell in both the domestic and foreign markets.

Finally, the model equilibrium is defined by a zero-profit condition, as in Melitz (2003). Firms do not know their productivity draw before they enter the differentiated product sector, but they do know their expected level of operating profit and the expected costs associated with each strategy. Assume further that firms must sink a fixed entry cost ($F_E$), denominated in labor units, to enter the industry. Firms will continue to enter until their expected profit, net of their expected fixed costs, exactly equals the fixed cost of entry.

Defining expected operating profits as $E[\pi]$, this condition can be expressed as:

$$(17) \quad E_i[\pi] - w_i E[F_x(\theta)] - w_i E[\delta(\theta)] = w_i F_E$$

Equations (15), (16) and (17) allow $\theta^A, \theta^B$ and the equilibrium mass of industry entrants ($M$) to be defined in terms of model parameters. Making the appropriate series of substitutions yields an expression defining the export cut-off $(\theta^A)$ only in terms of model parameters (see appendix 3):

$$(18) \quad (\theta^A)^{-1} F_x \left\{ \frac{(2s+1)\lambda(q_L) - [1+\tau^{1-\sigma}]s\lambda(q_H)}{\lambda(q_L)(s+1)\tau^{1-\sigma}} \right\} + (\theta^A)^{-(s+1)} F_x$$
\[ + (\theta^A)^{(s+1)} \left[ \left( \frac{\lambda(q_H) - \lambda(q_L)}{\lambda(q_L)} \right) \left( 1 + \tau^{\sigma - 1} \right) \right]^{s+1} \frac{F_x^{s+1}}{(q_H - q_L)^s} = F_E \]

The model yields no algebraic closed-form solution, but it is still possible to demonstrate the uniqueness and existence of the equilibrium. Call the left-hand side of (18) \( H(\theta^A) \). Assume parameters are fixed such that the first bracketed term in \( H(\theta^A) \) is strictly non-negative. It is straightforward to see that \( H(\theta^A) \) approaches some positive value as \( \theta^A \to \theta \). It can also be seen that \( H(\theta^A) \) monotonically approaches zero as \( \theta^A \to \infty \). As long as \( F_E \) is not too high, equation (18) identifies the unique equilibrium value of \( \theta^A \) for this model.

Having identified \( \theta^A \), it is possible to derive an expression to identify the corresponding equilibrium cut-off for \( HE \):

\[ \theta^B = \theta^A - \frac{\lambda(q_L)}{F_x[1+\tau^{\sigma - 1}]} \frac{q_H - q_L}{\lambda(q_H) - \lambda(q_L)} \]

A unique expression identifying \( \theta^B \) only in terms of model parameters can also be found by making a series of substitutions similar to those used to derive (18). (See appendix 3). The resulting expression is:

\[ (\theta^B)^{-1}[q_H - q_L] \left\{ \frac{(2s+1)\lambda(q_L) - [1+\tau^{1-\sigma}]s\lambda(q_H)}{[\lambda(q_H) - \lambda(q_L)][1+\tau^{1-\sigma}](s+1)} \right\} + (\theta^B)^{-(s+1)}[q_H - q_L] \]

\[ + (\theta^B)^{-(s+1)} \left( \frac{[q_H - q_L]\lambda(q_L)\tau^{1-\sigma}}{[\lambda(q_H) - \lambda(q_L)][1 + \tau^{1-\sigma}]} \right)^{s+1} F_x^{-s} = F_E \]

Define \( H(\theta^B) \) as the left-hand side of (19). Once again, it can be seen that \( H(\theta^B) \) defines a unique equilibrium value of \( \theta^B \) as long as \( F_E \) is not too high. The equilibrium mass of entrants to the differentiated products sector can also be found using (17) and the equilibrium values of \( \theta^A \) and \( \theta^B \):
The determination of the equilibrium cutoffs using (18) and (19) is illustrated in Figure 3. Equilibrium cut-offs can be found where $H(\theta^A) = H(\theta^B) = F_E$. Equilibrium exists as long as $F_E$ is not too large, so that the points of intersection occur at some $\theta^B > \theta^A \geq \theta$. The range of productivity in the support of $G(\theta)$ is, therefore, divided by the unique equilibrium productivity cut-offs shown in Figure 3.

The full model equilibrium is illustrated in Fig. 4 in productivity and profit space. The payoffs associated with each strategy are shown as a concave function of $\theta$. While $LN$ is constant, $LE$ and $HE$ are both monotonically increasing in productivity. Strategies $LE$ and $HE$ are everywhere steeper in slope than $LN$, but these payoff functions are shifted downward due to their associated fixed costs. The payoff to strategy $HE$ is sloped everywhere more steeply than $LE$, so this strategy will come to dominate over higher ranges of $\theta$. Profits earned by firms over the relevant range of $\theta$ can be seen as the upper envelope of the $LN$, $LE$ and $HE$ functions for $\theta \geq \theta_\ast$.

2.5 Determining the Prevailing Intermediate Strategy

These results demonstrate the existence and uniqueness of the model equilibrium when either intermediate strategy emerges. However, it is not yet clear how to determine which intermediate strategy will prevail. Intuitively, the relative magnitudes of the trade and the certification costs will determine how “quickly” firms begin exporting or certifying their output. If certification is expensive, relative to the additional profit that firms receive from
selling high-quality output, firms in the lower ranges of $\theta$ will be more likely to sink $F_X(\theta)$ and enter export markets, instead. Conversely, if exporting is expensive relative to the additional profit from selling output in the export market, firms in the lower ranges of $\theta$ will be more likely to sink $\delta(\theta)$ and increase output quality.

