

# Sustainability of Regional Food Reserves When Default Is Possible

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## Abstract

We model a regional grain reserve as a game of two countries that agree to pool together a fraction of their grain to cope with production risk, but that can also repudiate their obligations at any moment. The reserve can be operated as a “credit union” or an “insurance union”. We find that although risk sharing is more effective when production shocks are negatively correlated, the regional reserve is more sustainable when the correlation is positive. We also find that an “insurance” game can be more sustainable than a “credit” game.

**Key words:** multilateral reserve; grain; food crisis; default; game theory

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# 1 Introduction

The food price surge of 2007/2008 has revamped interest in public food reserves in Africa and Asia, as a tool to deal with excessive price volatility and with emergencies triggered by extreme weather or other disasters. Many governments from these continents have been expanding their food stock capacities, and by 2012 about 70 percent of African countries and over 90 percent of Asian countries maintained national food reserves (Demeke et al. 2014).

The interest in storage policies is not limited to national reserves. It has been argued that a coordinated, supra-national reserve could better address emergency situations because (i) economies of scale can save cost, (ii) independent management could prevent governments from using reserves for political gains, and (iii) the provision of fora for collective agreements could help avoid trade restrictions during major food crises. Consequently, the development of multilateral buffer stocks is now strongly supported by the international community. To date, major efforts to implement regional reserves are being taken by the Economic Community of West African States (ECOWAS), the Association of Southeast Asian Nations (ASEAN), and by the South Asian Association for Regional Cooperation Food Bank (SFB) (Briones 2011; G20 Working Group 2011; Iqbal and Amjad 2010).

Yet doubts persist regarding the feasibility of regional reserves. In particular, there are concerns about implementation procedures, financial sustainability, implications for free trade, and the commitment of member countries, especially those that maintain their own national reserves (Demeke et al. 2014; G20 Working Group 2011; Gilbert 2011; Robinson 2011).

In practice, a multilateral reserve can operate in either of two ways: as a “credit union” or as an “insurance union”. In the first case, countries keep a given amount of grain in the join

reserve, from which they can borrow when in distress. In the latter, the grain contributed to the joint reserve is considered a premium, and countries in distress receive grain as indemnity, not as a loan. Regardless of how it operates, a multilateral reserve is intrinsically a financial agreement, and participating countries are subject to the risk of financial loss, in the form of defaulted grain loans or unpaid indemnities.

This essay analyzes the stability of a bilateral buffer stock, asking what circumstances may lead countries to default. A bilateral agreement is modeled as a discrete-time, infinite-horizon dynamic game, where the players are two countries and “nature” (as food production in subject to idiosyncratic shocks). At each time period, each country must decide how much grain to store in the regional reserve, and whether to withdraw from the regional agreement. The payoff for each country is given by the lifetime discounted utility from consumption for its representative agent. To keep the model tractable, I assume that the countries do not trade with each other and that once a country reneges on its obligations the joint reserve is permanently dissolved. Different versions of the game are considered, changing the rules governing borrowing limits (in the credit union) and indemnities (insurance union). Nash Markov perfect equilibria are computed by numerical methods, and the model is simulated to compute the expected duration of the agreement. In the end, the objective of this paper is to determine what set of rules should be incorporated in regional reserve agreements to increase their financial viability.

The main results are as follow. The stability of the joint reserve ultimately depends on the magnitude of the food transfers (be it indemnities or loans) required from one country at a certain time period: the bigger the required transfer, the higher the incentive for the paying country to renege on its obligations. This implies that if transfers are proportional to the difference in production shocks, then the reserve will be more stable if production

shocks are positively correlated than otherwise, because on average the required transfers would be smaller. Nonetheless, smaller transfers also mean that the reserve is less effective at isolating countries from shocks.

I also find that an “insurance” game can be more sustainable than a “credit” game.

