Systemic Risk, Index Insurance and the Sustainability of Joint Liability Group Credit: A Dynamic Game Framework

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Abstract

Extending credit to a group of borrowers and holding them jointly liable for meeting the group's collective debt obligation has been widely promoted by development economists as an effective means to offer small loans to the multitude of poor in developing countries who lack adequate collateral to securitize their loan. However, joint liability group credit has failed to thrive in rural areas of the developing world, particularly among the 600 million smallholder farmers who practice rain-fed agriculture. In this paper, we analyze a stochastic dynamic game model of a smallholder joint liability group credit arrangement in which individual group members are exposed to common systemic shocks deriving primarily from adverse weather such as droughts. We find that exposure to adverse systemic shocks undermine the effectiveness of group credit, because, when they occur, the incomes of all borrowers drop simultaneously, rendering them unable to fully repay, not only their own loans, but also the loans of other group members. We also find that bundling index insurance against the systemic shock with the loan can substantially improve the performance of group credit, benefiting lenders and borrowers alike.

Key words: Joint Liability Group Credit; Index Insurance; Microfinance; Agricultural Credit.

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1 Introduction

Smallholder farmers in developing countries can often find themselves trapped in persistent poverty due to their inability to access the credit needed to invest in modern production technologies that can generate higher sustainable levels of income (Barnett, Barrett, and Skees, 2008; Boucher, Carter, and Guirkinger, 2008). Lack of access to credit by poor farmers has been attributed to a variety of interrelated and mutually reinforcing factors, including: moral hazard and adverse selection problems that derive from high costs of monitoring contract performance and paucity of public credit history information; costly or ineffectual legal contract enforcement and ill-defined property rights; distorting government policies, such as frequent government ad hoc loan forgiveness mandates; and high transactions costs due to the remoteness and geographic sparseness of rural populations (Conning and Udry, 2007; Besley, 1994).

However, although many factors contribute to smallholder inaccessibility to credit, two, in combination, are the most pervasive: the inability of poor smallholders to present adequate collateral to secure their loans and the high correlation among their incomes induced by drought and other widespread adverse natural events that plague rain-fed agricultural production (Besley, 1994). These two factors, in combination, threaten lenders with insolvency by exposing them to potentially massive defaults on unsecured agricultural loans in the event of a drought or other natural catastrophe that adversely affects large numbers of smallholder borrowers simultaneously. Exposure to this risk forces lenders to offer smallholder loans at very high interest rates affordable to few, or simply to avoid lending to smallholders altogether.[1]

To address lack of collateral among the poor, development economists and practitioners over the past thirty years have promoted the use of joint liability group credit. Joint liability group credit is extended, not to an individual borrower, but to a group of borrowers who are held jointly liable for meeting the group's collective debt obligation. Joint liability group credit does not require borrowers to pledge assets as collateral. Rather, it imposes joint liability provisions, wherein the debts of borrowers in the group who fail to repay their loans in full must be repaid by other group members. Failure of the group to meet its collective debt obligation results in all group members

[1] In Sub-Saharan Africa today, agriculture employs about 55% of the population, yet accounts for only 1% of bank lending (World Bank, 2018).

being denied credit in the future. Joint liability group credit, in principle, employs peer monitoring and peer pressure to ensure high loan repayment rates, effectively using "social capital" as a collateral substitute (Stiglitz, 1990; Besley and Coate, 1995; Ghatak and Guinnane, 1999; Katchova, Miranda, and Gonzalez-Vega, 2006).

However, the successes of joint liability group credit have been largely limited to urban and peri-urban areas. The failure of joint liability group credit to thrive in rural areas can be attributed in part to higher transactions costs due to a more geographically dispersed and less accessible population of potential borrowers. However, high correlation among the incomes of smallholder farmers, due to exposure to common systemic shocks in the form of droughts and other widespread adverse natural events, presents the greatest challenge. Systemic shocks reduce the incomes of all group members simultaneously, rendering group members unable to repay, not only their own loans, but also the loans of other group members, causing the group to default (Simmons and Tantisantiwong, 2014; Farrin and Miranda, 2015; Miranda and Gonzalez-Vega, 2011).