This comparison can be made more concrete by examining equation (3.10) from appendix 3. Rearranging terms in (3.10) yields:

\[
\frac{\theta^B}{\theta^A} = \frac{(q_H - q_L)}{[\lambda(q_H) - \lambda(q_L)]} \frac{\lambda(q_L)}{F_X(1 + \tau^{\sigma-1})}
\]

Knowing $\theta^B > \theta^A$ implies:

\[
\frac{(q_H - q_L)}{[\lambda(q_H) - \lambda(q_L)]} > \frac{F_X(1 + \tau^{\sigma-1})}{\lambda(q_L)}
\]

Equation (22) is a sufficient condition for the $LE$ strategy to dominate $HN$. According to this expression, the cost of certification for a given level of productivity, $(q_H - q_L)$, relative to the additional profit from increasing output quality $(\lambda(q_H) - \lambda(q_L))$, must be higher than the cost of entering the export market $(F_X)$ relative to the benefits of selling low-quality output in both markets $(\lambda(q_L))$. This makes certification a less appealing option for firms in lower ranges of productivity, which leads them to adopt the $LE$ strategy over the $HN$ strategy.

3. Comparative Statics

Although the model yields no closed-form solution for the cut-off productivities, it is possible to derive comparative statics for the policy-relevant variables in the model. Assuming $q_H$ is set by an independent agency (such as the ISO), the parameters that might be of interest to policy-makers include $F_E$, $F_X$ and $\tau$. Deriving comparative static for a given cut-off $\theta^k$ with respect to some parameter $X$ requires evaluating the following expression:
(23) \[ \frac{\partial q(\theta^k)}{\partial \theta^k} \cdot d\theta^k + \frac{\partial q(\theta^k)}{\partial X} \cdot dX = 0, \]

where \( Q(\theta^k) \equiv H(\theta^k) - F_E \), for \( k = A, B \). The comparative static is therefore:

(24) \[ \frac{d\theta^k}{dX} = -\left[ \frac{\partial q(\theta^k)}{\partial X} \cdot \frac{\partial q(\theta^k)}{\partial \theta^k} \right] \]

As discussed above, and shown in Figure 3, \( H(\theta) \) is everywhere decreasing in \( \theta \), implying the denominator of the right hand side of (24) will always be negative. The sign of each comparative static is therefore determined by the partial differential of \( Q(\theta^k) \) with respect to the parameter in question.

### 3.1 Fixed Entry Costs

Recall \( F_E \) is the fixed cost of entering the differentiated products sector. Changing \( F_E \) is analogous to raising or lowering the barriers to entry for the industry. Deriving the comparative static requires calculating the following:

(25) \[ \frac{\partial q(\theta^A)}{\partial F_E} = \frac{\partial q(\theta^B)}{\partial F_E} = -1 \]

Substituting this result into (24) yields:

(26) \[ \frac{d\theta^A}{dF_E} < 0, \frac{d\theta^B}{dF_E} < 0 \]

Raising the barriers to entry to the differentiated products sector will increase rates of participation in both the voluntary standard and export markets. This result is driven by indirect effects that are not obvious from looking at the payoff functions. Examining (20), the equilibrium number of entrants is decreasing in \( F_E \) for all \( i = A, B \). An increase in \( F_E \) discourages entry, as expected. Fewer entrants implies a less competitive marketplace, which will raise profits at every productivity level for all successful entrants. Firms that
were previously just shy of the productivity cut-offs for exporting and certification will now find themselves sufficiently profitable to justify sinking the associated fixed costs.

3.2 Fixed Export Costs

Deriving comparative statics for fixed export costs \( F_E \) requires evaluating \( \frac{\partial Q(\theta^k)}{\partial F_X} \) for each \( k = A, B \):

\[
(27) \quad \frac{\partial Q(\theta^A)}{\partial F_X} = (\theta^A)^{-1} \left\{ \frac{(2s+1)\lambda(q_L)-[1+\tau^{1-\sigma}][s\lambda(q_H)]}{\lambda(q_L)(s+1)\tau^{1-\sigma}} \right\} + (\theta^A)^{-s+1} \\
+ (\theta^A)^{-s} \left[ \frac{[\lambda(q_H)-\lambda(q_L)]}{\lambda(q_L)} \right]^{s+1} \left( 1 + \tau^{\sigma-1} \right) \frac{F_X^s}{(q_H-q_L)^s} > 0
\]

\[
(28) \quad \frac{\partial Q(\theta^B)}{\partial F_X} = -s(\theta^B)^{-s+1} \left[ \frac{[\lambda(q_H)-\lambda(q_L)]}{\lambda(q_L)} \right]^{s+1} \left( 1 + \tau^{\sigma-1} \right)^{-s} F_X^{-s+1} < 0
\]

Combining (24), (27), and (28) yields:

\[
(29) \quad \frac{d\theta^A}{dF_X} > 0, \quad \frac{d\theta^B}{dF_X} < 0,
\]

Recalling \( \theta^A \) corresponds to an export cut-off, the sign of the corresponding comparative static should not be surprising. Raising \( F_X \) makes exporting more expensive. Firms that were previously indifferent between exporting and not exporting will choose to serve only the domestic market.

The sign on the comparative statics for \( \theta^B \) is less intuitive. This represents the certification cut-off. A (small) change in \( F_X \) will not induce a change in exporting behavior for firms in the region of \( \theta^B \). An increase in \( F_X \) will lower the profits associated with the HE strategy, but it will not lower profits relative to those associated with the \( LE \) strategy. Changes in \( F_X \) therefore have no direct effect on a firm’s optimal certification strategy. The relationship between the certification cut-offs and \( F_X \) instead operates through the CES price indices. Given \( \frac{d\theta^A}{dF_X} > 0 \), raising \( F_X \) will reduce the number of foreign firms entering the
home market. This will make the home market less competitive overall and raise profits for domestic firms. Given a higher profit at every level of productivity, domestic firms with \( \theta \) previously just below the certification cut-off will now be willing to adopt the voluntary certification.