## 2 The model

Two countries,  $A$  and  $B$ , form a joint reserve to deal with their idiosyncratic production shocks. However, since they are unable to commit, either country can walk away from the agreement at any time, potentially causing a loss (unpaid loan or indemnity) to the other country. Once any of the countries defaults, the joint reserve is permanently closed.

### A country in autarky

Each year the country starts with  $a_t$  units of grain, consisting on current production  $\tilde{q}_t$  and previous year stock  $s_{t-1}$ :

$$a_t \equiv \tilde{q}_t + (1 - \phi)s_{t-1} \tag{1}$$

Here  $\phi$  represents the unit cost of storage. The random production level follows an i.i.d. non-standard beta distribution with support  $[1 - \lambda, 1 + \lambda]$ , where

$$\frac{\tilde{q} - (1 - \lambda)}{2\lambda} \sim \text{Beta}(\alpha, \alpha). \tag{2}$$

Notice that the two parameters of the beta distribution are equal, rendering a symmetric distribution, and that  $\mathbb{E} \tilde{q} = 1$ .

The country has an infinitely-lived representative consumer with instant utility  $u(c)$  and

constant discount factor  $\delta$ . The country's objective is to maximize the lifetime utility of its consumer by storing grain:

$$V(a) = \max_{s \in [0, a]} \{u(a - s) + \delta \mathbb{E} V(a')\} \quad (3)$$

$$\text{subject to } a' = (1 - \phi)s + \tilde{q}'$$

I assume that the utility function exhibits constant relative risk aversion  $\rho$ .

### Game version 1: Coinsurance contract

The two countries agree to share a fraction  $\psi$  of their availabilities. While they both honor their agreement, their post-redistribution availabilities  $\hat{a}$  become

$$\hat{a}_A = (1 - \psi)a_A + \psi a_B \quad (4)$$

$$\hat{a}_B = \psi a_A + (1 - \psi)a_B \quad (5)$$

There are two possible discrete states  $i$ , with  $i = 0$  denoting that the reserve is still open and  $i = 1$  otherwise. Furthermore, while the reserve is still open country  $c \in \{A, B\}$  has two discrete options  $j_c$ : either cooperate ( $j_c = 0$ ) or default ( $j_c = 1$ ). Countries observe the private availabilities (and therefore the contractual transfer  $\psi(a_A - a_B)$ ) at the beginning of the year, decide whether to cooperate, transfer the grain (if both are cooperating) and then they separately decide how much to store and to consume. If any of the countries defaults then  $\hat{a}_c = a_c$  (no transfer takes place).

Assuming that the joint reserve is still open, let  $v_{j_A, j_B}^c(a_A, a_B)$  be the payoff for country  $c$  when countries choose  $j_A, j_B$ . Since countries go back to autarky if any of them defaults,

		Country B	
		$j$	
Country A	Cooperate	$W^A(a_A, a_B)$ , $W^B(a_A, a_B)$	$V^A(a_A)$ , $V^B(a_B) - \sigma$
	Default	$V^A(a_A) - \sigma$ , $V^B(a_B)$	$V^A(a_A) - \sigma$ , $V^B(a_B) - \sigma$

Figure 1: Default in the insurance game: Normal form representation

it follows that  $v_{j_A, j_B}^c(a_A, a_B) = V^c(a_c) - \sigma j_c$  whenever  $j_A + j_B \neq 0$ . Here,  $\sigma > 0$  represents a “stigma” penalty paid by country  $c$  if it defaults. Denote by  $W^c(a_A, a_B) = v_{0,0}^c(a_A, a_B)$  the payoff for country  $c$  when both countries cooperate. Then the value of the joint reserve for country  $c$ ,  $U^c(a_A, a_B)$ , is given by the expected payoff of the game represented in Figure 1.

