

# **A Heterogeneous Agent Model of Strategic Credit Card Default over the Business Cycle**

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September 25, 2015

## **Abstract**

We examine the economy-wide consequences of strategic credit card default using a structural dynamic model of heterogeneous agents subject to both idiosyncratic and systemic unemployment shocks. We begin by deriving the steady-state distribution of wealth and aggregate credit-worthiness, net savings, and loan default rates in the absence of cyclical movements in the economy. The analysis yields a deeper understanding of the strategic default behavior of borrowers. Specifically, the model produces non-monotonic, non-convex optimal savings/borrowing policies for credit-worthy agents that exhibit three distinct regimes in which the agent will: i) make the minimum required interest payment on her debt, if any, thus remaining credit-worthy; ii) default on her debt, thus being labeled credit-unworthy and being permanently barred from borrowing in the future; or iii) “max-out” her credit card, taking out the maximum allowable debt in anticipation of defaulting in the following period. We then introduce a business cycle with two states of the economy, a “normal state” of relatively low aggregate unemployment and a “recession state” with relatively high aggregate unemployment. We calibrate the model to match historically observed credit card charge-off rates between 1990 and 2011. Simulations of the business cycle model suggest that the charge-off rate does not respond immediately when a recession hits, but increases dramatically in the second and third period following the onset of a recession. When the recession ends, high default rates persist for one period before subsequently beginning to decline.

## 1. Introduction

Since the collapse of subprime mortgages in 2007 and during the ensuing Global Financial Crisis of 2007 to 2009, there has been a substantial increase in credit card charge-off rates, that is, the value of loans written off as uncollectible and charged against loss reserves as a percentage of total outstanding credit card debt. As shown in Figure 1, at the beginning of the recent Financial Crisis in 2007, the charge-off rate on credit card in the U.S. was 4.3%. However, by the second quarter of 2010, more than 10% of credit card debt was considered unrecoverable. The rise in default rates has been attributed to a variety of causes, including high unemployment rate, low lending standards, reductions in default costs, and to strategic “maxing out” of credit cards, that is, charging up to the allowable limit in anticipation of defaulting. The recent dramatic upswing in the charge-off rate, however, was not unprecedented. As shown in Figure 1, there have been several credit default cycles over the past two decades, all of which appear to be closely tied to variations in aggregate employment.

In order to manage losses caused by unsecured loan defaults and to stabilize financial markets, we need to develop a deeper understanding of the factors that contribute to individual default decisions. Specifically, we need to investigate how forward-looking consumers, when facing uncertainty, make repayment/default decisions while considering the direct and indirect cost associate with defaulting. We also need a deeper understanding of the role played by credit and liquidity constraints, employment uncertainty, credit worthiness, current and future aggregate economic conditions etc. Given an understanding of individual default decisions, we can then obtain a better understanding of how aggregate credit card default rates vary with aggregate employment conditions, aggregate savings, and aggregate borrowing.

Gross and Souleles 2002 and Cohen-Cole and Duyan-Bump 2008 argue that the aggregate default rate rises because the costs and “social stigma” effects associated with default have declined. Dick and Lehnert 2007 and Keys et al. 2008 find that banking deregulation has led to a decline in credit quality, increasing the overall loan default and bankruptcy rates. There are also several studies that have examined variation of aggregate default rate over the business cycle yielding inconclusive and contradictory findings regarding whether default is procyclical or countercyclical.

To better understand credit card default, we must better understand the factors that determine the strategic default behavior on unsecured debt in the U.S. and how the rate varies over the business cycle. To this end, we build a structural dynamic model of a population of heterogeneous agents subject to both idiosyncratic and systemic unemployment shocks. The typical, forward-looking agent can borrow on her credit card up to a certain credit limit and has an option to default in any period, subject to indefinite exclusion from the credit market in the future. The idiosyncratic employment shocks give rise to individuals that are heterogeneous with respect to asset holdings, debt levels, employment status, and “credit-worthiness”. In each period, the typical consumer maximizes expected discounted utility over an infinite time horizon by choosing how much to consume, how much to save, how much to borrow, and whether to default on her

credit card debt given savings, borrowing, employment status, credit-worthiness and the state of the economy.

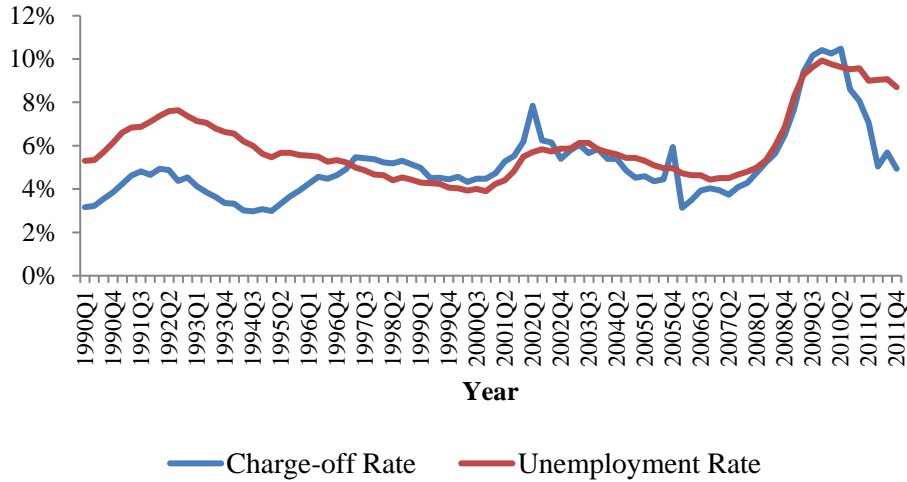


Figure 1 Charge-off Rate and Unemployment from 1990 to 2011

We first solve and simulate the structural model in the absence of a business cycle, allowing only for idiosyncratic employment shocks. In this context, we derive the steady-state distribution of wealth for credit-worthy and -unworthy agents and the steady-state aggregate net savings, credit worthiness, charge-off rates, and default percentage. The parameters of the model are calibrated so that the model replicates average rates of unsecured consumer borrowing and default in the U.S. economy from 1990 to 2011.

The model produces a non-monotonic, non-convex optimal savings/borrowing policy for credit-worthy agents that exhibits three distinct regimes in which the agent will: i) make the minimum required interest payment on her debt, if any, thus remaining credit-worthy; ii) default on her debt, thus being labeled credit-unworthy and being permanently barred from borrowing in the future; or iii) strategically “max-out” her credit card, i.e. taking out the maximum allowable debt in anticipation of defaulting in the following period.

We then introduce a business cycle in which the economy can assume two states: a “normal” state of relatively low aggregate unemployment (5%) and a “recession” state with relatively high aggregate unemployment (9%). We calibrate the other parameters of the model to match historically observed variation in credit card charge-off rates between 1990 and 2011. Simulations of the business cycle model suggest that the charge-off rate exhibits a lagged response to the outset of a recession, increasing dramatically in the second and third period following the onset of a recession. When the recession is over, high default rates also persist for one period, but subsequently decline to pre-recession levels.

The rest of the paper is organized as follows. Section 3 presents the stochastic dynamic model for strategic default with idiosyncratic shock. Section 4 provides the base-case parameterization. Base-case simulation results in the absence of a business cycle are

explained in Section 5. Sensitivity analysis is conducted in Section 6, Section 7, and 8. We then extend the model by introducing a business cycle in which aggregate employment varies over time. The parameterization of the business cycle model is discussed in section 9. Results for the business cycle model are presented in Section 10 and 11. Section 12 summarizes our findings. Additional details are given in Appendices A, B and C.

## **2. Review of Related Studies**

Here, we briefly review previous studies in the area of credit card default. The relationship between employment and credit card default is controversial. Ausubel (1997) finds that credit card defaults are countercyclical using quarterly data from 1973 to 1996; specifically, he finds that upturns in the credit card charge-offs coincide downturns in the U.S. economy. Agarwal and Liu (2003) using data between 1995 and 2001 find that higher aggregate unemployment leads to a rise in 90-day delinquency rates. Their results also indicate credit limit is negatively related to delinquency. Other studies, however, do not find an established relationship between employment and default. Using PSID data, Fay, Hurst and White (2002) show that the willingness to file for bankruptcy increases with the financial benefit associated with filing, providing evidence of strategic behavior in bankruptcy filing decisions. However, they did not find a relationship between county level unemployment and consumer bankruptcy decisions. Lopes (2008), on other hand, developed a model to explain the procyclical nature of aggregate default rates from 1980 to 2002. This paper differs from all of these models in that we explicitly introduce a real business cycle that embraces both individual and systematic employment shock. This framework allows us to examine the aggregate default rate along the business cycle while controlling for other individual characteristics and economic factors.

A number of papers have suggested that the “social stigma” associated with default is a major determinant of default rates. Fay et al. (1998) use the aggregate filing rate in household’s state of residence in the past three years as the reverse proxy for the level of bankruptcy stigma. The authors find the likelihood of filing for bankruptcy rises when stigma falls. Gross and Souleles (2002) investigate how the propensity to default changes over time and find there has been a decline in “social stigma”. Athreya (2004) shows that, as the nonpecuniary cost of bankruptcy or stigma effect falls, the bankruptcy rate increases, but the debts discharged in bankruptcy shrink dramatically. Cohen-Cole and Duygan-Bump (2008) also find that stigma plays an important role in observed bankruptcy trends. The authors find that social stigma seems to increase among the very poor and less educated groups, and stigma seems to decrease the relatively rich and well educated.

Strategic usage of credit cards has also drawn some attention in the literature. Of specific interest is the practice of some borrowers to assume the maximum allowable debt on their credit cards and default in the subsequent billing cycle (or a “max-out” behavior). Only a few studies have studied this behavior among borrowers. Dunn and Kim (1999) using survey data find that the number of cards on which the consumers has reached the borrowing limit is a major predictor of default. Johnson’s (2004) empirical analysis of credit card usage by college students indicates that there is a widespread problem of maxing out on credit card debt. The author cited Dr. Robert Manning’s study and pointed

out that 60% of freshmen and 75% of upperclassmen had maxed out their credit cards at least once. Although strategic “maxing-out” behavior is frequently reported in newspapers and magazines, and discussed among industry analysts, little formal theoretical and empirical analysis has been published in the economic and financial literature. Our structural model is among the first to capture max-out behavior and explain its relationship with defaults, both at the level of the individual and in the aggregate.

A number of other studies on credit card debt and default merit mention here. Agarwal, Liu, and Mielnicki (2002) studied the cost associated with default. They find that garnish allowances negatively affect an individual’s decision to file for bankruptcy and that homestead exemption levels are statistically significant and positively correlated to bankruptcy decisions. They also show that borrower’s financial creditworthiness and macro shocks are important factors in a consumer’s decision to default. Lopes (2008) finds that social stigma and credit limits are critical in explaining bankruptcy rates. The existence of a steady-state equilibrium of unsecured debt default was established by Chatterjee et al (2007), who also characterizes the circumstances under which a household defaults. Bellotti and Crook (2009) estimate a survival model of default using macroeconomic time series data, finding that macroeconomic variables such as bank interest rates and earnings were significant in explaining default. Specifically, the interest rate is positively related to default while increase in earnings leads to lower default risk. Bellotti and Crook (2010) further build dynamic models of default including macroeconomic conditions for UK portfolios of credit cards and use Monte Carlo simulations for stress testing of credit portfolios. The model structure of the heterogeneous agent in our paper is also similar in some respects to those developed in Aiyagari (1994), Krusell and Smith (1997, 1998), and Miranda and Gonzalez-Vega (2009).

### 3. A Model of Strategic Default

#### *The Representative Agent*

An infinitely-lived agent facing employment uncertainty must decide each period how much to consume, how much to borrow or save, and whether to default on any debt obligations she may have. At the beginning of each period, the agent possesses net savings  $s$ , with  $s < 0$  indicating that the agent is carrying debt. The agent may be “employed”,  $i = 1$ , receiving a wage income that is normalized to  $y_1 = 1$ , or “unemployed”,  $i = 0$ , receiving an unemployment benefit  $y_0 < 1$ . The agent also may be “credit-worthy”,  $j = 1$ , indicating she has access to credit, or “credit-unworthy”,  $j = 0$ , indicating otherwise.