This result supports the hypothesis that lowering trade barriers will lead domestic firms to lower production standards in order to stay competitive. From the firm’s perspective, if the only effect of a change in trade policy is to raise market competitiveness, then the firm will be less willing to make costly investments in higher production standards. It should be noted, however, that the above result is specifically for trade costs that are not related to output quality or to total output. This likely applies to a relatively small set of non-tariff barriers (NTBs), but it is an important result to consider as trade negotiations continue to orient around these types of policies.

3.3 Transportation Costs

Raising transportation costs (\( \tau \)) increases the per-unit costs a domestic firm must pay to sell output in the foreign country. This makes comparative statics for transportation costs of particular interest because they are a close analogy to tariff barriers. The comparative static for the export cut-off \( \theta^A \) is unambiguous. The partial differential of \( Q(\theta^A) \) with respect to (\( \tau \)) is:

\[
\frac{\partial q(\theta^A)}{\partial \tau} = (\sigma - 1)\tau^{\sigma - 2} \frac{(\theta^A)^{-1} F_X}{(s+1)\lambda(q_L)} \left[ (2s + 1)\lambda(q_L) - s\lambda(q_H) \right] \\
+ (s + 1)(1 + \tau^{\sigma - 1})^s (\sigma - 1)\tau^{\sigma - 2} \frac{((\theta^A)^{-1}\lambda(q_H) - \lambda(q_L))F_X}{\lambda(q_L)} \left[ q_H - q_L \right]^s > 0
\]

Substituting (3) into (24) yields:

\[
\frac{d\theta^A}{d\tau} > 0
\]
As with $F_X$, raising transportation costs unambiguously raises the export cut-off. The intuition behind this result is simple: raising the costs associated with shipping each unit to the foreign market makes domestic firms less willing to engage in export markets.

The effect of changes in $\tau$ on the certification decision is ambiguous. Partially differentiating $Q(\theta^B)$ with respect to $\tau$ yields:

$$\frac{\partial Q(\theta^B)}{\partial \tau} = (\sigma - 1)[1 + \tau^{1-\sigma}]^{-2}\tau^{-\sigma}\left\{\frac{(\theta^B)^{-1}(q_H-q_L)\lambda(q_L)}{\lambda(q_H)-\lambda(q_L)}\right\}^{\frac{2s+1}{s+1}}$$

$$-(s + 1)(\sigma - 1)[1 + \tau^{\sigma-1}]^{-(s+2)}\tau^{\sigma-2}\left\{\frac{(\theta^B)^{-1}(q_H-q_L)\lambda(q_L)}{\lambda(q_H)-\lambda(q_L)}\right\}^{s+1} F_X^{-s}$$

It is not possible to sign (32) without imposing further restrictions on the relative magnitudes of certain model parameters. It has been established that increases in $F_X$ lower the certification cut-off for export-competing firms ($\theta^B$). This result derives entirely from the indirect effects of higher fixed export costs on domestic market competitiveness. While export-competing firms considering certification must pay $F_X$, the effect of an increase in $F_X$ is the same whether they sell high-quality or low-quality goods. There is no direct change in the relative profitability of the $LE$ and $HE$ strategies. The same is not true for $\tau$. To see this, differentiate (10) with respect to $\tau$ (ignoring indirect effects through $\tilde{P}$):

$$\frac{\partial \pi_F(q_\omega)}{\partial \tau} = (1 - \sigma)\tau^{-\sigma}p^{1-\sigma}\lambda(q_\omega)\frac{E}{p^{1-\sigma}}$$

Given $\sigma > 1$ and $\lambda(q_H) > \lambda(q_L)$, profits in the foreign market fall faster for sellers of high-quality goods as $\tau$ increases. This result does not depend on differences in the elasticity of demand; rather, it derives from the relatively larger market share held by firms selling high-quality goods in foreign markets. Ignoring changes in $\tilde{P}$, increases in $\tau$ will change the relative profitability of the $LE$ and $HE$ strategies. While firms will still benefit from the indirect effects of decreased domestic market competitiveness, the direct effect will be to
discourage investment in the voluntary certification. If the latter effect is sufficiently large, then an increase in $\tau$ will decrease the rate of certification adoption among export-competing firms, implying $d\theta^B/d\tau > 0$. Rearranging terms in (32) yields:

\[ (33) \quad \theta^B > \left\{ \frac{(q_H - q_L)\lambda(q_L)\tau^{1-\sigma}}{[\lambda(q_H) - \lambda(q_L)]f_X[1+\tau^{1-\sigma}]} \right\} \left\{ \frac{(s+1)^2}{2s+1} \right\}^{\frac{1}{2}}, \]

so the sign of the comparative static will depend on the value of $\theta^B$ from which the model deviates. This is illustrated in Figure 5. Deviating from a relatively low equilibrium value of $\theta^B$, an increase in $\tau$ would decrease the certification cut-off. Deviating from a relatively high equilibrium value of $\theta^B$, an increase in $\tau$ would increase the certification cut-off.

[Place Figure 5 approximately here]

This result suggests that there is no reason a priori to assume that lowering tariff barriers will increase or decrease production standards in a given country. As shown in equation (33), the effect will depend on market conditions, technological constraints, and the existing policy regime. In general, we are more likely to see lower tariff barriers put upward pressure on standards in industries where consumers place a large premium on high quality goods, where the increase in marginal costs to produce high quality goods is small, and where existing trade barriers are relatively high. Note that parameter values in (33) must still obey the condition in (22) for this result to hold.