When both countries cooperate the joint reserve would be open the following year, so

$$W^c(a_A, a_B) = \max_{s_c \in [0, \bar{a}_c]} \{u(\hat{a}_c - s_c) + \delta \mathbb{E} U(a'_A, a'_B)\} \quad (6)$$

$$\text{subject to } a'_c = (1 - \phi)s_c + \tilde{q}'_c$$

## Game version 2: Credit contract

In the credit version of the game, the countries still redistribute  $l = \psi(a_B - a_A)$  units of grain, but as a loan that must be repaid the following period. In this case the post-redistribution availabilities are:

$$\hat{a}_A = (1 - \psi)a_A + \psi a_B - l \quad (7)$$

$$\hat{a}_B = \psi a_A + (1 - \psi)a_B + l \quad (8)$$

		Country B	
		Cooperate	Default
Country A	Cooperate	$W^A(a_A, a_B, l), W^B(a_A, a_B, l)$	$V^A(a_A) , V^B(a_B) - \sigma$
	Default	$V^A(a_A) - \sigma , V^B(a_B)$	$V^A(a_A) - \sigma , V^B(a_B) - \sigma$

Figure 2: Default in the credit game: Normal form representation

where  $l$  represents the grain loan from the previous year. This version of the game is represented in Figure 2.

### 3 Solving the model

The solution algorithm builds upon Vedenov and Miranda 2001 and Miranda and Fackler 2002, who solve dynamic games with continuous policy variables, to incorporate a discrete choice (in my case, cooperate or default). For each country in isolation, the value function  $V$  can be approximated beginning with a guess  $V^{(0)}$  as to what the value of a given availability might be. Given this provisional guess for the autarky value function, it is possible in principle to solve Bellman equation (3) to compute a new iteration  $V^{(1)}$ . This process may be repeated indefinitely until the sequences of iterates converges to the one true value function. Convergence is guaranteed by the same Contraction Mapping Theorem that is used to establish the existence and uniqueness of  $V$ . Although the value functions  $V$  do not possess a known closed form expression, they may be computed to any desired degree of accuracy using standard projection methods. In particular, I construct a finite-dimensional approximation  $V^{(0)}(a) \approx \sum_{h=1}^H c_h \phi_h(a)$ , where  $\phi_h$  is a Chebyshev polynomial, and  $c_h$  its corresponding collocation coefficient .

To find the subgame perfect Nash equilibria of the dynamic games, I start with a guess  $W^{A,(0)}$ ,  $W^{B,(0)}$  for value obtained by players  $A$  and  $B$  when both of them cooperate. I use these initial guess, together with the autarky value functions previously obtained, to compute a first estimate  $U^{A,(0)}$  and  $U^{B,(0)}$  of the expected payoffs of the games in Figures 1 and 2. In the next step, I use the estimated expected payoffs and (6) to obtain new iterates  $W^{A,(1)}$ ,  $W^{B,(1)}$ . As before, this process is repeated until the required value functions converged to a game solution. However, in this case the algorithm faces a potential complication: When solving a dynamic game with discrete choices (“default” and “cooperate” in my model), the game may have zero, one, or two Nash equilibria in pure strategies, depending on the state variables<sup>1</sup>. Thus, instead of converging to a unique solution, the iterations could oscillate indefinitely around multiple equilibria. To guarantee a unique solution, in solving the game I restrict  $U^A$  and  $U^B$  to the payoff associated with the mixed-strategy Nash equilibrium of each subgame.

Furthermore, as in the case of the autarky value function, the closed form solution of  $U$  and  $W$  is unknown, so I approximate these functions with Chebyshev polynomials.

As stated in (2), the marginal distribution of production in countries  $A$  and  $B$  is an affine transformation of a beta distribution. To consider cases of correlated production shocks, I discretized the joint distribution of production using copulas.

To illustrate the model, I set baseline parameters as shown in Table 1.

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<sup>1</sup>For example, for certain combinations of availabilities the game may take the form of a battle-of-the-sexes game, while for others it may be a matching-pennies game.