Given her employment state  $i$  and her net savings  $s$  at the beginning of the period, the agent must decide how much net savings  $x$  to carry over to the following period. Debt commands an interest rate  $r_d$  and savings earn a risk-free interest rate  $r_s$ , where  $r_d \geq r_s$ . As such, if the agent carries over net savings  $x$  from this period to the next, her net savings at the beginning of next period will be

$$s' = g(x) \equiv \begin{cases} (1 + r_d)x, & x < 0 \\ (1 + r_s)x, & x \geq 0. \end{cases} \quad (1)$$

The agent may save an unlimited amount and, if credit-worthy, may borrow up to an amount  $b > 0$ . A credit-worthy agent who begins the period in debt also has the option to default on her debt obligations. A defaulting credit-worthy agent completely erases her debt, but is immediately declared credit-unworthy. A credit-unworthy agent is barred from borrowing again until her credit is reinstated, which may occur from one period to the next with probability  $\mu \in (0,1]$ , independent of her employment state and net savings. The expected duration of credit-unworthiness state once the agent has defaulted then is simply  $1/\mu$ .

The agent maximizes the present value of current and expected future utility of consumption over an infinite time horizon. The agent's dynamic decision problem is thus characterized by a pair of Bellman equations whose value functions specify the maximum expected present value of lifetime utility  $V_{ij}(s)$  attainable by the agent, given her net savings  $s$ , employment state  $i$ , and credit worthiness state  $j$ :

$$V_{i0}(s) = \max_{x \geq 0} \{u(y_i + s - x) + \delta \sum_{i',j'} q_{ii'} \mu_{j'} V_{i'j'}(g(x))\}, \quad (2)$$

$$V_{i1}(s) = \max \{V_{i0}(0) - \sigma, \max_{x \geq -b} \{u(y_i + s - x) + \delta \sum_{i'} q_{ii'} V_{i1}(g(x))\}\}, \quad (3)$$

for  $s \geq -(1 + r_d)b$ . Here,  $\delta \in (0,1)$  is the agent's per-period discount factor;  $q_{ii'}$  is the probability that the agent's employment state will be  $i'$  next period, given it is  $i$  this period;  $\mu_1 = \mu$  and  $\mu_0 = (1 - \mu)$  are the probabilities of having and not having credit reinstated, respectively;  $\sigma \geq 0$  is a "social stigma" penalty suffered by the agent if she defaults; and  $u$  is the agent's utility, a twice continuously differentiable function of current consumption, with  $u' > 0, u'' < 0$  and  $u'(0) = -\infty$ . Consumption equals labor income  $y_i$  plus net withdrawals from savings  $s - x$ .

Bellman equations (2)-(3) capture the borrowing and saving decisions that must be made by credit-unworthy and credit-worthy agents, respectively. In equation (2), a credit-unworthy agent is not permitted to borrow and thus faces the borrowing constraint  $x \geq 0$ . In equation (3), a credit-worthy agent must decide whether to default on her debt obligation. If she defaults, she erases her debt but immediately becomes credit-unworthy, accepting the value  $V_{i0}(0)$  associated with a credit-unworthy agent with no debt but additionally incurring the social stigma penalty  $\sigma$  attached to defaulting. If she does not default, she remains credit-worthy and faces the borrowing constraint  $x \geq -b$ . A credit-worthy agent thus make a decision of whether to default on her credit card by comparing the value of defaulting with the value of continuing to make a payment.

Let  $\tilde{V}_i$  denote the agent's choice-contingent value function, that is, the value associated with being credit-worthy and employment state  $i$  and choosing not to default (whether optimal or not). Then  $\tilde{V}_i$  is characterized by the functional equation

$$\tilde{V}_i(s) = \max_{x \geq -b} \{u(y_i + s - x) + \delta \sum_{i',j'} q_{ii'} \max\{V_{i'o}(0) - \sigma, \tilde{V}_{i'}(g(x))\}\} \quad (4)$$

for  $s \geq -(1 + r_d)b$ . The choice-contingent value function for a non-defaulting credit-worthy agent  $\tilde{V}_i$  is related to the credit-worthy agent's unconditional value function  $V_{i1}$  via the relation

$$V_{i1}(s) = \max\{V_{i0} - \sigma, \tilde{V}_i(s)\}. \quad (5)$$

The choice-contingent value function can be used to accurately compute the critical net savings level  $s_i^*$  below which a credit-worthy agent in employment state  $i$  will default, which is fully characterized by the nonlinear equation

$$V_{i0}(0) - \sigma = \tilde{V}_{i1}(s_i^*) \quad (6)$$

We let  $x_{ij}(s)$  denote optimal net savings carryover for an agent who begins the period in employment state  $i$  and credit-worthiness state  $j$  with net savings  $s$ . Also, we let  $g_{ij}(s) \equiv g(x_{ij}(s))$  denote the net savings that the agent will hold at the beginning of the following period, given she begins the period in employment state  $i$  and credit-worthiness state  $j$  with net savings  $s$ .

The Bellman equations (2) and (3) are solved using the method of collocation (Judd 1998; Miranda and Fackler 2002). The collocation method calls for the value functions  $V_{ij}(s)$  to be approximated using a linear combination of  $n$  known basis functions  $\varphi_k$ :

$$V_{ij}(s) \approx \sum_{k=1 \dots n} c_{ijk} \varphi_k(s)$$

The  $2 \times 2 \times n$  unknown coefficients  $c_{ijk}$  are then fixed by requiring the value function approximants to satisfy the Bellman equations (2) and (3), not at all net savings levels  $s$ , but rather at  $n$  judiciously chosen collocation nodes  $s_k$ . The collocation method replaces the Bellman functional equations with a set of  $2 \times 2 \times n$  nonlinear equations with  $2 \times 2 \times n$  unknowns that are solved using Newton's method. The collocation method can generate highly accurate approximate solutions to the Bellman equation, provided the basis functions and collocation nodes are chosen judiciously and their number  $n$  is set adequately high. For this dissertation, we chose cubic spline basis functions and equally-spaced nodes to compute approximate solutions for the Bellman equations. The solution was computed using the CompEcon 2010 Toolbox routine *dpsolve* (Miranda and Fackler, 2002).

## The Economy

The economy is composed of a large number of agents who behave as the representative agent, but who otherwise are heterogeneous with respect to net savings, employment, and credit-worthiness as a result of having experienced distinct, idiosyncratic employment transition and credit reinstatement shocks. Employment transitions and credit reinstatements are presumed to be independent and fully diversifiable across agents in the economy.

Each period, a proportion  $\rho$  of employed agents in state  $j$  are “separated” from their jobs and become unemployed.<sup>1</sup> Given the aggregate unemployment state  $\gamma$  and the probability  $q_{ii'}$  that an agent in employment state  $i$  this period will be in employment state  $i'$  next period, it follows that

$$q_{10} = \rho \quad (7)$$

$$q_{01} = \rho \frac{1-\gamma}{\gamma} \quad (8)$$

Here,  $q_{10}$  is the probability an employed agent becomes unemployed and  $q_{01}$  is the probability an unemployed agent becomes employed. Clearly, it is required that

$0 \leq \rho \leq \frac{\gamma}{1-\gamma}$ . The number of periods an agent expects to remain in employment state  $i$ , if currently in employment state  $i$  is  $\psi_i = \frac{1}{1-q_{ii}}$ . The representative agent’s conditional employment transition probabilities  $q_{ii'}$ , and the aggregate unemployment rate must satisfy

$$\gamma = \gamma q_{00} + (1 - \gamma)q_{10} \quad (9)$$

That is, the aggregate unemployment rate equals the aggregate unemployment rate times the conditional probability that an unemployed agent remains unemployed, plus the aggregate employment rate times the conditional probability that an employed agent becomes unemployed.

Let  $P_{ij}(s)$  denote the proportion of agents in the economy who, in steady-state, are in employment state  $i$  and credit worthiness state  $j$ , with initial net savings less than or equal to  $s$ . Since employment transitions and credit reinstatements are independent and fully diversifiable across agents in the economy, it must be that, for  $i' = 0,1$  and  $s' \geq -(1 + r_d)b$ :

$$P_{i'0}(s') = \sum_i q_{ii'} \left( \mu_0 P_{i0}(g_{i0}^{-1}(s')) + P_{i1}(s_i^*) \cdot I(s' \geq g_{i0}(0)) \right)$$

$$P_{i'1}(s') = \sum_i q_{ii'} \left( \mu_1 P_{i0}(g_{i0}^{-1}(s')) + \max\{0, P_{i1}(g_{i1}^{-1}(s')) - P_{i1}(s_i^*)\} \right)$$

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<sup>1</sup> Separation includes quits (voluntary separations), layoffs and discharges (involuntary separations).



Where  $I(\cdot)$  is the Boolean indicator function and

$$g_{ij}^{-1} \equiv \max\{s \mid g_{ij}(s) \leq s'\}$$

Here,  $P_{ij}(g_{ij}^{-1}(s'))$  is the proportion the proportion of agents in the economy who are in employment state  $i$  and credit-worthiness state  $j$ , and who next period will have initial net savings less than or equal to  $s'$ ;  $P_{i1}(s_i^*)$  is the proportion of agents in the economy who are in employment state  $i$  and default on their debt.

Given the steady-state distribution of net savings, the steady-state per-capita saving, debt, and default for agents in employment state  $i$  and credit-worthiness state  $j$  are, respectively,

$$\bar{s}_{ij} = \frac{\int_0^{\infty} s P_{ij}(ds)}{P_{ij}(\infty)},$$

$$\bar{d}_{ij} = -\frac{\int_{-\infty}^0 s P_{ij}(ds)}{P_{ij}(\infty)},$$

and

$$\bar{f}_{i1} = -\frac{\int_{-\infty}^{s_i^*} s P_{i1}(ds)}{\int_{-\infty}^0 s P_{i1}(ds)}.$$

#### 4. Base-Case Parameterization

For our base-case simulations, we calibrate the parameters of our model so as to match the characteristics of the U.S. credit card market over the period 1990 to 2011 (Table 1). More specifically, we assume the coefficient of absolute risk aversion  $\alpha$  is constant and equal to 3.0, the benchmark value used by Carroll (1997) and Lopes (2008). We allow for differential interest rates in the model, one for risk-free rate of return on savings, and one for the interest rate on the agent's revolving debt. The risk-free rate of return is assumed equal to 3%, the average of the three-month Treasury bills rate from 1990 to 2011. The interest rate on credit cards is assumed equal to 12%, the average real rate on credit cards from 1994 to 2011. Income when employed, without loss of generality, is normalized to 1. Unemployed agents are presumed to receive an unemployment benefit of 0.4, which is consistent with observed rates of state unemployment benefits on the order of 40% to 50% of earnings while employed.

An agent's employment state is assumed to follow a first-order, two-state Markov chain with transition probabilities chosen to as to match a 6% unemployment rate and an average of 9 weeks of unemployment duration observed in the U.S. economy from 1990 to 2011 using data from the Bureau of Labor Statistics. Given the aggregate unemployment rate and expected unemployment duration, the conditional probability that

a currently unemployed agent will remain unemployed in the next period is 15.3%, and the probability of an employed agent remaining employed is 94.6%; the employment separation rate, which includes quits, layoffs and discharges out of employed agents, is thus at 5.3%.

We set the credit limit to 0.36 based on 2009 Survey of Consumer Finance, which gives a median household credit limit of \$18,000 and a median household income of \$49,800, that is, 36% of the income of a representative agent when employed. The credit-reinstatement rate  $\mu$  is set at 0.14, corresponding to an average of a 7 year penalty without access to credit as punishment for default.

There are different ways to measure credit card default. Some papers, such as Chatterjee et.al (2007) and Lopes (2008), use the annual consumer bankruptcy rate. However, the personal bankruptcy rate does not adequately reflect the economic impacts of credit card default, because it cannot distinguish the size of the loans that are being defaulted upon. Another measure of credit card default, which we find preferable, is the credit card charge-off rate.<sup>2</sup> Charge-offs, the value of loans written off as uncollectible and charged against loss reserves, are measured net of recoveries as a percentage of average loans. The FDIC requires that revolving debt accounts be charged off if they are under to up to 180 days of delinquency. The charge-off rate has the advantage over bankruptcy rate as it is less sensitive to changes in laws and regulations over time and by state. In our model we set the value of stigma so that the annual average of charge-off rate or loan default rate equals approximately 5%, the average value from 1990 to 2011. The annual subjective discount factor is set to 0.94.