4 Conclusions

The results presented here improve our understanding of the relationship between participation in international markets and the adoption of a credible voluntary standard. The theoretical model is complementary to Sheldon and Roe (2009), and builds on existing work in the HFT framework by Podhorksy (2010, 2013) by incorporating fixed export
costs and transportation costs. This allows for the derivation of comparative statics for the adoption of a voluntary standard given a change in trade policy. Adoption of the voluntary standard allows firms to overcome an otherwise binding information asymmetry problem similar to the one described in Akerlof (1970) and meet consumer demand for high-quality goods. The model treats quality as a credence attribute, so the framework is broadly applicable to topics of concern in debates over trade policy including product safety, sustainability and labor practices.

Changes in trade policy have the expected effects on firms’ export decisions; raising fixed trade costs or transportation costs decreases the proportion of firms willing to enter export markets. The model can only provide a qualified answer to the question of whether or not lower trade barriers lead to higher production standards in the presence of a voluntary standard. The effect of a change in trade policy on certification adoption depends on the policy instrument in question. Lowering fixed export costs makes the firm’s domestic market more competitive, meaning lower profit levels at every level of productivity. Given the high fixed costs associated with certification, firms that were previously indifferent will choose not to certify. However, lowering transportation costs can encourage certification adoption among export-competing firms. Lowering transportation costs will increase the profits firms earn in the foreign market. The absolute gains from a decrease in \( \tau \) will be greater for producers of high-quality goods due to the higher revenues they earn in the foreign country. Firms that were previously indifferent will therefore choose to adopt the voluntary standard to reap these higher profits.

Transportation costs are a close analogy to tariff barriers, so the latter result is the most relevant in the debate over whether or not trade liberalization can raise production
standards. The answer presented here is a qualified “yes,” but the ambiguity of the results might also explain why empirical analysis has produced conflicting results in different contexts. The model predicts that the relationship between tariffs and production standards, mediated by voluntary certification, will depend on industry- and country-specific characteristics. These include consumer willingness to pay for high quality goods, the effect of quality choice on production costs, and the status quo level of trade barriers. The model can also help inform future empirical analysis by explaining why firm size, sunk costs and export participation are correlated with the adoption of voluntary standards.

There are several key extensions that would significantly expand the set of model predictions. First, being unable to characterize the equilibrium with both export and import-competing certified firms makes it more difficult to apply the model results to a given country context, where these two cases are likely to coexist. This result stems from the fact that heterogeneity is confined to a single dimension. Both fixed export costs and certification costs are a function of the same productivity parameter (θ). As long as fixed export costs are independent of quality and certification costs are independent of export status, the model will generate two mutually exclusive equilibria: one where firms choose certification conditional on exporting, and one where firms choose certification conditional on not exporting. This can be avoided by extending firm heterogeneity to two dimensions, as in Kugler and Verhoogen (2012), but this substantially complicates the analysis. More simply, it would be sufficient to assume higher fixed export costs for high quality goods or higher fixed certification costs for exporters.

The model would also be improved by relaxing the assumption of strict symmetry between the two countries. The comparative statics implicitly assume policymakers
implement identical policy changes in both countries. It would be beneficial to see whether or not these results change when policymakers act unilaterally. Relaxing the symmetry assumption would also allow the model to illustrate trade between a small, developing country and a large, developed country. This might change the underlying relationship between liberalization and certification. It would also be of particular interest because voluntary standards have been so widely adopted in the developing world. Developing countries often lack the political institutions necessary to implement strict legal standards for product safety, environmental protection or labor practices. Voluntary certification provides firms with an incentive to raise standards independent of the action of local regulators.

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Tables and Figures

Table 1: Payoff Functions for Firm Strategies

<table>
<thead>
<tr>
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<th>No Certification (Low Quality)</th>
<th>Certification (High Quality)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Exports</td>
<td>( \pi_i(q_L) ) ((LN))</td>
<td>( \pi_i(q_H) - \delta(\theta) ) ((HN))</td>
</tr>
<tr>
<td>Exports</td>
<td>( \pi_i(q_L) + \pi_j(q_L) - F_x(\theta) ) ((LE))</td>
<td>( \pi_i(q_H) + \pi_j(q_H) - \delta(\theta) - F_x(\theta) ) ((HE))</td>
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**Fig. 1.** Determination of the Certification Cut-Off Productivity. Notes: Firms with productivity at or above \( \theta^* \) will choose to adopt the voluntary certification and market high-quality goods. Firms with productivity below \( \theta^* \) will not adopt the certification and market low-quality goods.
Fig. 2. Determination of the Export Cut-Off Productivity. Notes: Firms with productivity at or above $\theta^X$ will choose to enter the export market. Firms with productivity below $\theta^X$ will serve only the domestic market.
Fig. 3. Determination of Equilibrium Productivity Cut-Offs. Notes: Firms with productivity in the range $[\theta^A, \theta^B)$ will choose the LN strategy. Firms with productivity in the range $[\theta^B, \infty)$ will adopt the LE strategy. Firms with productivity in the range $[\theta^B, 0)$ will adopt the HE strategy.
Fig. 4. Equilibrium Payoffs in the LN/LE/HE Case. Notes: Payoffs to firms are given by the upper envelope of the individual payoff functions for each strategy. As before, firms with productivity in the range $[\theta, \theta^A)$ will choose the LN strategy. Firms with productivity in the range $[\theta^A, \theta^B)$ will adopt the LE strategy. Firms with productivity in the range $[\theta^B, \infty)$ will adopt the HE strategy.
Fig. 5. The Effect of Rising Transportation Costs on the Certification Cut-off. Notes: As transportation costs rise from $\tau$ to $\tau'$, the certification productivity cut-off rises from $\theta^B$ to $\theta^{B'}$. This implies that higher variable trade costs will decrease adoption of the voluntary standard, decreasing the average quality of output.
Appendix