Symbol	Description	Player 1	Player 2
$\rho$	risk aversion	2.00	2.00
$\phi$	storage cost	0.05	0.05
$\delta$	discount factor	0.95	0.95
$\sigma$	stigma	0.05	0.05
$\alpha$	beta distribution parameter	1.25	1.25
$\lambda$	max. production shock	0.30	0.30
$\psi$	shared availab.		0.15
$\varrho$	production correlation		0.00

Table 1: Model baseline parameters

## 4 Results

Figure 3 presents the value function and the optimal storage policy for a country in autarky (note that the solution is the same for both countries because they are identical). To facilitate the discussion, assume that the initial reserve is empty, so that availability equals the current production level. A country in autarky does not keep any grain in storage whenever production is below average (expected production equals one). If production is above average, the country consumers roughly one-third of the excess production and stores the remaining grain.

When the countries are in the joint reserve, they expect to transfer  $|\psi(a_B - a_A)|$  units of grain, either as an indemnity or as a loan. In both cases, one may anticipate that the incentive for a country to renege on the agreement grows with the size of the transfer. This result is confirmed in Figure 4, which shows the probability of the dissolution of the joint reserve (that is, of at least one country reneging) as a function of the initial availabilities, for the insurance game (top left) and the credit game when initial debt is zero (top right), A owes 0.05 units to B (bottom left) and B owes 0.05 units to B (bottom right). In terms of the first two plots, the required transfer is given by the distance from the 45-degree line.