Table 1 Base-Case Parameterization

Parameter	Value	Description
$\alpha$	3.0	Constant relative risk aversion
$r_s$	3%	Risk-free asset rate of return
$r_d$	12%	Borrowing interest rate
$b$	0.36	Borrowing limit
$y_0$	0.4	Unemployment benefit
$y_1$	1	Income for employed agents
$\gamma$	6%	Unemployment rate
$\psi$	1.2	Expected duration of unemployment
$\rho$	5.3%	Employment separation rate
$\mu$	14%	Credit reinstatement rate
$\delta$	0.94	Per-period discount factor
$\sigma$	0.52	Stigma effect
Target	5%	Loan default rate or charge off rate

<sup>2</sup> These are annualized charge-off rates are annualized. Board of Governor's charge-off rates are computed by taking net charge-offs (gross charge-off minus recoveries) for a quarter and dividing by the average level of loans outstanding over the quarter. The percentage is multiplied by 4 to obtain an annualized rate.

## 5. Base-Case Model Results

### *Option Value of Default*

The option to default on debt provides additional value to a dynamically optimizing agent. The value of the option rises with the level of debt. Executing the default option immediately increases the agent's consumption by relieving the agent from the responsibility of having to service debt in the current period or repay it in a future period. However, default carries some costs. In particular, defaulting excludes the agent from borrowing for an indefinite period of time (averaging 7 periods), until credit is reinstated. Defaulting also imposes non-pecuniary social costs arising from being stigmatized in one's community for not meeting debt obligations.

The option to default produces a non-convexity in the agent's value function. Figure 3 shows the value functions of credit-worthy and credit-unworthy agents. Credit-worthy agents, who have access to credit with an option to default, have at least the same or higher discounted expected lifetime utility across all the net savings than their credit-unworthy counterparts. The option value of default decreases as wealth accumulates. Stigma will further lower the option value of default.

The reservation default value  $V_{i,0} - \sigma$  in employment state  $i$  is the lower bound for a credit-worthy agent, as it is always attainable by defaulting. The critical initial savings level,  $S_i^*$ , below which a credit-worthy agent in employment state  $i$  will default, is determined by equation (6). Under the base-case scenario, the critical initial savings level for a credit-worthy agent to default is -0.359 for unemployed agent, while an employed credit-worthy agent will never default.

Table 2 Reservation Value for Default, Credit Worthy Agent

	Unemployed	Employed
$V_{i,0} - \sigma$	-13.41	-10.43

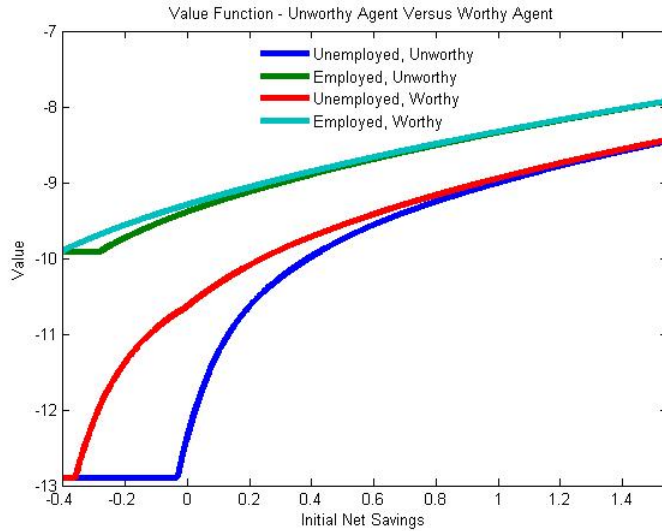


Figure 2 Expected Present Value of Lifetime Utility for Unworthy Unemployed, Unworthy Employed, Worthy Unemployed, and Worthy Employed Agents

### *Optimal Savings Policy*

Figure 4 illustrates the optimal savings policy for employed and unemployed credit-worthy agents. As seen in Figure 4, the optimal policy of the credit-worthy employed agent is non-decreasing, implying that an agent with higher initial savings will have higher ending savings. The only exception is a small flat segment at zero ending savings. The flat segment is a consequence of having different borrowing and savings rates, which leads to a range of initial savings at which it is optimal neither to save nor to borrow.

In contrast, as seen in Figure 4, the optimal savings policy of the credit-worthy unemployed agent is neither monotonic nor convex. Specifically, the optimal savings policy for a credit-worthy unemployed agent exhibits three distinct regimes: i) the agent makes the required minimum interest payment on her debt, if any, thus remaining credit-worthy; ii) the agent defaults on her debt, thus being labeled credit-unworthy and being permanently excluded from borrowing in the future; and iii) the agent “maxes out” her credit card debt, that is, taking out the maximum allowable debt in anticipation of defaulting in the following period.

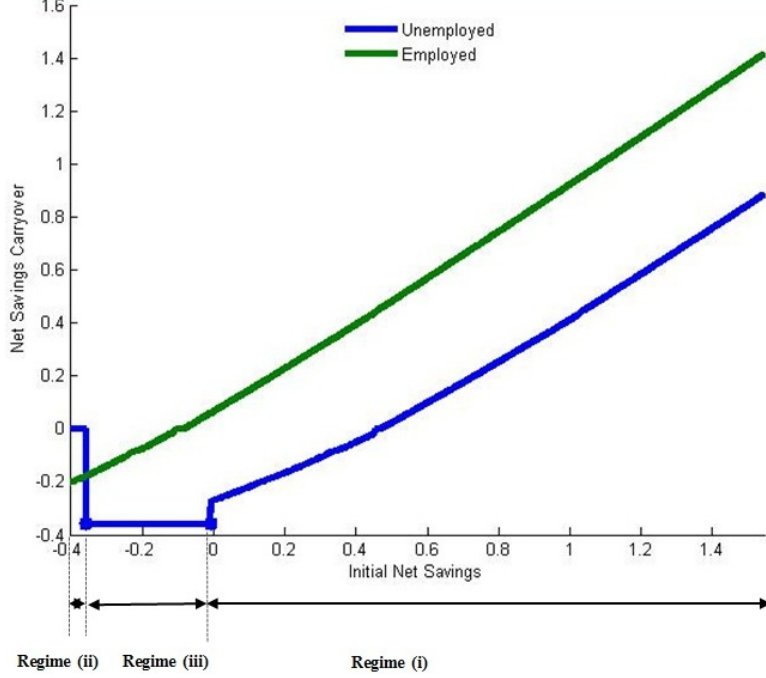


Figure 3 Optimal Savings/Borrowing Policy for Employed and Unemployed Credit-Worthy Agent

Table 3 Critical Net Savings Levels and Max-out Levels for a Credit Worthy Agent under Base-Case Parameters

	Unemployed	Employed
Critical Default Levels	-0.359	N/A
Critical Max-out Levels	-0.007	N/A

A striking result in Figure 4 is the existence of regime (iii). In regime (iii) an unemployed agent will borrow up to the maximum allowable debt, i.e. “max out” her total credit card debt, with the intention of defaulting in the following period if she continues to be unemployed. If it exists,  $s_i^{maxout}$ , the level of initial savings at or below which an agent in employment state  $i$  will borrow the maximum allowable amount  $b$ , is characterized by

$$u'(y_i + s_i^{maxout} + b) = \delta \sum_{i'j'} q_{ii'} \mu_{j'} V'_{i'j'}(-b), \quad (10)$$

where  $s_i^{maxout} \geq -b$ , such that

$$s_i^{maxout} = u'^{-1} \left( \delta \sum_{i'j'} q_{ii'} \mu_{j'} V'_{i'j'}(-b) \right) - y_i - b, \quad i = 0,1$$

When  $s \in [s_i^*, s_i^{maxout}]$ , given employment state  $i$ , credit-worthy agent is acting strategically, honoring her debt obligation this period, but only for to borrow the maximum allowable amount  $b$ , anticipating defaulting the following period should she remain unemployed. Under the base-case parameters, only unemployed agents exhibit the default and “max out” regimes. As seen in Table 3, the critical default level is -0.359 and critical max-out savings level is -0.01 for unemployed agents. Under different parameterizations, default and “max out” behavior will also be observed among the employed, as discussed in the sensitivity analysis section.

### *Steady State Analysis*

Using Monte Carlo methods, we simulated the representative agents and computed a numerical approximation for the steady-state cumulative distribution of net savings, employment status, and credit worthiness. We also computed the steady-state distributions using exact numerical approaches. Both methods were found to generate the same steady state distribution, up to unavoidable Monte Carlo sampling error, for net savings as shown in Figure 5. The details regarding the computation of the steady-state distribution are given in Appendix B.

Credit-worthy agents save less in steady state than credit-unworthy agents. In steady state, the aggregate charge-off rate is 5%, which means that 5% of loan amounts will not be repaid or recovered. Since only unemployed agents default, the loan default rate among the unemployed is as high as 37%, under the baseline parameters. In steady-state, 95% of agents are credit-worthy.

Table 4 summaries the long-run per-capita savings and debt and loan charge-off rates by employment state. Credit-worthy agents save more assets than unworthy agents. Employed worthy agents save more and borrow less than unemployed worthy agents. The average debt for unemployed borrowers is 6.5 times their unemployment benefit, whereas the average debt for employed borrowers is only 15% of their income. In the long run, about 1.9% of individuals default every period and credit-unworthy agents on average regain access to credit after seven years.

Table 4 Long-Run Per-capita Asset and Debt, Charge-off and Default Rates, by State

	Unemployed, Unworthy	Employed, Unworthy	Unemployed, Worthy	Employed, Worthy	All States
Average Asset	0.189	0.272	0.255	0.290	0.288
Average Debt	0	0	0.030	0.012	0.013
Average Debt for Borrowers	N/A	N/A	2.610	0.153	0.144
Loan Default Rate	N/A	N/A	0.386	0	0.050
Default Percentage	N/A	N/A	0.148	0	0.019

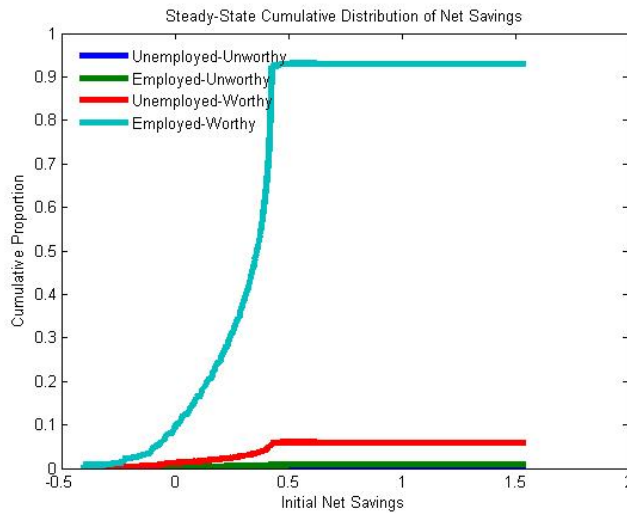


Figure 4 Steady-State Cumulative Distribution of Net Savings by Employment and Credit State

## 6. Parametric Sensitivity Analysis

### *How Does Stigma Effect Affect Strategic Default Decisions?*

In the model, “stigma” is a one-time utility penalty suffered by a credit-worthy agent when she defaults on her debt obligations. An increase in stigma reduces the reservation default value and makes default less desirable when all other parameters kept the same; this is clear in Equation (10). If the stigma associated with default is sufficiently high, the agent will ever default.

As seen in Table 5, the critical net savings level at which a credit-worthy agent defaults decreases with stigma both for employed and unemployed agents. The critical default levels for unemployed agents are less sensitive to stigma than for employed agents. This leads to switches in the relative positions of the critical default levels for employed and unemployed. For instance, when  $\sigma$  is 0.2, the critical default value for an employed agent exceeds that of an unemployed agent; however, if  $\sigma$  increases sufficiently, the critical default value for an unemployed agent exceeds that of an employed agent. Thus, stigma plays such a critical role that changes the default behavior of both employed and unemployed.