1: Eliminating $HN$ from the $LN$/$LE$/$HE$ Case

It must be shown that whenever any subset of firms chooses to export low-quality products, it must be that no firm would choose to sell high-quality products in their home market. If some firms choose the $LE$ strategy, then there must exist some $\theta$ s.t.:

$$\pi_i(q_L) < \pi_i(q_L) + \pi_j(q_L) - w_iF_X(\theta)$$

or,

(1.1) \hspace{1cm} w_iF_X(\theta) < \pi_j(q_L)

This same range of $\theta$ must also satisfy:

$$\pi_i(q_H) + \pi_j(q_H) - w_iF_X(\theta) - w_i\delta(\theta) < \pi_i(q_L) + \pi_j(q_L) - w_iF_X(\theta)$$

or

(1.2) \hspace{1cm} [\pi_i(q_H) - \pi_i(q_L)] + [\pi_j(q_H) - \pi_j(q_L)] < w_i\delta(\theta)

Equations (A1a) and (A2a) jointly imply that the $HN$ strategy is strictly dominated. In other words, they imply:

$$\pi_i(q_H) - w_i\delta(\theta) < \pi_i(q_L) + \pi_j(q_L) - w_iF_X(\theta)$$

Rearranging terms in (A3):

(1.3) \hspace{1cm} [\pi_i(q_H) - \pi_i(q_L)] - w_i\delta(\theta) < \pi_j(q_L) - w_iF_X(\theta)

Equation (1.2) implies the left-hand side of (1.3) is strictly negative, given the result from (7) and (8) that operating profit is everywhere increasing in quality. Equation (1.1) implies the right-hand side of (1.3) is strictly positive. This ensures (1.3) holds as long as (1.1) and (1.2) are true. Combined with the concavity and monotonicity of the payoffs described in the matrix, this ensures that the No Exports/Certification strategy will be
strictly dominated over the whole range of $\theta$.

2. Eliminating $LE$ from the $LN/HN/HE$ Case

It must be shown that, whenever any subset of firms chooses to sell high-quality products only in the domestic market, it must be that no firm would choose to export low-quality products. If some firms choose the No Export/Certification strategy, then there must exist some $\theta$ s.t.:

$$\pi_i(q_l) < \pi_i(q_h) - w_i \delta(\theta)$$

or,

$$(2.1) \quad w_i \delta(\theta) < \pi_i(q_h) - \pi_i(q_l)$$

This same range of $\theta$ must also satisfy:

$$\pi_i(q_H) + \pi_j(q_H) - w_i F_X(\theta) - w_i \delta(\theta) < \pi_i(q_H) - w_i \delta(\theta)$$

or

$$(2.2) \quad \pi_j(q_H) < w_i F_X(\theta)$$

Equations (2.1) and (2.2) jointly imply that the Export/No Certification strategy is strictly dominated. In other words, they imply:

$$\pi_i(q_L) + \pi_j(q_L) - w_i F_X(\theta) < \pi_i(q_H) - w_i \delta(\theta)$$

Rearranging terms:

$$(2.3) \quad \pi_j(q_L) - w_i F_X(\theta) < \pi_i(q_H) - \pi_i(q_L) - w_i \delta(\theta)$$

Equation (2.2) implies the right-hand side of (2.3) is strictly negative. Equation (2.1) implies the right-hand side of (2.3) is strictly positive. This ensures (2.3) holds as long as (2.1) and (2.2) are true. Combined with the concavity and monotonicity of the
payoffs described in the matrix, this ensures that the Exports/No Certification strategy will be strictly dominated over the whole range of $\theta$.

3. Definition of the Model Equilibrium in the LN/LE/HE Case

Equations (15), (16) and (17) can be used to demonstrate the existence and uniqueness of the model equilibrium in the case where the strategies designate $LN$, $LE$, and $HE$ dominate. It is first necessary to establish several preliminary results. Take the definition of the quality-adjusted CES price index:

$$\bar{P}_i^{1-\sigma} = \int_{\omega \in \Omega_i} \lambda(q_\omega) \cdot p_i(\omega)^{1-\sigma} d\omega$$

For the two-country case, it can be expressed as:

$$\bar{P}_i^{1-\sigma} = M_i \left\{ \int_{\theta_i^A}^{\theta_i^B} \lambda(q_L) \cdot p_i^{1-\sigma} g(\theta) d\theta 
+ \int_{\theta_i^A}^{\theta_i^B} \lambda(q_L) \cdot p_i^{1-\sigma} g(\theta) d\theta \right\}$$

$$+ M_j \left\{ \int_{\theta_j^A}^{\theta_j^B} \lambda(q_L) \cdot (\tau p_j)^{1-\sigma} g(\theta) d\theta 
+ \int_{\theta_j^A}^{\theta_j^B} \lambda(q_H) \cdot (\tau p_j)^{1-\sigma} g(\theta) d\theta \right\}$$

Note the asymmetry between the domestic and foreign contributions to the price index: the index for country $i$ includes all country $i$ firms, but only includes the subset of country $j$ firms that opt into exporting. For simplicity, assume there are two symmetric countries, in the sense that $L_i = L_j$. This implies (3.2) can be rewritten as:

$$\bar{P}^{1-\sigma} = Mp^{1-\sigma} \left\{ \lambda(q_L) \int_{\theta}^{\theta_B} g(\theta) d\theta 
+ \lambda(q_H) \int_{\theta_B}^{\theta} g(\theta) d\theta \right\}$$

$$+ M(\tau p)^{1-\sigma} \left\{ \lambda(q_L) \int_{\theta}^{\theta_B} g(\theta) d\theta 
+ \lambda(q_H) \int_{\theta_B}^{\theta} g(\theta) d\theta \right\}$$

Recalling the definition of the distribution function $G(\theta)$, this can also be rewritten as:
For convenience, define:

\[(3.4) \quad Q_i = \lambda(q_L)G(\theta_i^B) + \lambda(q_H)[1 - G(\theta_i^B)]\]