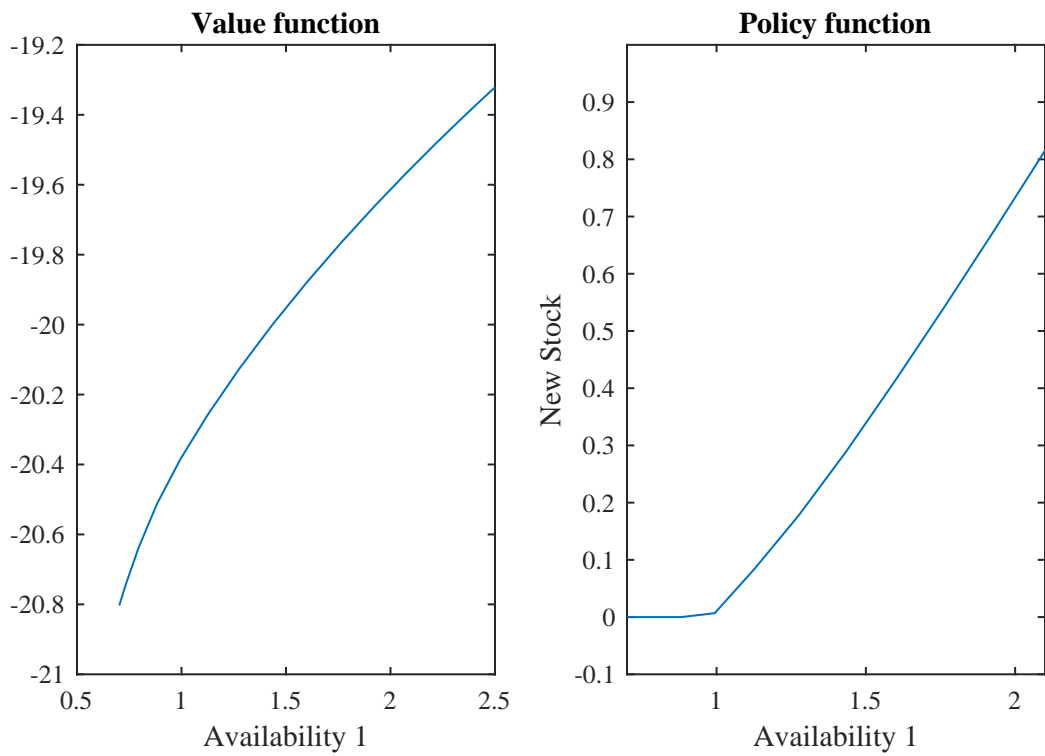


Figure 3: Value and policy functions for a country in autarky

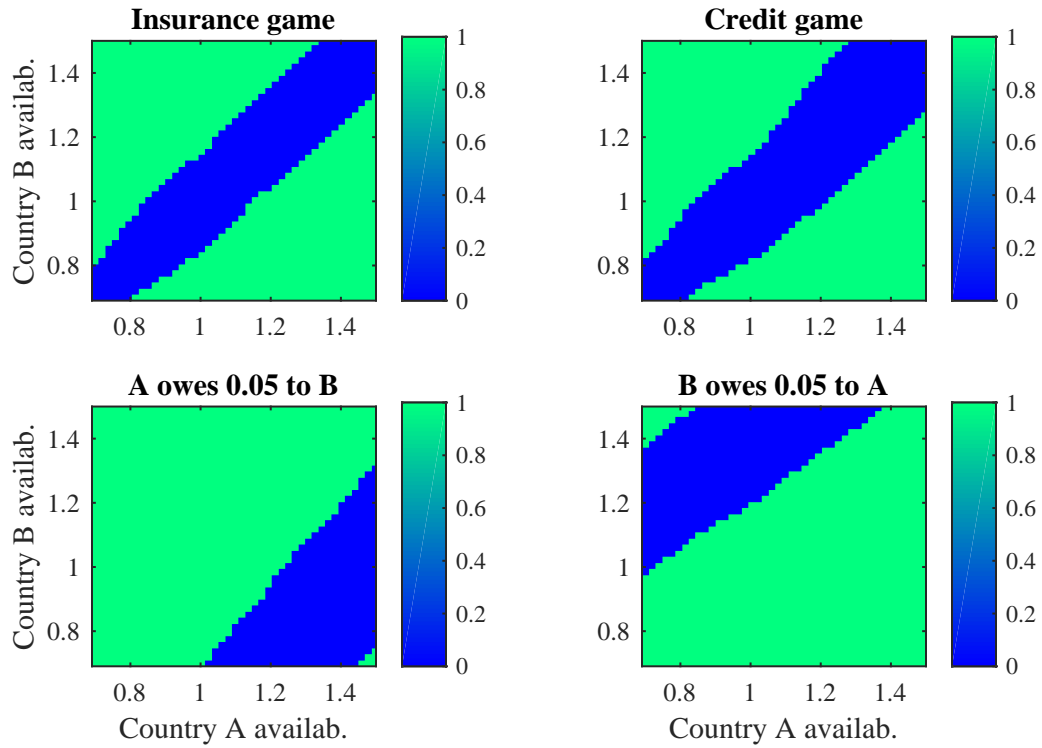


Figure 4: Probability of dissolution of the joint reserve

Notice that in all cases the transition from the no-default regions to the default-with-probability-one regions is sharp, without taking intermediate default probabilities (as would be the case in a mixed-strategy Nash equilibrium). This is due to the design of the game: at any given state, only the country that must pay the transfer (or give a loan) has any incentive to default. For the other country, cooperating is the dominant strategy. Knowing this dominant strategy and for any given availability of the receiving country, there is a level of availability for the paying country at which point it becomes indifferent between transferring the grain and paying the stigma penalty<sup>2</sup>. Furthermore, notice that if country A owes 0.05 units of grain and its production is below average it will certainly default on its loan (an equivalent result holds if country B is the debtor).

I next address the issue of the long-term sustainability of the joint reserve. To this end, I run a Monte Carlo simulation of the insurance and the credit games, consisting of ten thousand independent replications of a 30-period simulation, starting from a state where each country has one unit of grain in availabilities (and there is no outstanding debts in the case of the credit model). For each simulated period, I compute the survival rate as the share of replications where the joint reserve is still operating. The results are presented in Figure 5.