The critical savings level at which agents max out credit card debt also decreases with stigma  $\sigma$ , both for employed and unemployed agents. Notice that when the value of  $\sigma$  is low, for example 0.2, there is a range of initial net savings levels over which an employed agent will “max out” their debt in anticipation of defaulting the following period (See Figure 6). However, with higher values of stigma  $\sigma$ , max-out behavior disappears for employed agents. Moreover, as stigma rises, the range of initial savings over which

“maxing out” occurs shrinks. Therefore, an increase in stigma not only deters default in the current period but also discourages “maxing-out” behavior.

Stigma affects the steady-state distributions of net savings in the economy. For credit-worthy agents, savings increase and debt decreases with stigma  $\sigma$  because, given higher cost associated with default, credit-worthy individuals will postpone default and save more for precautionary reasons. The charge-off rate declines dramatically as stigma increases. Sensitivity analysis indicates that the loan default rate drops from 37% to 5% as  $\sigma$  increases from 0.51 to 0.52. This sudden drop occurs because the model changes from a regime where credit-worthy employed agents default to one where they do not default. Among the unemployed, the loan default rate drops from 74% when  $\sigma$  is 0.2 to 37% when  $\sigma$  is 0.52, though the loan default rate of the unemployed stays high. The credit-worthiness rate and the default percentage of the total population also increase with stigma.

Table 5 Sensitivity Analysis—Stigma Effect

	$\sigma=0$	$\sigma=0.2$	$\sigma=0.4$	$\sigma=0.52$ (Base Case)	$\sigma=0.6$
Critical Default Level-Unworthy, Unemployed	-0.356	-0.357	-0.358	-0.359	-0.363
Critical Default Level-Unworthy, Employed	-0.254	-0.274	-0.350	N/A	N/A
Max-out Level-Worthy, Unemployed	0.219	0.188	0.069	-0.010	-0.020
Max-out Level-Worthy, Employed	-0.082	-0.232	N/A	N/A	N/A
Average Asset-Unworthy, Unemployed	0.24	0.26	0.27	0.21	0.18
Average Asset-Unworthy, Employed	0.29	0.30	0.31	0.28	0.27
Average Asset-Worthy, Unemployed	0.13	0.23	0.27	0.27	0.27
Average Asset-Worthy, Employed	0.15	0.27	0.30	0.29	0.30
Average Asset-All agents	0.18	0.27	0.30	0.29	0.30
Average Debt-Worthy, Unemployed	0.05	0.03	0.03	0.03	0.03
Average Debt-Worthy, Employed	0.02	0.01	0.01	0.01	0.01
Average Debt-All agents	0.02	0.01	0.01	0.01	0.01
Loan Default Rate-Unemployed	0.91	0.74	0.51	0.37	0.33
Loan Default Rate-Employed	0.92	0.70	0.46	0.00	0.00
Loan Default Rate-All agents	0.92	0.71	0.47	0.05	0.04
Default Percentage of Population -All agents	0.81	0.23	0.16	0.02	0.02



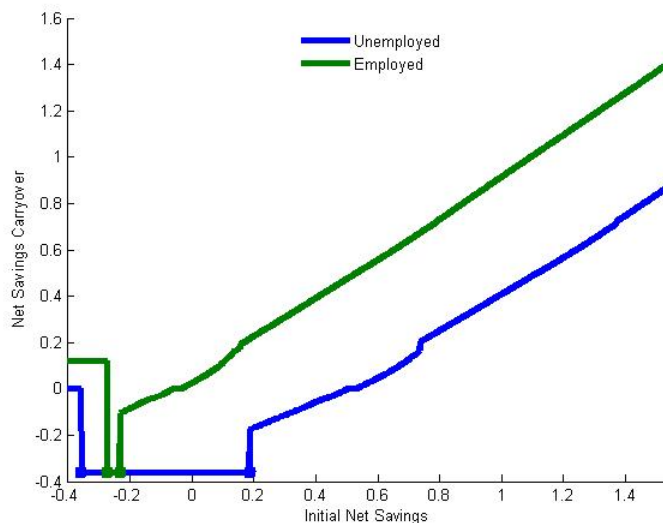


Figure 5 Optimal Policies for Unemployed Worthy Agent and Employed Worthy Agent When Stigma=0.2

### ***Borrowing Limit***

The borrowing limit has a profound impact on an individuals' strategic default behavior. On the one hand, a greater borrowing limit increases the value of the default option such that agents are less willing to default, especially unemployed agents who need borrowing to smooth consumption. On the other hand, a greater borrowing limit increases incentives for agents to carry more debt and face higher interest payments, leading to higher default risk. Thus, the two competing forces affect the default rate in the opposite directions.

The borrowing limit also affects the optimal savings policy. As the credit limit is extended, critical default levels and critical max-out levels both decrease. Moreover, when the credit limit is low, employed agents do not exhibit default and “max out” regimes. However, “max-out” behavior is present for employed and unemployed agents with increased credit limits. This implies that higher borrowing limit motivates agents in financial distress to “max out” debt and default in the subsequent period.

In steady state, the long run average savings level decreases as precautionary savings motive falls, and average debt level increases accordingly. Although a monotonic relationship is not observed between the default rate and the borrowing limit, higher borrowing limits imply higher default rates, as seen in Table 6. When the borrowing limit is low, for example a 20% of a representative agent’s employed income, the value of the default option is low, and thus the default rate is higher. When the borrowing limit is at high, for example, 0.7, the value of the default option is greater; however, borrowing costs also increase, eventually motivating borrowers to default. Thus, the charge-off rate when borrowing limit is set at 0.7 is as high as 47%.

Table 6 Sensitivity Analysis—Borrowing Limit

	b=0.2	b=0.36 (base case)	b=0.5	b=0.7
Critical Default Level-Unworthy, Unemployed	-0.211	-0.359	-0.498	-0.698
Critical Default Level-Unworthy, Employed	N/A	N/A	-0.454	-0.634
Max-out Level-Worthy, Unemployed	0.085	-0.010	0.001	-0.076
Max-out Level-Worthy, Employed	N/A	N/A	N/A	-0.436
Average Asset-Unworthy, Unemployed	0.19	0.21	0.26	0.254
Average Asset-Unworthy, Employed	0.30	0.28	0.30	0.29
Average Asset-Worthy, Unemployed	0.36	0.25	0.20	0.09
Average Asset-Worthy, Employed	0.40	0.29	0.23	0.10
Average Asset-All agents	0.40	0.29	0.23	0.13
Average Debt-Worthy, Unemployed	0.01	0.03	0.05	0.09
Average Debt-Worthy, Employed	0.00	0.01	0.02	0.05
Average Debt-All agents	0.00	0.01	0.02	0.04
Loan Default Rate-Unemployed	0.56	0.37	0.38	0.37
Loan Default Rate-Employed	0.00	0.00	0.31	0.48
Loan Default Rate-All agents	0.09	0.05	0.32	0.47
Individual Default Percentage-All agents	0.04	0.02	0.10	0.19

### *Interest Rate*

The model has two different interest rates —a rate at which one may borrow on a credit card and risk-free rate of return on savings. As the borrowing rate increases, individual agents save more and borrow less. As expected, a higher borrowing rate leads to more loan defaults. Change in the borrowing rate alters the policy function, as the higher rate motivates employed agents to default. Specially, the model experiences a structural change from a regime where credit-worthy employed agents do not default to one where they do default, as  $r_d$  increases from 12% to 14%.

The other interest rate in the model is the risk-free rate of return on savings, which is set to 0.03 under the baseline parameterization. As the risk-free rate increases, credit-worthy agents save more and borrow less. Access to credit and the option to default become less valuable because agents can take advantage of the high risk-free return and accumulate more buffer stock savings, which substitute for borrowing as a means of insuring against income shocks. As shown in Table 8, the simulation results indicate there is a positive relationship between loan defaults and the risk-free asset return rate.

Table 7 Sensitivity Analysis—Borrowing Interest Rate

	$r_d=0.08$	$r_d=0.10$	$r_d=0.12$ (base case)	$r_d=0.14$	$r_d=0.16$
Critical Default Level-Unworthy, Unemployed	-0.362	-0.360	-0.359	-0.359	-0.359
Critical Default Level-Unworthy, Employed	N/A	N/A	N/A	-0.400	-0.395
Max-out Level-Worthy, Unemployed	-0.023	-0.014	-0.010	0.010	0.026
Max-out Level-Worthy, Employed	N/A	N/A	N/A	N/A	N/A
Average Asset-Unworthy, Unemployed	0.19	0.19	0.21	0.27	0.27

Average Asset-Unworthy, Employed	0.27	0.27	0.28	0.31	0.31
Average Asset-Worthy, Unemployed	0.24	0.25	0.25	0.28	0.29
Average Asset-Worthy, Employed	0.27	0.28	0.29	0.32	0.33
Average Asset-All agents	0.27	0.28	0.29	0.32	0.33
Average Debt-Worthy, Unemployed	0.03	0.03	0.03	0.02	0.02
Average Debt-Worthy, Employed	0.01	0.01	0.01	0.01	0.01
Average Debt-All agents	0.02	0.01	0.01	0.01	0.01
Loan Default Rate-Unemployed	0.34	0.34	0.37	0.47	0.50
Loan Default Rate-Employed	0.00	0.00	0.00	0.43	0.47
Loan Default Rate-All agents	0.04	0.04	0.05	0.43	0.48
Individual Default Percentage-All agents	0.02	0.02	0.02	0.13	0.14

Table 8 Sensitivity Analysis—Risk-Free Asset Return Rate

	$r_s=0.01$	$r_s=0.02$	$r_s=0.03$ (base case)	$r_s=0.04$	$r_s=0.05$	$r_s=0.06$
Critical Default Level-Unworthy, Unemployed	-0.361	-0.360	-0.359	-0.359	-0.359	-0.359
Critical Default Level-Unworthy, Employed	N/A	N/A	N/A	-0.397	-0.387	-0.376
Max-out Level-Worthy, Unemployed	-0.010	-0.009	-0.010	0.002	0.014	0.027
Max-out Level-Worthy, Employed	N/A	N/A	N/A	N/A	N/A	N/A
Average Asset-Unworthy, Unemployed	0.16	0.17	0.21	0.30	0.34	0.43
Average Asset-Unworthy, Employed	0.26	0.25	0.28	0.35	0.40	0.47
Average Asset-Worthy, Unemployed	0.16	0.20	0.25	0.38	0.59	1.03
Average Asset-Worthy, Employed	0.18	0.23	0.29	0.43	0.64	1.10
Average Asset-All agents	0.18	0.03	0.29	0.42	0.63	1.09
Average Debt-Worthy, Unemployed	0.04	0.04	0.03	0.02	0.01	0.00
Average Debt-Worthy, Employed	0.02	0.02	0.01	0.01	0.00	0.00
Average Debt-All agents	0.02	0.02	0.01	0.01	0.00	0.00
Loan Default Rate-Unemployed	0.29	0.33	0.37	0.37	0.40	0.44
Loan Default Rate-Employed	0.00	0.00	0.00	0.32	0.36	0.36
Loan Default Rate-All agents	0.04	0.04	0.05	0.33	0.36	0.37
Individual Default Percentage-All agents	0.01	0.02	0.02	0.12	0.13	0.14

## 7. The Extended Model: A Heterogeneous Agent Model for Credit Card Default over the Business Cycle

In this section, we extend the model discussed in the preceding sections by introducing a business cycle that generates variation in aggregate employment over time. Specifically, agents will be subject to both systematic and idiosyncratic employment shocks. The goal of the section is to gain insights into how individual consumption, borrowing and saving behavior and how aggregate default and charge-off rates vary over the business cycle and how their dynamics are affected by the frequency, severity and duration of “recessions.” We also examine the impacts of changes of other parameters, including risk aversion, interest rates, debt limits unemployment benefit, the credit-reinstatement rate, and the “social stigma” attached to defaulting on debt.