Over the past thirty years, development economists and practitioners have taken a keen interest in the use of index insurance to address weather-related risks faced by smallholders in developing countries (Miranda and Farrin, 2012; Jensen and Barrett, 2017). Unlike conventional insurance, which indemnifies the policyholder for defined and verifiable losses, index insurance provides payouts based on the observed value of a specified "index" that is objectively observable and strongly correlated with the losses of the insured, and which additionally cannot be influenced by the actions of the insured. The most widely used indexes in agricultural insurance contract designs in developing countries are rainfall measured at nearby ground meteorological stations or vegetation indexes computed from satellite observations. Index insurance is free of many of the moral hazard and adverse selection problems that render conventional indemnity insurance inviable for agriculture in developing countries, and can be delivered at much lower administrative costs because it does not require individual farm-level rate-setting and loss verification, making it more affordable to poor smallholders (Miranda, 1991).

However, index insurance suffers from "basis risk", the failure of index insurance payouts to adequately match the income losses of the insured, rendering index insurance of limited value to the insured (Jensen, Barrett, and Mude, 2016). Basis risk remains a pervasive obstacle to the provision of effective sustainable index insurance products designed for smallholder farm-level risk management. Efforts to design index insurance contracts to improve coverage for individual farmers have enjoyed limited success. To date, all index insurance pilot programs designed specifically for smallholder farm-level risk management throughout the developing world have failed to achieve scale and sustainability, and exist only where supported by massive external subsidies (Smith, 2016; Binswanger-Mkhize, 2012).

Researchers, however, are beginning to examine the use of index insurance, not as a means of managing smallholder farm-level risk, but rather as a means of managing the risk borne by agricultural lenders. This is typically accomplished by requiring smallholders to purchase index insurance to receive a loan. Recent theoretical research finds that bundling index insurance with individual smallholder loans can substantially reduce the aggregate rate of default on the lenders loan portfolio in the event of a widespread drought or other indexed event. In principle, this reduces lenders' risk of insolvency, allowing them to offer a greater volume of loans at lower interest rates (Marr et al., 2016; Giné and Yang, 2009; Farrin and Miranda, 2015; Miranda and Gonzalez-Vega, 2011; Barnett, Barrett, and Skees, 2008). However, whether this is true remains an open question subject to further empirical testing (Carter, Cheng, and Sarris, 2016).

This paper is devoted to gaining a deeper understanding of how exposure to systemic shocks such as droughts affect the performance of smallholder group credit, as opposed to individual credit. It also explores whether bundling index insurance with joint liability group credit can improve its performance and sustainability, and whether the smallholder or the lender shuould be the named beneficiary of these payours. To this end, we develop a stochastic dynamic game model of two smallholders who have been offered a joint liability group loan and who are exposed to droughts that reduce both their incomes simultaneously. We establish the existence and uniqueness of the Markov perfect equilibrium that simultaneously solves the smallholders' interrelated Bellman equations and, because it lacks a known closed-form solution, compute it numerically for reasonable parameterizations of the model. We then simulate the model under a wide range of parameterizations to ascertain how the performance and sustainability of joint liability group credit depends on: 1) the severity and frequency of systemic shocks; 2) whether the loan is bundled with an index insurance contract, and, if so, whether the borrower or the lender receives the payouts; and 3) other factors, including the size of the loan, the expected rate of return on the investment financed by the loan, the rate of interest charged by the lender on the loan, overall income volatility, the coverage provided by the index insurance contract, and the premiums charged for the index insurance contract.

We find that systemic risk undermines the performance of joint liability group credit contracts by increasing the rate of default beyond what it would otherwise be if group member incomes were uncorrelated. We also find that bundling index insurance with joint liability group credit contracts increases both the supply of and demand for joint liability group credit. Finally, assigning index insurance payouts to the lender, rather than the borrower, further reduces defaults, increases the lenders rate of return per loan and creates incentives for lenders to charge lower interest rates on smallholder loans.