In the baseline, the survival rate for a joint reserve is essentially the same for the insurance and the credit contracts. This could be anticipated from the results in Figure 4, which show that when there are no outstanding debts the default regions for both versions of the game overlap significantly. Here, I find that the probability of keeping the reserve open sinks sharply after the second period, falling below 5 percent in just seven periods.

In Figure 6 I study the effect of production correlation on the sustainability of the reserve.

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<sup>2</sup>The boundary between the default and cooperation regions is smooth; it looks pixelated in the figures due to the discretization of the state space.

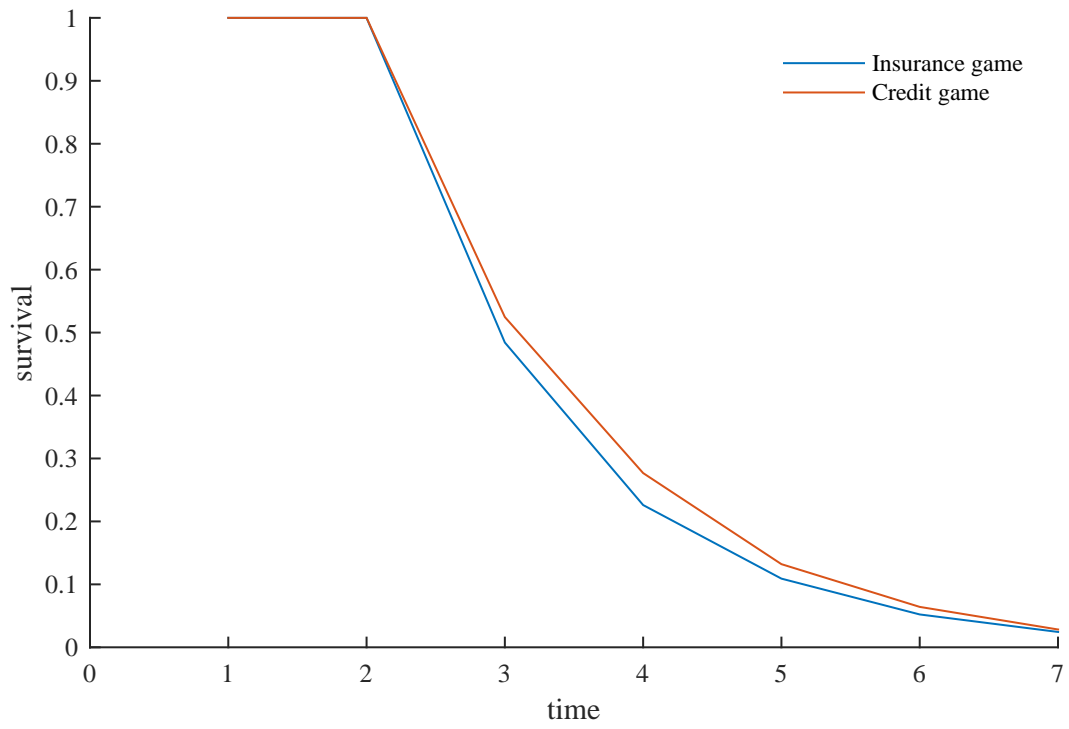


Figure 5: Survival function for the joint agreement

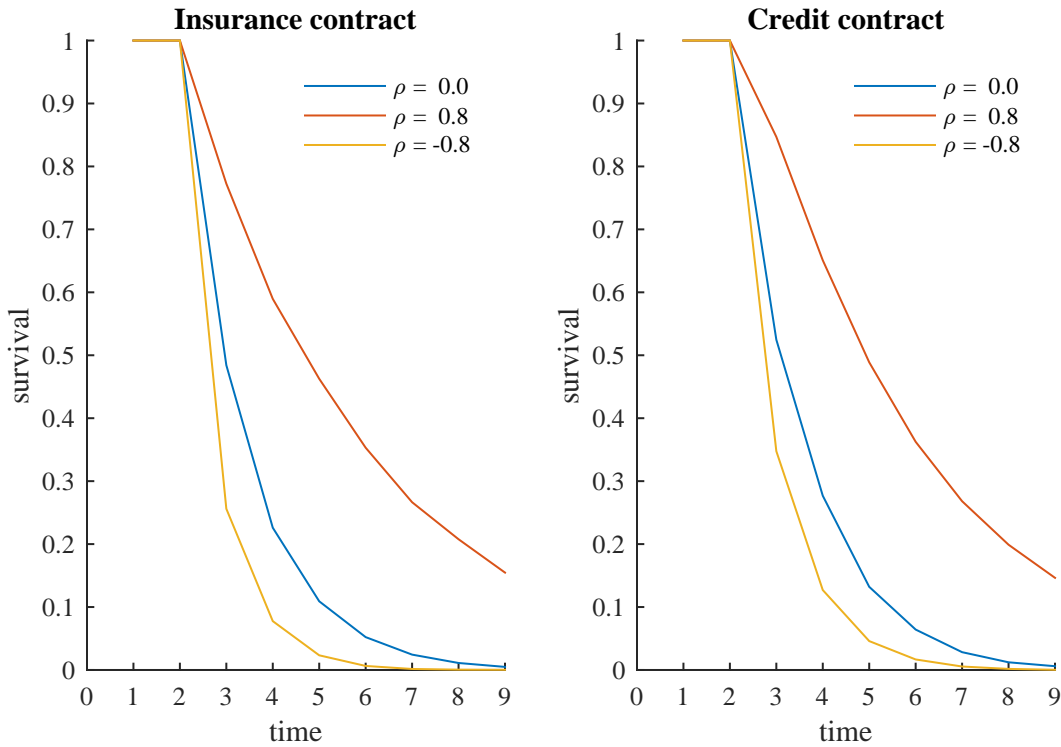


Figure 6: The effect of production correlation on survival of the reserve

As a general rule, (when default is not possible) two agents can more easily cope with risk when their shocks are negatively correlated. However, if default is possible and the transfer of wealth is proportional to the difference in shocks, then the risk-sharing mechanism is more stable when shocks are *positively* correlated. In this case, since the transfers of grain are typically smaller, there is less incentive to default; by the same token, smaller transfers also hinder the ability of the countries to effectively smooth consumption.

Given that the likelihood of default depends crucially on the magnitude of the grain transfers between countries, one way to improve the stability of the joint reserve is by reducing the fraction  $\psi$  of the gap in availabilities that is transferred. For example, in Figure 7 I compute the probability of default for a given combination of availabilities, assuming to