This represents the average quality level produced in a given country. Substituting from (C4), (C3) becomes:

\[\bar{p}_{1-\sigma} = Mp_{1-\sigma}\{Q + \tau_{1-\sigma}(Q - \lambda(q_L)G(\theta^A))\}\]

Or,

\[\bar{p}_{1-\sigma} = Mp_{1-\sigma}\{(1 + \tau_{1-\sigma})Q - \tau_{1-\sigma}\lambda(q_L)G(\theta^A)\}\]

From (2):

\[\pi_i(\omega_i) = p_i(\omega)\frac{E_i}{\sigma\bar{p}_{1-\sigma}}\]

This is the profit a firm from country \(i\) earns by selling output with quality \(q_\omega\) in country \(i\). Allowing for symmetry and substituting from (3.5) yields:

\[\pi_i(\omega_i) = \lambda(q_\omega)\frac{E_i}{\sigma\bar{p}_{1-\sigma}}\frac{E}{\sigma M(1 + \tau_{1-\sigma})Q - \tau_{1-\sigma}\lambda(q_L)G(\theta^A)}\]

Similarly, the profits a firm in country \(i\) earns by selling output with quality \(q_\omega\) in country \(j\) can be expressed by substituting (3.5) into (9) and allowing for symmetry yields:

\[\pi_j(\omega_i) = \tau_{1-\sigma}\lambda(q_\omega)\frac{E}{\sigma M(1 + \tau_{1-\sigma})Q - \tau_{1-\sigma}\lambda(q_L)G(\theta^A)}\]

Substitute this result into (15):

\[\tau_{1-\sigma}\lambda(q_L)\frac{E}{\sigma M(1 + \tau_{1-\sigma})Q - \tau_{1-\sigma}\lambda(q_L)G(\theta^A)} = w_i F_\chi(\theta^A)\]

Rearranging terms yields:

\[\frac{L}{\sigma M(1 + \tau_{1-\sigma})Q - \tau_{1-\sigma}\lambda(q_L)G(\theta^A)} = \frac{F_\chi(\theta^A)}{\tau_{1-\sigma}\lambda(q_L)}\]
Substituting (3.6) and (3.7) into (15) and rearranging terms yields:

\[ \frac{L \cdot [\lambda(q_H) - \lambda(q_L)] [1 + \tau^{1-\sigma}]}{\sigma M [(1 + \tau^{1-\sigma}) Q - \tau^{1-\sigma} \lambda(q_L) g(\theta^A)]} = \delta(\theta^B) \]

Or,

\[ \frac{L}{\sigma M [(1 + \tau^{1-\sigma}) Q - \tau^{1-\sigma} \lambda(q_L) g(\theta^A)]} = \frac{\delta(\theta^B)}{[\lambda(q_H) - \lambda(q_L)] [1 + \tau^{1-\sigma}]} \]

Equating (3.8) and (3.9) yields:

\[ \frac{\delta(\theta^B)}{[\lambda(q_H) - \lambda(q_L)] [1 + \tau^{1-\sigma}]} = \frac{F_x(\theta^A)}{\tau^{1-\sigma} \lambda(q_L)} \]

This expression defines \( \theta^B \) in terms of \( \theta^A \) and model parameters, and vice-versa.

Defining the equilibrium requires deriving an expression that defines one of the variables of interest only in terms of model parameters. Given (3.7), it is only necessary to derive one additional expression defining \( \theta^A \) and \( \theta^B \) as a function of model parameters.

Finding such an expression requires making use of (15). The expected operating profit term \( E_i[\pi] \) can be expressed as:

\[ E_i[\pi] = \int_{q_L}^{\theta^A} \pi_i(q_L) g(\theta) d\theta + \int_{\theta^A}^{\theta^B} \left[ \pi_i(q_L) + \pi_j(q_L) \right] g(\theta) d\theta + \int_{\theta^B}^{\theta^H} \left[ \pi_i(q_H) + \pi_j(q_H) \right] g(\theta) d\theta \]

Substituting from (3.6) and (3.10) and allowing for symmetry allows (3.11) to be rewritten as:

\[ E_i[\pi] = \frac{L}{\sigma M [(1 + \tau^{1-\sigma}) Q - \tau^{1-\sigma} \lambda(q_L) g(\theta^A)]} \left\{ \lambda(q_L) \int_{q_L}^{\theta^A} g(\theta) d\theta + [1 + \tau^{1-\sigma}] \lambda(q_L) \int_{\theta^A}^{\theta^B} g(\theta) d\theta + [1 + \tau^{1-\sigma}] \lambda(q_H) \int_{\theta^B}^{\theta^H} g(\theta) d\theta \right\} \]

Or,
\[ E_i[\pi] = \frac{E}{\sigma M(1 + \tau^{1-\sigma})Q - \tau^{1-\sigma}\lambda(q_L)G(\theta^A)} \{\lambda(q_L)G(\theta^A) + \\
[1 + \tau^{1-\sigma}]\lambda(q_L)[G(\theta^B) - G(\theta^A)] + [1 + \tau^{1-\sigma}]\lambda(q_H)[1 - G(\theta^B)] \}
\]

And finally, after substituting from (3.5):
\[ E_i[\pi] = \frac{E}{\sigma M} \]

Substituting this into (15) yields:
\[ \frac{E}{\sigma M} - w_iE[F_\chi(\theta)] - w_iE[\delta(\theta)] = w_iF_E \]

or,
\[ (3.12) \quad \frac{L}{\sigma M} - E[F_\chi(\theta)] - E[\delta(\theta)] = F_E \]