As in the preceding sections, imagine an agent facing employment uncertainty must decide each period how much to consume, how much to borrow or save, and whether to default on any debt obligations she may have. At the beginning of each period, the agent possesses net savings  $s$ , with  $s < 0$  indicating that the agent is carrying debt. The agent may be “employed”,  $i = 1$ , receiving a wage income that is normalized to  $y_1 = 1$ , or “unemployed”,  $i = 0$ , receiving an unemployment benefit  $y_0 < 0$ . The economy may be in a “normal” state,  $j = 1$ , in which case the aggregate unemployment rate is  $\gamma_1$ , or a state of “recession”,  $j = 0$ , in which case the aggregate unemployment rate is  $\gamma_0 > \gamma_1$ . The agent also may be “credit-worthy”,  $k = 1$ , indicating she has always met her debt obligations, or “credit-unworthy”,  $k = 0$ , indicating that she has defaulted on her debt obligations at some point in the past.

Given her state at the beginning of the period, the agent must decide how much net savings  $x$  to carry over to the following period, with  $x < 0$  indicating borrowing. The agent may save an unlimited amount and, if credit-worthy, may borrow up to an amount  $b > 0$ . Debt commands an interest rate  $r_d$  and risk-free asset earns a rate of return  $r_s$ , where  $r_d \geq r_s$ . As such, if the agent carries over net savings  $x$  from this period to the next, her net savings at the beginning of next period will be

$$s' = g(x) \equiv \begin{cases} (1 + r_d)x, & x < 0 \\ (1 + r_s)x, & x \geq 0. \end{cases} \quad (11)$$

A credit-worthy agent who begins the period in debt may choose to default on her debt obligations. A defaulting credit-worthy agent completely erases her debt, but is immediately declared credit-unworthy. A credit-unworthy agent is barred from borrowing again until her credit is reinstated, which may occur from one period to the next with probability  $\mu \in (0,1]$ , independently of her employment state and net savings and the state of the economy.

The agent's employment state and the state of the economy are joint realizations of an exogenous Markov chain. We denote by  $\theta_{jj'}$  the probability that the economy will be in state  $j'$  next period, given it is in state  $j$  this period. We denote  $q_{iji'j'}$  the probability that an agent's individual employment state will be  $i'$  next period, given it is  $i$  this period, the

economy is in state  $j$  this period, and the economy is in state  $j'$  next period. And we denote by  $\tilde{q}_{iji'j'} = \theta_{jj'}q_{iji'j'}$  the probability that an agent's individual employment state will be  $i'$  and the economy state will be  $j'$  next period, given that they are  $i$  and  $j$  respectively, this period.

The agent maximizes the present value of current and expected future utility of consumption over an infinite time horizon. The agent's dynamic decision problem is thus characterized by a pair of Bellman equations whose value functions specify the maximum expected present value of lifetime utility  $V_{ijk}(s)$  attainable by the agent, given her employment state  $i$ , the aggregate economy state  $j$  her credit worthiness state  $k$ , and her net savings  $s$ :

$$V_{ij0}(s) = \max_{x \geq 0} \{u(y_i + s - x) + \delta \sum_{i'j'k'} \tilde{q}_{iji'j'} \mu_{k'} V_{i'j'k'}(g(x))\} \quad (12)$$

$$V_{ij1}(s) = \max\{V_{ij0}(0) - \sigma, \max_{x \geq -b} \{u(y_i + s - x) + \delta \sum_{i'j'} \tilde{q}_{iji'j'} V_{i'j'1}(g(x))\}\} \quad (13)$$

for  $s \geq -(1 + r_d)b$ . Here, consumption equals labor income  $y_i$  plus net withdrawals from savings  $s - x$ ;  $\delta \in (0,1)$  is the agent's per-period discount factor;  $\mu_1 = \mu$  and  $\mu_0 = 1 - \mu$  are the probabilities of a credit-unworthy agent having and not having her credit reinstated, respectively;  $\sigma \geq 0$  is a “social stigma” penalty suffered by the agent if she defaults; and  $u$  is the agent's utility, a twice continuously differentiable function of current consumption, with  $u' > 0$ ,  $u'' < 0$ , and  $u'(0) = -\infty$ .

Bellman Equations (12)-(13) capture the borrowing and saving decisions that must be made by credit-unworthy and credit-worthy agents, respectively. In equation (12), a credit-unworthy agent is not permitted to borrow and thus faces the borrowing constraint  $x \geq 0$ . In equation (13), a credit-worthy agent must decide whether to default on her debt obligation. If she defaults, she erases her debt but immediately becomes credit-unworthy, accepting the value  $V_{ij0}(0)$  associated with a credit-unworthy agent with no debt but additionally incurring the social stigma penalty  $\sigma$  attached to defaulting. If she does not default, she remains credit-worthy and faces the borrowing constraint  $x \geq b$ . The critical net savings level  $s_{ij}^*$  below which a credit-worthy agent in employment state  $i$  and economy state  $j$  will default is characterized by indifference between defaulting and not defaulting:

$$V_{ij1}(0) - \sigma = \max_{x \geq -b} \{u(y_i + s - x) + \delta \sum_{i'j'} \tilde{q}_{iji'j'} V_{i'j'1}(g(x))\}. \quad (14)$$

Let  $x_{ijk}(s)$  denote optimal net savings carryover for an agent who begins the period in employment state  $i$ , economy state  $j$ , and credit-worthiness state  $k$ , with net savings  $s$ . Also, let  $g_{ijk}(s) = g(x_{ijk}(s))$  denote the net savings that the agent will hold at the beginning of the following period, given she begins the period in employment state  $i$ , economy state  $j$ , and credit-worthiness state  $j$ , with net savings  $s$ .

## 8. Aggregate Economy and Transitional Probabilities for the Extended Model

We assume that the economy is composed of a large number of agents who behave as the representative agent, but who otherwise are heterogeneous with respect to net savings, employment, and credit-worthiness as a result of having experienced distinct, idiosyncratic employment transition and credit reinstatement shocks over time. It is assumed that individual employment state transitions from one period to the next are subject to the systematic movements of the economy, but are independent and fully diversifiable across agents in the economy, *conditional* on changes in the state of the economy.

In order to fully specify the model, the following transitions and probabilities need to be specified:  $\theta_{jj'}$ , the probability that the economy will be in state  $j'$  next period, given it is in state  $j$  this period, and  $q_{ij'ij}$ , the probability that an agent's individual employment state will be  $i'$  next period, given it is  $i$  this period, the economy state is in state  $j$  this period, and the economy is in state  $j'$  next period, and  $\eta_j$  the individual unemployment duration in state  $j$ . Each period, a proportion  $\rho_j$  of employed agents in state  $j$  are "separated" from their jobs and become unemployed.<sup>3</sup> The following twenty transition probabilities must be satisfied.

$$\gamma_{j'} = \gamma_j q_{0j0j'} + (1 - \gamma_j) q_{1j0j'}, \quad i = 1,2; j = 1,2 \quad (15)$$

$$\psi_j = \frac{1}{1 - \theta_{jj}}, \quad j = 1,2 \quad (16)$$

$$\eta_j = 1 + \sum_{j'} \theta_{jj'} q_{0j0j'} \eta_{j'}, \quad j = 1,2 \quad (17)$$

$$\rho_j = q_{1j0j}, \quad j = 1,2 \quad (18)$$

$$1 = \sum_{j'} \theta_{jj'}, \quad j = 1,2 \quad (19)$$

$$1 = \sum_{i'} q_{ij'ij}, \quad i = 1,2; j = 1,2; j' = 1,2 \quad (20)$$

Equation (16) states that the aggregate unemployment rate next period equals the aggregate unemployment rate this period times the conditional probability that an unemployed agent remains unemployed, plus the aggregate employment rate this period times the conditional probability that an employed agent becomes unemployed. Equation (17) describes the relationship between the duration of the economy state and probability of transition of economy states. The individual's unemployment duration in state  $j$  is characterized by Equation (18). Equation (18) states the definition of employment separation rate  $\rho_j$ . Equation (19) and (20) are the probability requirements for state transition probabilities and individual transition probabilities respectively.

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<sup>3</sup> Separation includes quits (voluntary separations), layoffs and discharges (involuntary separations).

Let  $P_{tik}(s)$  denote the proportion of agents in the economy who, at time  $t$ , are in employment state  $i$  and credit worthiness state  $k$ , with initial net savings less than or equal to  $s$ . Since employment transitions and credit reinstatements are fully diversifiable across agents in the economy, if the economy jumps from state  $j$  in period  $t$  to state  $j'$  in period  $t + 1$ , it must be that

$$P_{t+1,i'0}(s') = \sum_i q_{ijivj'} \left( \mu_0 P_{ti0}(g_{ij0}^{-1}(s')) + P_{ti1}(s_{ij}^*) \cdot I(s' \geq g_{ij0}(0)) \right)$$

$$P_{t+1,i'1}(s') = \sum_i q_{ijivj'} \left( \mu_1 P_{ti0}(g_{ij0}^{-1}(s')) + \max\{0, P_{ti1}(g_{ij1}^{-1}(s')) - P_{ti1}(s_{ij}^*)\} \right)$$

For  $i' = 0,1$  and  $s' \geq -(1 + r_d)b$ , where  $I(\cdot)$  is the Boolean indicator function and

$$g_{ijk}^{-1} \equiv \max\{s \mid g_{ijk}(s) \leq s'\}$$

Here  $P_{tij}(g_{ijk}^{-1}(s'))$  is the proportion of agents in the economy who in period  $t$  are in employment state  $i$  and credit-worthiness state  $k$ , and who next period will have initial net savings less than or equal to  $s'$ ;  $P_{ti1}(s_{ij}^*)$  is the proportion of agents in the economy who in period  $t$  are in employment state  $i$  and default on their debt.

In period  $t$ , the per-capita saving and debt for agents in employment state  $i$  and credit-worthiness state  $k$  are, respectively,

$$\bar{s}_{ij} = \frac{\int_0^\infty s P_{ij}(ds)}{P_{ij}(\infty)},$$

and

$$\bar{d}_{ij} = -\frac{\int_{-\infty}^0 s P_{ij}(ds)}{P_{ij}(\infty)};$$

The default and charge-off rates for agents in employment state  $i$  are, respectively,

$$\bar{f}_{ti} = \frac{P_{ti1}(s_{ij}^*)}{P_{ti1}(0)}$$

and

$$\bar{l}_{ti} = -\frac{\int_{-\infty}^{s_{ij}^*} s P_{ti1}(ds)}{\int_{-\infty}^0 s P_{ti1}(ds)}.$$

and  $P_{t01}(\infty) + P_{t11}(\infty)$  is the proportion of agents in the economy who are creditworthy.

Bellman Equation (12)-(13) may be written equivalently, and more succinctly as

$$V_{ijk}(s) = \max_{\substack{l=0,1 \\ x \geq -bkl}} \{ u(y_i + ls - x) - (1-l)\sigma + \delta \sum_{i'j'k'} Q_{ijk i' j' k' l} V_{i' j' k'}(g(x)),$$

for  $s \geq -(1+r_d)b$ , where

$$Q_{ijk i' j' k' l} = \pi_{jj'} q_{ij' j'} \mu_{kk' l}$$

and

$$\mu_{kk' l} = \begin{cases} 1 - \mu, & \text{if } k = 0, k' = 0, l = 0, \\ 1, & \text{if } k = 1, k' = 0, l = 0, \\ \mu, & \text{if } k = 0, k' = 1, l = 0, \\ \mu, & \text{if } k = 0, k' = 1, l = 1, \\ 1 - \mu, & \text{if } k = 0, k' = 0, l = 1, \\ 0, & \text{if } k = 1, k' = 0, l = 1, \\ \mu, & \text{if } k = 0, k' = 1, l = 1, \\ 1, & \text{if } k = 1, k' = 1, l = 1, \end{cases}$$

Here  $l = 0$  indicates “default” and  $l = 1$  indicates otherwise.

## 9. Parameterization for the Extended Model

Consistent with the base-case parameters presented in section 3, we employ a base-case parameterization for the business cycle model presented in Table 9 below.