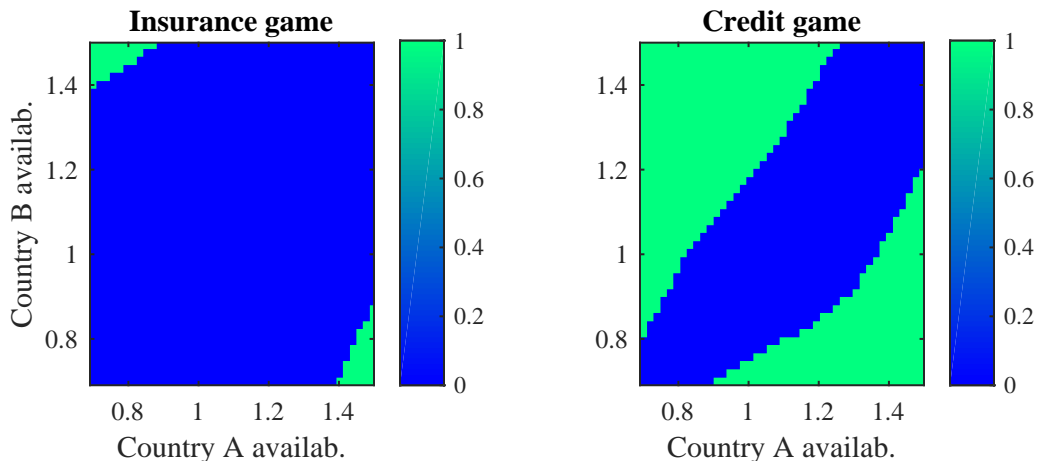


Figure 7: Probability of dissolution with smaller transfers

initial debts, when  $\psi = 0.12$ . As expected, the no-default regions increase relative to their counterpart in Figure 4, for both types of contracts. Notice, however, that the insurance game is much more responsive to this change of policy, as reflected in Figure 8: while the survival of the credit contract improves modestly, the survival rate of the insurance contract remains close to one for at least nine periods

## 5 Conclusion

This paper illustrates how the stability of a joint food reserve is affected by *i*) the type of financial contract (insurance or credit), *ii*) the size of the financial transfers (indemnity or loans) and *iii*) the correlation of production shocks. . I find that because bigger financial obligations drive countries to default, the joint reserve is more sustainable when production shocks are positively correlated.

I also show that depending on the predetermined level of pooling (parameter  $\psi$  in the model), a mutual insurance contract may be more sustainable than a credit contract.

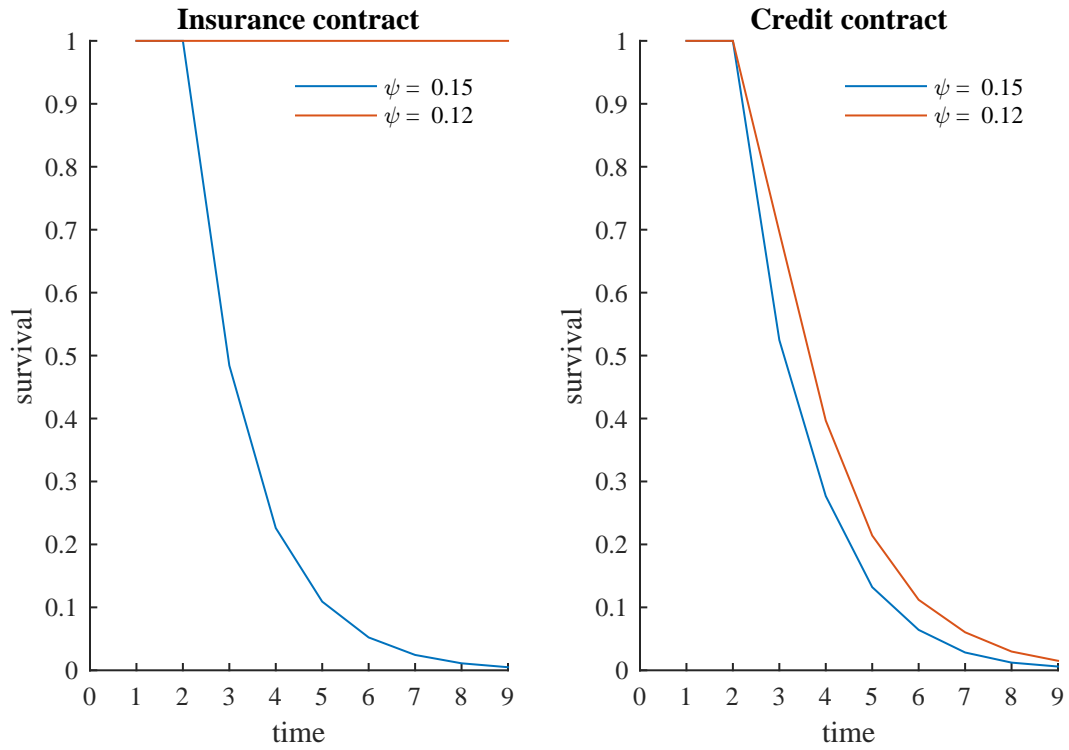


Figure 8: The effect of transfer size on survival of the reserve



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