Equation (3.12) can be further simplified by evaluating the expected values of the fixed export and certification costs. Because only a subset of firms will sink \(F_\chi(\theta)\) and \(\delta(\theta)\), the remaining terms in (3.12) must be evaluated as conditional expectations. The expected fixed export costs are therefore:
\[ (3.13) \quad E[F_\chi(\theta)] = E[F_\chi(\theta)|\theta \geq \theta^A] = \int_{\theta^A}^{\infty} F_\chi(\theta)\mu(\theta) d\theta \]

Where \(\mu(\theta) = \frac{g(\theta)}{1-G(\theta^A)}\). Substituting this expression and (14) into (3.13) yields:
\[ E[F_\chi(\theta)] = \frac{F_\chi}{1-G(\theta^A)} \int_{\theta^A}^{\infty} \theta^{-1} g(\theta) d\theta \]

From the definition of \(G(\theta), g(\theta) = s\theta^{-(s+1)}\). This implies:
\[ E[F_\chi(\theta)] = \frac{sF_\chi}{(\theta^A)^{s+1}} \int_{\theta^A}^{\infty} \theta^{-(s+2)} d\theta \]

And finally,
\[ (3.14) \quad E[F_\chi(\theta)] = \frac{s}{s+1} \frac{F_\chi}{\theta^A} = \frac{s}{s+1} F_\chi(\theta^A) \]

A similar expression for the expected certification costs can also be derived:
\[ (3.15) \quad E[\delta(\theta)] = E[\delta(\theta)|\theta \geq \theta^B] = \int_{\theta^B}^{\infty} \delta(\theta)\mu(\theta) d\theta \]
Which implies:

\[(3.16) \quad E[\delta(\theta)] = \frac{s}{s+1} \frac{[q_H - q_L]}{\theta^B} = \frac{s}{s+1} \delta(\theta^B)\]

Substituting (3.14) and (3.16) into (3.12) yields:

\[(3.17) \quad \frac{L}{\sigma M} - \frac{s}{s+1} F_x(\theta^A) - \frac{s}{s+1} \delta(\theta^B) = F_E\]

This expression is now in terms of all three endogenous variables: \(M, \theta^X\) and \(\theta^C\).

Substituting (3.8) into (C17):

\[(3.18) \quad \frac{F_x(\theta^A)}{\tau^{1-\sigma} \lambda(q_L)} \left\{ (1 + \tau^{1-\sigma})Q - \tau^{1-\sigma} \lambda(q_L)G(\theta^A) \right\} - \frac{s}{s+1} F_x(\theta^A) - \frac{s}{s+1} \delta(\theta^B) = F_E\]

Recalling the definition of \(Q\) from (3.4), (3.18) is now an expression in terms of only \(\theta^A, \theta^B\) and model parameters. Combining (3.18) with (3.10) will define the equilibrium value of either \(\theta^A\) or \(\theta^B\) in terms of only model parameters. Before proceeding to this final expression, it is possible to simplify the bracketed term on the left-hand side of (3.18) by substituting from (3.4) and the definition of \(G(\theta)\).

\[(3.19) \quad \left\{ (1 + \tau^{1-\sigma}) \left[ \lambda(q_L) - G(\theta^B) \left( \lambda(q_H) - \lambda(q_L) \right) \right] - \tau^{1-\sigma} \lambda(q_L)G(\theta^A) \right\} =
\left\{ (1 + \tau^{1-\sigma}) \left[ \lambda(q_L) - (1 - (\theta^B)^{-s}) \left( \lambda(q_H) - \lambda(q_L) \right) \right] - \tau^{1-\sigma} \lambda(q_L)(1 - (\theta^A)^{-s}) \right\} =
\left\{ \lambda(q_L) + (\theta^B)^{-s} \left( \lambda(q_H) - \lambda(q_L) \right)(1 + \tau^{1-\sigma}) + \tau^{1-\sigma} \lambda(q_L)(\theta^A)^{-s} \right\}\]

Substituting (3.19) into (3.18) yields:

\[(3.20) \quad \frac{F_x(\theta^X)}{\tau^{1-\sigma} \lambda(q_L)} \left\{ \lambda(q_L) + (\theta^B)^{-s} \left( \lambda(q_H) - \lambda(q_L) \right)(1 + \tau^{1-\sigma}) + \tau^{1-\sigma} \lambda(q_L)(\theta^A)^{-s} \right\}
- \frac{s}{s+1} F_x(\theta^A) - \frac{s}{s+1} \delta(\theta^B) = F_E\]

The last two terms on the left-hand side of (C20) can be rewritten as:

\[\frac{s}{s+1} \left\{ F_x(\theta^A) + \delta(\theta^B) \right\}\]
Substituting (3.10) into this expression yields:

\[
\frac{s}{s+1} \left\{ F_x(\theta^A) + F_x(\theta^A) \frac{\lambda(q_H)-\lambda(q_L)}{\lambda(q_L)} \left[ 1 + \tau^{1-\sigma} \right] \right\} \\
= \frac{s}{s+1} \left\{ F_x(\theta^A) \frac{\lambda(q_H)}{\lambda(q_L)} [1 + \tau^{1-\sigma}] - F_x(\theta^A)\tau^{1-\sigma} \right\}
\]

Replacing this expression in (3.20) and collecting terms yields:

\[
F_x(\theta^A)\tau^{1-\sigma} + F_x(\theta^A) \frac{\lambda(q_H)-\lambda(q_L)}{\lambda(q_L)} \left[ 1 + \tau^{1-\sigma} \right] (\theta^B)^{-s} + F_x(\theta^A)(\theta^A)^{-s} \\
- \frac{s}{s+1} \left\{ F_x(\theta^A) \frac{\lambda(q_H)}{\lambda(q_L)} [1 + \tau^{1-\sigma}] - F_x(\theta^A)\tau^{1-\sigma} \right\} = F_E
\]

or equivalently,

\[
\frac{F_x(\theta^A)}{s+1} \left\{ \tau^{1-\sigma} \left[ 2s + 1 - s \frac{\lambda(q_H)}{\lambda(q_L)} \right] - s \frac{\lambda(q_H)}{\lambda(q_L)} \right\} + F_x(\theta^A)(\theta^A)^{-s} \\
+ F_x(\theta^A) \frac{\lambda(q_H)-\lambda(q_L)}{\lambda(q_L)} [1 + \tau^{1-\sigma}] (\theta^B)^{-s} = F_E
\]