Table 9 Base-Case Parameter Values for Business Cycle Model

Parameter	Value	Description
$\alpha$	3.0	Constant relative risk aversion
$r_s$	3%	Risk-free asset rate of return
$r_d$	12%	Borrowing interest rate
$b$	0.36	Borrowing limit
$y_0$	0.4	Unemployment benefit
$y_1$	1.0	Income for employed agents
$\mu$	14%	Credit reinstatement rate
$\delta$	0.94	Per-period discount factor
$\sigma$	0.53	Stigma effect

To capture the key features of the business cycle, we chose values for the unemployment rate, employment separation rate, expected individual unemployment duration and expected duration for each state of the economy to match conditions prevailing in the U.S. economy between 1990 and 2011. We set the unemployment rate for a “normal economy” at 5%, which is the average unemployment rate from 1990 to 2011 when the economy is not in recession; we set the unemployment rate for a “recession economy” at



9%, which is the peak level for the most recent recession from December 2007 to June 2009. The expected duration of unemployment is set at 1.1 periods in a normal economy, corresponding to 5 weeks, and to 1.5 periods in a recession economy, corresponding to 25 weeks. To set the transition probabilities for the economy state, the expected duration is set at eight periods for a normal economy, and at two periods for a recession economy. These base case parameter values are documented in Table 10. Given the above parameter values, sixteen transition probabilities for individuals and four transition probability for the economy states can be all pinned own according to equation (15) to (20).

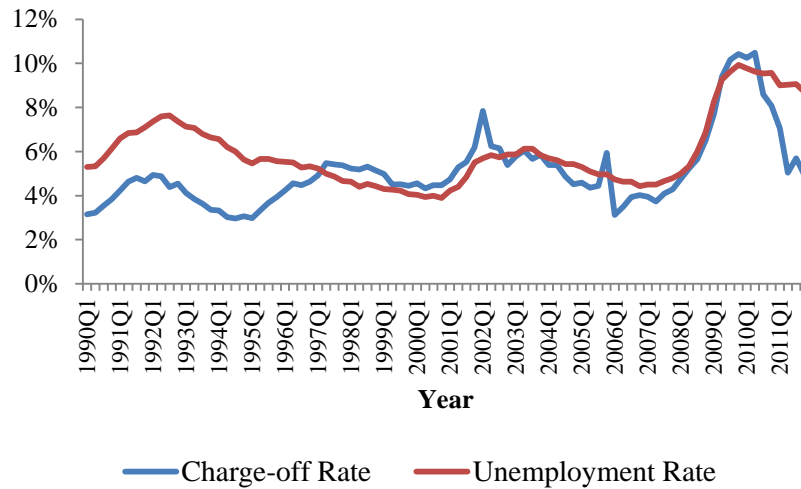


Figure 6 Charge-off Rate and Unemployment Rate from 1990 to 2011

Table 10 Parameter Values for Recession Economy and Normal Economy

	Recession State	Normal State
Unemployment rate	$\gamma_0 = 0.09$	$\gamma_1 = 0.05$
Expected Individual Unemployment Duration	$\eta_0 = 1.5$	$\eta_1 = 1.1$
Employment Separation Rate	$\rho_0 = 0.047$	$\rho_1 = 0.067$
Expected Duration of Economy State	$\psi_0 = 2$	$\psi_1 = 8$

## 10. Different States of the Economy

In this section, we compare and contrast how the behavior of the representative agent and the aggregate savings, debt and loan default rates vary between normal and recession states.

### *Savings Policy for Normal and Recession Economies*

The optimal savings/borrowing functions for representative agent in normal states and recessions are presented in Figure 8 and Figure 9, and the critical default values are given in Table 11. As shown in Figure 8, the state of the economy has a minimal impact on an employed worthy agent's optimal policy function. Employed worthy individuals will not default on their debt obligations at any initial net savings levels, regardless of the state of the economy. Unemployed worthy individuals on the other hand, are more sensitive to the state of aggregate economy. An unemployed worthy agent in recession has a higher critical default value,  $-0.356$ , as compared to the critical value of  $-0.360$  in a normal economy. Moreover, unemployed credit-worthy agents max out at a higher initial net savings in a normal economy than in a recession. This is because the optimal policy function for the unemployed shifts to the left as the economy transits from a normal state to a recession. At each initial savings level, unemployed worthy agents in a recession have stronger incentives to accumulate precautionary savings due to the higher likelihood of being unemployed, and due to longer unemployment spells. Consequently, they save more and borrow less. Notice that when an unemployed agent is in the max-out regime, her next move, i.e. whether to default or not, depends on the realization of her next period's employment status. If she becomes employed, the agent will not default; if she remains unemployed, however, she defaults immediately. Also, in a recession, an unemployed worthy individual has a higher likelihood of remaining unemployed; see Table 12.

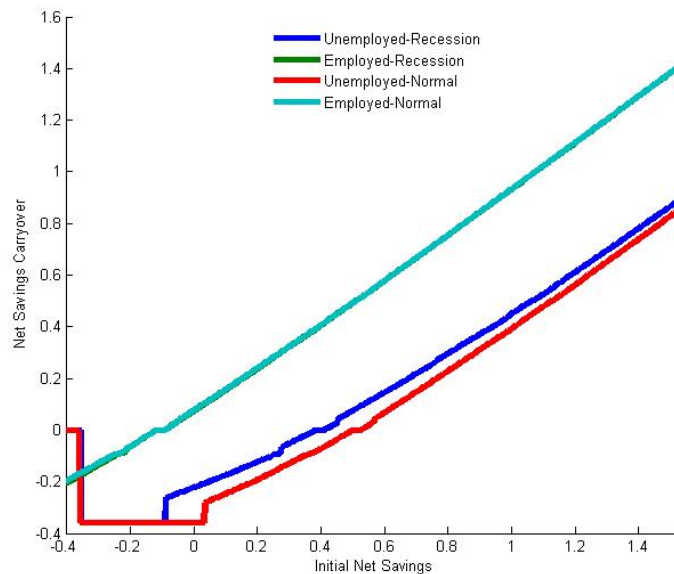


Figure 7 Optimal Policy Functions for Credit-Worthy Agents in Recession Economy and Normal Economy

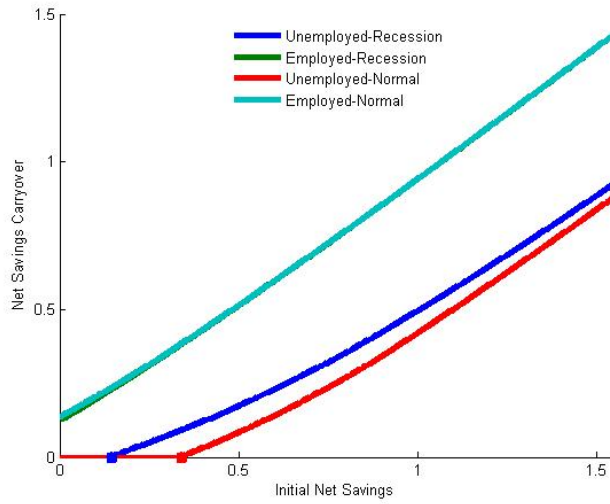


Figure 8 Optimal Policy Functions for Credit-Unworthy Agents in Recession Economy and Normal Economy

Table 11 Critical Default Levels and Max-out Levels for Credit-Worthy Agents by Employment State and Aggregate Economy State

	Unemployed Recession	Unemployed Normal	Employed Recession	Employed Normal
Critical default level	-0.356	-0.36	N/A	N/A
Critical max-out level	-0.087	0.029	N/A	N/A

Table 12 Transition Probabilities within Each State of the Economy for All Agents

	Unemployed, Recession	Employed Recession	Unemployed, Normal	Employed, Normal
Unemployed, Recession	34%	68%	N/A	N/A
Unemployed, Normal	N/A	N/A	11%	89%

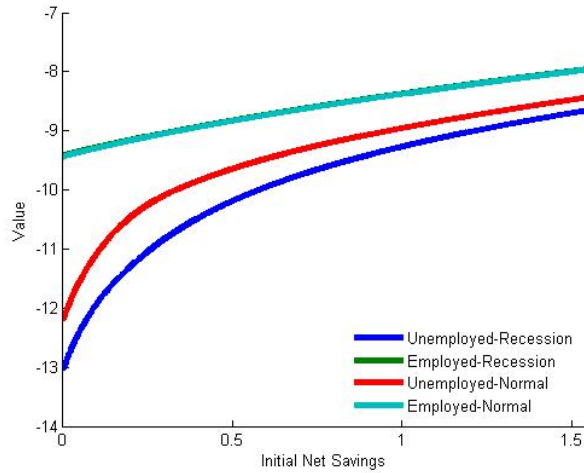


Figure 9 Expected Present Value of Lifetime Utility for Unworthy Agents by Employment State and Economy State

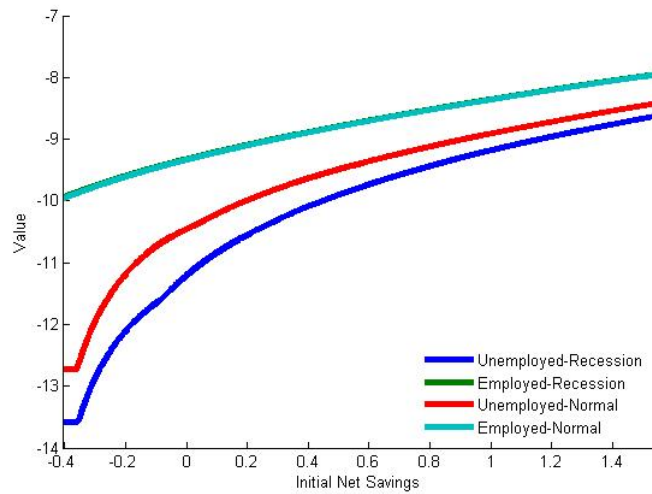


Figure 10 Expected Present Value of Lifetime Utility for Worthy Agents by Employment State and Economy State

***Saving, Debt and Default under Polar Cases***

To understand the effects of economy-wide shocks on aggregate measures of savings, borrowing, and default we simulate the economy under the admittedly unlikely scenario that the economy has been in normal state for a very long period of time (50 periods) and then enters a recession and remains in one for a long period of time (50 periods); see Figure 12.

As seen in Figure 13, agents in a normal economy save 21% more and borrow half as much than during a recession. During a long run of normal conditions, the savings level stabilizes at 35% of employment income, which is normalized to 1. When the economy enters a long recession, savings drop to 29% of the employment income, and borrowing on credit card doubled. Here, we match the average loan default rate with the average charge-off rate at 4% for a normal state, and 9% for recession. The simulated loan default rate path is provided in Figure 15. As seen in this figure, loan default rates are around 4% in normal states and 9% in recession. The change in charge-off rates when the economy changes from a normal to a recession economy is characterized by its path from  $t=51$  to  $t=55$ , during which the loan default rate increases gradually and stabilizes around 9%, and the simulated values of charge-off rates are 3.6%, 4.6%, 8.5% 10.4% , and 9.6% in each period respectively.

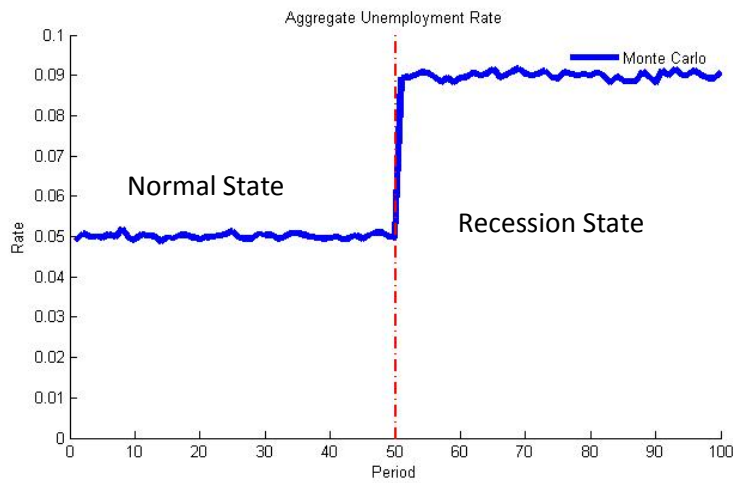


Figure 11 Aggregate Unemployment for Normal Economy and Recession

Table 13 Average Characteristics for Asset, Debt, and Default by Economy State, Employment State and Credit-Worthiness

	Unemployed, Unworthy	Employed, Unworthy	Unemployed, Worthy	Employed, Worthy	All
<i>Normal State</i>					
Average Asset	0.22	0.27	0.34	0.36	0.35
Average Debt	0.00	0.00	0.013	0.008	0.008
Loan Default Rate	N/A	N/A	0.40	0.00	0.04
Default Percentage	N/A	N/A	0.11	0.00	0.008
<i>Recession State</i>					
Average Asset	0.12	0.25	0.23	0.30	0.29
Average Debt	0.00	0.00	0.045	0.013	0.016
Loan Default Rate	N/A	N/A	0.38	0.00	0.09
Default Percentage	N/A	N/A	0.18	0.00	0.035

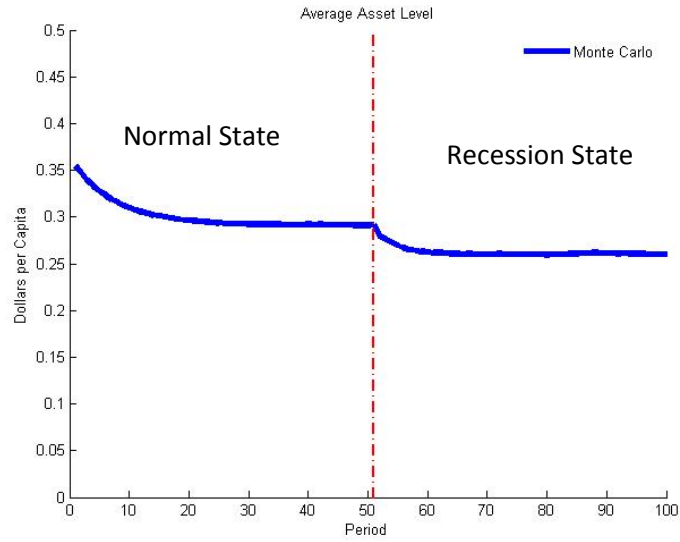


Figure 12 Per-Capita Asset Levels for Normal Economy and Recession Economy

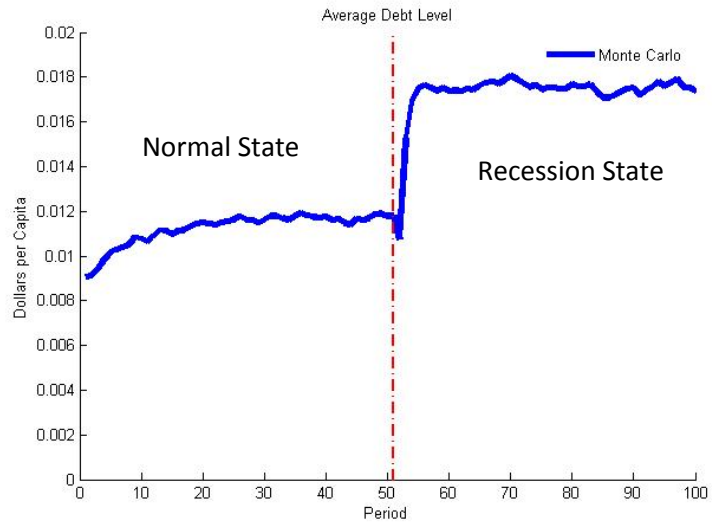


Figure 13 Per-Capita Debt Level for Normal Economy and Recession Economy

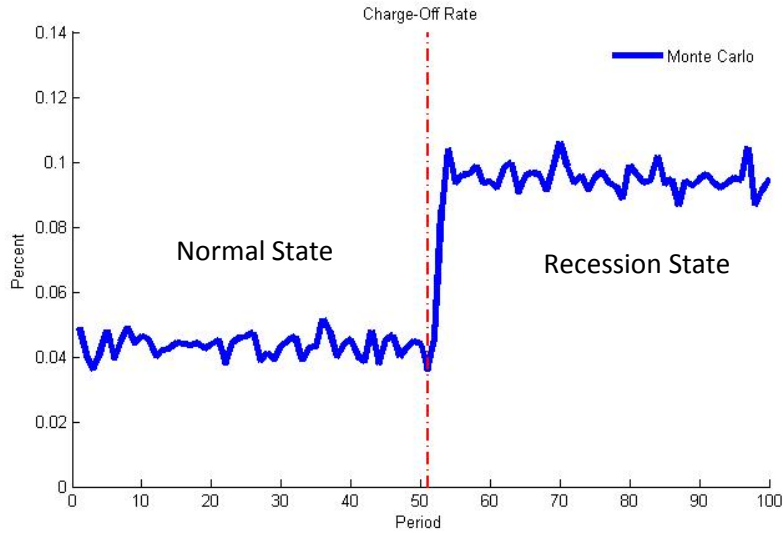


Figure 14 Charge-Off Rate for Normal and Recession Economies

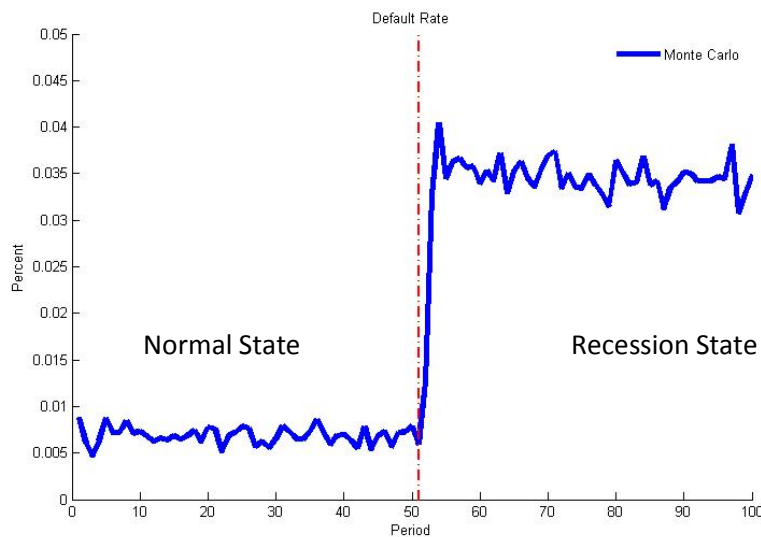


Figure 15 Individual Default Percentage for Normal and Recession Economies

## 11. Dynamics of Aggregate Default

In this section, we examine the dynamics of aggregate default rate when the state of the economy is subject to the aggregate employment shocks. We set the expected duration for normal state at eight periods, and the expected duration for recession at two periods. Using the Monte Carlo Methods, we simulated the stochastic model with idiosyncratic and systematic employment shock for a total of 40 periods. The simulated history of the economy is presented in Figure 18; in the figure recessions are highlighted using red dots.

As seen in Figure 18, the economy is in recession, with an unemployment rate of 9%, in periods  $t = 1, 12, 13, 19, 20, 21, 23, 27, 28, 29$ ; and is in a normal state with an unemployment rate of 5% in all other periods. Thus, the economy experiences a two-period recession in periods  $t = 12$  to  $13$  and a three-period recession in periods  $t = 19$  to  $21$ .

Figure 19, Figure 20, and Figure 21 present the aggregate per-capita savings, per-capita debt and the loan default rate over a simulated history. Figure 18 indicates that when a recession hits, the asset level is not affected in the first period, but reacts in the second and the third period of a recession. Per-capita savings decrease after the return to normalcy for one period and then recover quickly to a new baseline of 29% of normal annual income.

Figure 20 indicates that when the economy transits from a normal state to a recession, per-capita debt does not react immediately, and even decreases a bit in the second period. This is because of the change in the behavior of unemployed credit-worthy agents; specifically, when the economy enters a recession, the policy function for unemployed worthy agents shift to the left, implying that they behave more conservatively and borrow less. However, as the recession deepens, borrowing on debt increases dramatically by more than 50% and more individuals borrow up to the maximum allowable debt. The upward momentum in borrowing continues for one or two more periods, even though the economy has emerged from recession.

Figure 21 indicates that when a recession hits, the loan default rate is not affected in the first period, but subsequently increases to around 10% if the recession is long (as in periods  $t=20, 21$  and  $22$ ). The upward momentum in loan default rate continues after the conclusion of a recession for one period, but then falls back to a lower level. The figure shows that in a recession, the charge off rate can rise to 14% or 15%, three times that under a normal state. The charge-off rate drops quickly to around 4% as the economy emerges from recession. This is consistent with the empirical data for charge-off rates. Even though the most recent recession ended in the 2<sup>nd</sup> quarter of 2009, the charge-off rate continued to climb from 9% in 2009 quarter 2 to 11% in 2010 quarter 2.