Substituting from (3.7) and (14) again yields:

(3.21) \]

\[
(\theta^A)^{-1} F_x \left\{ \frac{(2s+1)\lambda(q_L)-[1+1-\sigma]s\lambda(q_H)}{\lambda(q_L)(s+1)\tau^{1-\sigma}} \right\} + (\theta^A)^{-(s+1)} F_x \\
+ (\theta^A)^{-(s+1)} \left[ \frac{\lambda(q_H)-\lambda(q_L)}{\lambda(q_L)} [1 + \tau^{1-\sigma}] \right]^{s+1} \frac{F_x}{(q_H-q_L)^s} = F_E
\]

Equation (3.19) expresses the equilibrium export cut-off for the LN/LE/HE case \( \theta^A \) in terms of only model parameters. It is possible to derive a similar expression to identify \( \theta^B \) using only model parameters. Substituting (3.10) into (3.20) yields:

(3.22) \]

\[
\frac{L}{\sigma M} - \frac{s}{s+1} \left\{ \delta(\theta^B) + \delta(\theta^B) \frac{\lambda(q_L)\tau^{1-\sigma}}{\lambda(q_H)-\lambda(q_L)[1+1-\sigma]} \right\} = F_E
\]

From (3.6):

\[
\frac{L}{\sigma M} = \frac{\delta(\theta^B)[(1+1-\sigma)Q+\tau^{1-\sigma}]{\lambda(q_L)}G(\theta^A)}{[\lambda(q_H)-\lambda(q_L)][1+1-\sigma]}
\]
Substituting (3.10) into (3.19) yields:

\[(3.23) \quad \{(1 + \tau^{1-\sigma})Q - \tau^{1-\sigma} \lambda(q_L)G(\theta^A)\} = \]
\[
\left\{\lambda(q_L) + (\theta^B)^{-s}(\lambda(q_H) - \lambda(q_L))(1 + \tau^{1-\sigma}) + \tau^{1-\sigma}\lambda(q_L)(\theta^B)^{-s} \left[\frac{[q_H - q_L]\lambda(q_L)\tau^{1-\sigma}}{\lambda(q_H) - \lambda(q_L)[1 + \tau^{1-\sigma}]}\right]^S\right\}
\]

Substituting (3.23) (3.9) and then into (3.22) yields:

\[
\frac{\delta(\theta^B)}{\lambda(q_H) - \lambda(q_L)[1 + \tau^{1-\sigma}]} \left\{\lambda(q_L) + (\theta^B)^{-s}(\lambda(q_H) - \lambda(q_L))(1 + \tau^{1-\sigma}) + \tau^{1-\sigma}\lambda(q_L)(\theta^B)^{-s} \left[\frac{[q_H - q_L]\lambda(q_L)\tau^{1-\sigma}}{\lambda(q_H) - \lambda(q_L)[1 + \tau^{1-\sigma}]}\right]^S\right\} - \frac{s}{s+1} \delta(\theta^B) \left\{1 + \frac{\lambda(q_L)\tau^{1-\sigma}}{\lambda(q_H) - \lambda(q_L)[1 + \tau^{1-\sigma}]}\right\} = F_E
\]

Or,

\[(3.24) \quad (\theta^B)^{-1}[q_H - q_L] \left\{\frac{(2s+1)\lambda(q_L) - [1 + \tau^{1-\sigma}]\lambda(q_H)}{[\lambda(q_H) - \lambda(q_L)(1 + \tau^{1-\sigma})[s+1]}\right\} + (\theta^B)^{-s+1}[q_H - q_L]
\]
\[
+ (\theta^B)^{-s+1} \left[\frac{[q_H - q_L]\lambda(q_L)}{[\lambda(q_H) - \lambda(q_L)]}\right]^{s+1} \left[[1 + \tau^{\sigma-1}]^{-(s+1)}F_x\right]^{-s} = F_E
\]

Equation (3.24) defines the equilibrium certification productivity cut-off ($\theta^B$) in terms of only model parameters.

Note that additional assumptions are required to ensure some intermediate range of firms between $\theta^A$ and $\theta^B$ choose to adopt the LE strategy. Specifically, rearranging terms in (3.7) yields:

\[(3.25) \quad \frac{\theta^B}{\theta^A} = \frac{(q_H - q_L)}{[\lambda(q_H) - \lambda(q_L)]} \frac{\lambda(q_L)}{F_x(1 + \tau^{\sigma-1})}
\]

Knowing $\theta^B > \theta^A$ implies:

\[(3.26) \quad \frac{(q_H - q_L)}{[\lambda(q_H) - \lambda(q_L)]} > \frac{F_x(1 + \tau^{\sigma-1})}{\lambda(q_L)}
\]

Equation (C26) is a sufficient condition for the LE strategy to dominate HN.

According to this expression, the cost of certification for a given level of productivity
\((q_H - q_L)\), relative to the additional profit from increasing output quality \((\lambda(q_H) - \lambda(q_L))\), must be higher than the cost of entering the export market \((F_X)\) relative to the benefits of selling low-quality output in both markets \((\lambda(q_L))\). This makes certification a less appealing option for firms in lower ranges of productivity, which leads them to adopt the \(LE\) strategy over the \(HN\) strategy.
References