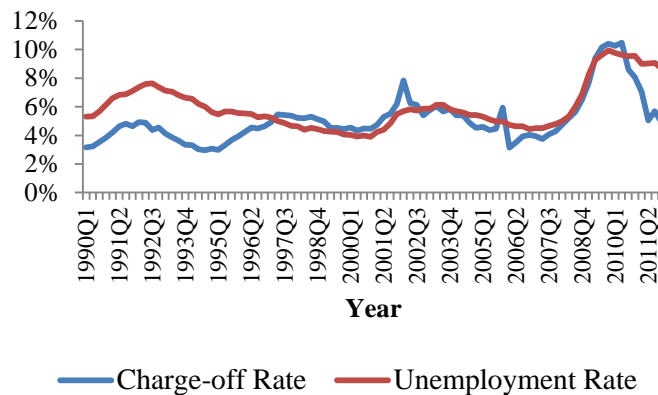


Figure 16 Empirical Charge-off Rate and Unemployment Rate from 1990 to 2011



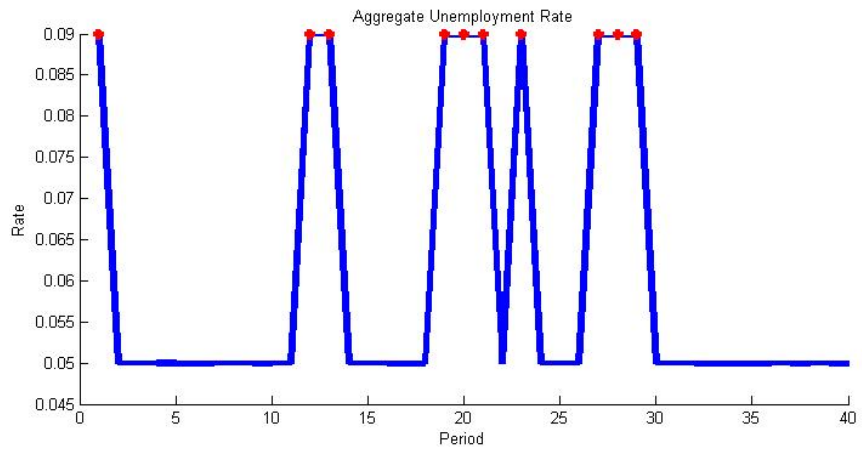


Figure 17 Simulated Aggregate Unemployment Rate

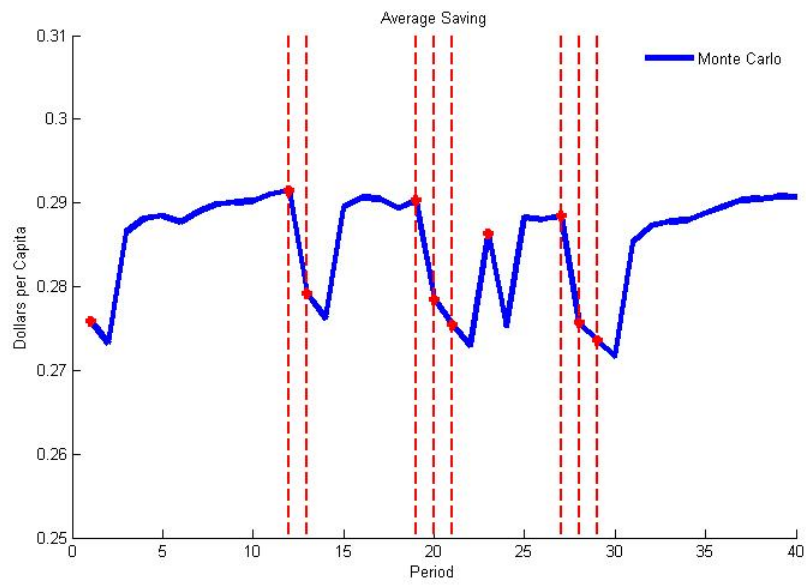


Figure 18 Simulated Path for Average Savings for 40 Periods

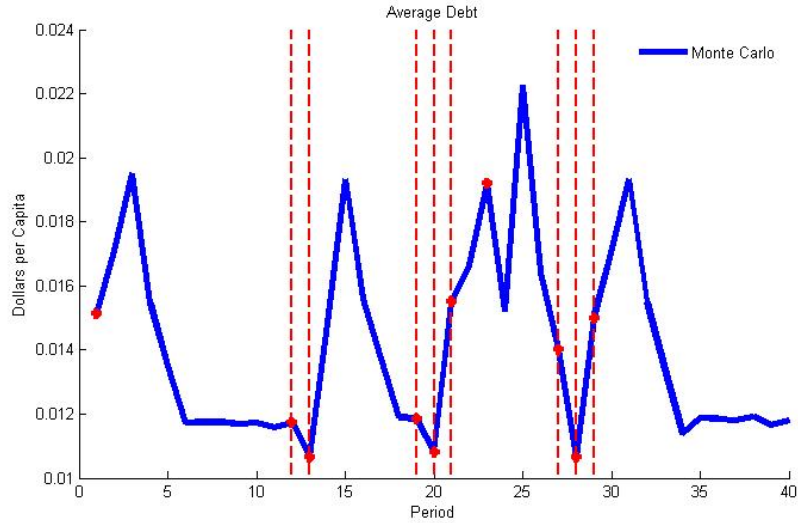


Figure 19 Simulated Path for Average Debt for 40 Periods

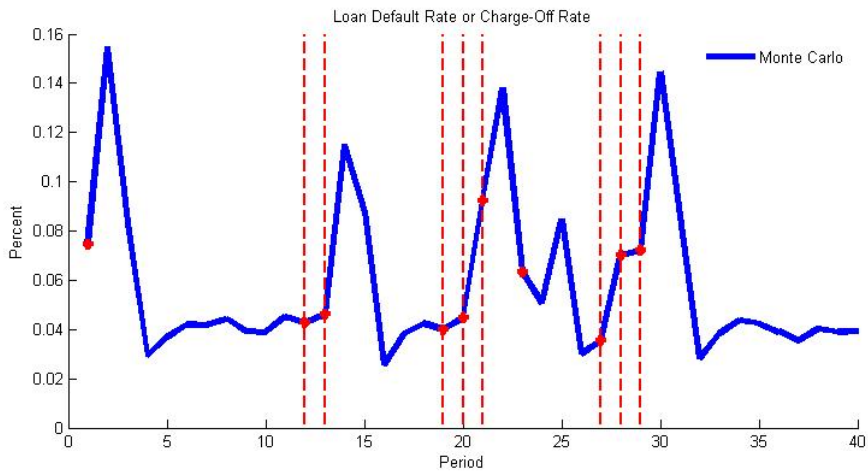


Figure 20 Simulated Path for Charge-Off Rates for 40 Periods

## 12. Summary

In this paper, we have investigated strategic credit card default by building a structural dynamic model that matches the key empirical characteristics of credit card default in the U.S from 1990 to 2011. The model features heterogeneous agents who are subject to both idiosyncratic employment shocks and systemic aggregate business-cycle employment shocks. The representative agent is a forward-looking consumer who can borrow on her credit card up to a credit limit, given that she is “credit-worthy.” She has an option to default in any period. In each period, the individual decides how much to consume, how much to save, how much to borrow and whether to default, given initial

asset holding, debt level, employment status, and credit-worthiness, and the state of the economy.

Solving the Bellman equations that characterize optimal behavior provided us with a deeper understanding of how individuals act strategically on credit card debt repayment/default decisions, and how individuals' consumption, borrowing and savings change depending on their asset levels, debt levels and credit-worthiness, and how aggregate savings, borrowing and loan default rate vary over the business cycle.

Our major findings are as follows: First, the option to default provides additional insurance and improves the representative agent's welfare. The individual will therefore consume more, save less and demand more credit. Second, the optimal borrowing/savings policies generated by the structural model indicate that unemployed credit-worthy agents exhibit three regimes in which the individual will i) make the minimum required interest payment on her debt, if any, thus remaining credit-worthy; ii) default on her debt, thus being labeled credit-unworthy and being permanently barred from borrowing in the future; or iii) strategically "max-out" her credit card, taking out the maximum allowable debt in anticipation of defaulting in the following period if remain unemployed.

Stochastic simulations for the calibrated model suggest that social "stigma", borrowing limit, and interest rates all have a profound impact on average asset holding, borrowing and loan default. An increase in the social "stigma" penalty attached to defaulting results in fewer loan defaults. Our simulations further indicate that increasing the borrowing limit stimulates borrowing; however, the increased burden of interest payment leads to higher default rate and more frequent "maxing out" total debt. Another finding is that as the borrowing rate on credit card and the rate of return on risk-free asset increases, loan defaults increase as well.

When the economy enters a recession, it takes three to four periods to reach the maximum loan default rate. Agents in a normal state have about 21% more savings than representative agents in a recession, and at the same time, they hold only half amount of the debt as of the average debt holding in recession. Simulations of the business cycle suggest that the charge-off rate is not affected immediately when a recession hits, but increases dramatically in the second and third period of a recession. When the recession is over, high default rates persist for one more period and subsequently decline.

### **13. Conclusions**

Our work contributes to the knowledge of consumer default in two major ways: (1) we have explained the major determinants and implications of strategic "maxing-out" of credit card debt; and (2) we have obtained a richer explanation of how the aggregate default rate varies along the business cycle.

"Maxing-out" behavior is clearly a strategic first move for consumers who are considering default. Although "maxing-out" activities have been observed by lenders and

reported by credit bureaus, our paper is a pioneer in explaining this phenomenon using a theoretical framework with structural dynamic model. As demonstrated by the optimal borrowing/savings policy functions for the credit worthy agents, once an unemployed worthy agent has reached the borrowing limit on credit cards, he is very likely to default in the next period. In lending practice, lenders have already noticed the “max-out” activities. Once a consumer charges a card up to the limit, his/her credit scores will fall, signaling a greater default risk. However, credit card max-out does not necessarily lead to default, because default decision also depends on the employment status of the borrower.

Employment information is only collected when credit cards are initially approved, and lenders do not have updated information on employment status. One way a lender could manage loan losses is to charge “maxed out” accounts higher interest rates or endogenously price loans with respect to potential “max-out” behavior. For instance, if lenders increase interest rates on “maxed out” accounts, our model predicts lower expected loss for lenders. The reason is that if the borrower is unemployed next period, the lender will lose the total credit limit plus the interest rate, and if the borrower is employed next period, the borrower will not default and continue to make interest payment; thus the expected loss for lender is reduced.

Another way to reduce losses due to default, lenders or policy makers may consider, is debt restructuring for borrowers who have maxed out and are about to default. Debt restructuring could take the form of forgiving part of the outstanding debt or postponing payment schedules in order to restore the borrowers’ ability to service their debt and stay credit-worthy. For example, for an unemployed borrower who is about to default, unsecured debt lenders recoup nothing from her once she defaults; however, a reduction in debt amount can change the consumer’s optimal behavior, as suggested by the optimal borrowing/savings policy function, such that she can avoid default. Once the borrower’s employment status is improved and she is no longer near default, lenders can even charge a fee for debt restructuring as premium. No matter how debt restructuring is designed and carried out, the goal is to encourage borrowers to stay on payment schedules such that total expected loss of default is minimized.

The major concern with debt restructuring is the moral hazard issue. The April 2012 World Economic outlook points out that whether debt restructuring programs will be successful or not depends on careful design and implementation, because they face the risk of moral hazard that debtors will try to take advantage of the program when they initially apply loans. Also, government interventions will distort credit market as lenders respond to these programs, causing the market to move to a new equilibrium with higher interest rates or lower borrowing limit or less credit supply in the market. Future work in consumer default should explore the effectiveness of different types of debt restructuring programs and their impact on the credit market and individual borrowers.

In addition, the “maxing out” strategy (as long as it is not fraud) should not be seen as an immoral or irresponsible behavior. First, a consumer’s risk of default and “max out” should be all captured by contract terms such as specifications on borrowing limit and the interest rate associate with his/her account. Thus, even “max out” or default happens, it is

permissible under the law. Second, one of the most important functions of credit cards with the borrowing limit is to buffer against negative income shocks. It could be the optimal choice for a financially stressed unemployed borrower to take out all that can be borrowed from this resource, especially if she has made regular interest payments on the card in the past. However, the “max out” behavior could be a fraud if a new borrower simply take out the maximum loans and never intend to make a payment.<sup>4</sup> Fortunately, lenders can screen out those borrowers by applying stricter screening rules.

Although our model captures the “max-out” regime of strategic default, one limitation of our work is that we do not have access to longitudinal data on credit card default to empirically estimate the model. With detailed historical data on individual credit card usage and default, we can estimate the dynamic discrete choice model empirically as in Rust (1987) seminal paper.

A novel feature of our model is the explicit incorporation of a business cycle that allows us to explain the recent surge in charge-off rate during the Financial Crisis 2007-09. As discussed in the literature section, there has been no agreement on how aggregate unsecured debt default varies along the business cycle. The model of Lopes (2008) indicates that default is negatively correlated with expectations about unemployment. The author inferred the procyclicality of default from of a stationary model without a business cycle feature, arguing that lower default rates are associated with higher variation in transitory income, which in turn are associated with high aggregate. Thus the author concludes that default rates are low in economic downturn, and default rates are high in economic expansion. To justify these results and the procyclical nature of credit card default, Lopes (2008) uses data of bankruptcy rate and unemployment rate from 1980 to 2002 and find that personal bankruptcy filings have been increasing during that time period although unemployment rate is relatively low.

However, Lopes needed to make a leap of faith to draw the above conclusion. First without a business feature in the model or aggregate economy-wide shock, the inference of procyclicality of default out of a stationary model is unsound. Second, the author has confused the cyclical variation with the general upward trend in personal bankruptcy rates as the data are not detrended and are largely affected by exogenous changes. For example, the upward trend in bankruptcy filing could be due to substantial increases in credit supply during that period and lenders offer loan to a larger pool of credit worthy borrowers (Dick and Lehnert 2007).<sup>5</sup> Thus, it is not that consumers are more likely to default each year but simply that bankruptcy filing increases with the base population of credit-worthy individuals. On the other hand, bankruptcy rate is greatly affected by bankruptcy codes, which differ across time and across states. Cyclical variations of default over business cycle are much likely to be disguised by these exogenous factors.

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<sup>4</sup> The problem of moral hazard is not modeled in this paper.

<sup>5</sup> Dick and Lehnert (2007) find that banking deregulation leads to increases in overall lending and bankruptcy rate. The authors indicate that “increased competition led banks to extend credit on the extensive margin—by lending to new households, including higher and lower risk borrowers that were previously out of the credit market. It is in this manner that bank risk has decreased while bankruptcy rates increased.”

Our model has a real business cycle as in Krusell and Smith (1998) that is generated by aggregate employment shock, and it indicates default is countercyclical during the period of 1990 to 2011. We have used the credit card charge-off rate, the ratio of debt written off as uncollectible as of the total outstanding credit card debt, to quantify default. Charge-off rate is a better measure for credit card default in that it is less likely to be affected by changes in legislations and regulations related to bankruptcy and the supply of credit of unsecured debt and other types of loans such as auto loans and mortgages. The countercyclical variation in credit card default using charge-off data is clearly observed during the period from 1990 to 2011.<sup>6</sup> The correlation of charge-off rate and unemployment was 0.63. The correlation of bankruptcy rate and unemployment lagged one period was 0.58. These values support the finding of our model that aggregate default rate rises in recession with high unemployment rate and stays low in economic expansion with low unemployment rate.

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<sup>6</sup> During 1990 to 2011, personal bankruptcy rate has increased all the time except in 2005, right before and after the reform of the bankruptcy code. Thus, the undetrended data of bankruptcy filings will not reveal the variation of default rate with the aggregate economy.

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