Our investigation differs from the existing literature on joint liability group credit in two principal ways. First, to our knowledge, our study is the first to examine the sustainability of joint liability group credit in a fully dynamic game setting in which the costs of default are fully endogenized, allowing us to capture how the costs of defaulting to individual borrowers depends on interest rates, the severity and likelihood of systemic adverse shocks, and other factors.[2] Second, our study is the first to theoretically investigate the impacts of mandatory index insurance on joint liability group lending on both the demand and supply sides of the smallholder credit market.

The rest of the paper is organized as follows: Section 2 presents a two-person stochastic dynamic game model of joint liability group credit and establishes the existence and uniqueness of its equilibrium solution. Section 3 discusses the sustainability of joint liability group credit contracts. Section 4 documents how the model is parameterized to allow further analysis of joint liability group credit. Section 5 analyzes the impact of systemic risk on the sustainability of joint liability group credit contracts. Section 6 analyzes the potential benefits of bundling index insurance contracts with joint liability group credit. Section **??** summarizes our main findings and their implications for agrarian financial development policy in developing countries.

[2]Most existing theoretical studies of liability lending group credit assume an exogenously fixed cost of default to the borrower (Ahlin and Townsend, 2007; Simmons and Tantisantiwong, 2014; Varian, 1990).

2 A Model of Joint Liability Group Credit

Consider an autonomous infinitely-lived subsistence farmer who employs a traditional production technology and has no means to borrow or save. Each period, the farmer receives an exogenously determined random income $\tilde{y}$. Since the farmer cannot borrow or save, she consumes her entire income, yielding current utility $u(\tilde{y})$.[3] At any point in time, the farmer's present value of expected *future* utility of consumption is thus

$$A \equiv \sum_{\tau=1}^{\infty} \delta^{\tau} E_{\tilde{y}} u(\tilde{y}) = \frac{1}{\rho} E_{\tilde{y}} u(\tilde{y}) \tag{1}$$

where ρ is the farmer's per-period subjective discount rate and $\delta = 1/(1+\rho)$ the farmer's per period discount factor. Assume $\rho > 0$, so that $0 < \delta < 1$.

Suppose now that two identical farmers, indexed by i and $-i$, are offered a group loan. Under the terms of the loan, the lender provides each farmer with an in-kind loan in the form of "hi-tech" seed that raises their incomes by a multiplicative factor $\gamma > 1$ in all states of nature the following period. Both farmers' incomes depend on a common systemic factor $\tilde{z}$ and to mutually independent idiosyncratic shocks $\tilde{\epsilon}_i$ and $\tilde{\epsilon}_{-i}$ that are independent of the systemic factor. More specifically, if the farmers take out a group loan, farmer i's income the following period will be $\tilde{y}_i = \gamma\tilde{z}\tilde{\epsilon}_i$ and farmer $-i$'s income the following period will be $\tilde{y}_{-i} = \gamma\tilde{z}\tilde{\epsilon}_{-i}$.

As a condition of receiving the loan, each farmer nominally agrees to pay the lender an amount $L = (1+r)\kappa > 0$ the following period, where $r > 0$ is the interest rate on the loan and $\kappa > 0$ is the cost of the hi-tech seed. The two farmers, however, are held jointly liable for repaying both their loans. That is, if one farmer is unwilling to meet her entire debt obligation, the other farmer must repay his own loan and further cover his partner's deficit. If the two farmers fail to cover their collective debt obligation, both farmers are punished by being permanently banned from future credit and additionally suffer a nonpecuniary utility penalty $\phi \geq 0$ due to moral regret and/or decline in social standing. We refer to ϕ simply as the group's *creditworthiness*, noting that, other things being equal, the greater the creditworthiness, the less inclined both farmers are to default.[4]

[3] We assume that the utility function u is continuously differentiable, strictly increasing, and strictly concave on $(0, \infty)$, with $\lim_{y \to 0} u(y) = -\infty$ and $\lim_{y \to \infty} u(y) = \infty$.

[4] Based on the "positive assortative matching" arguments of Ghatak (1999), Ghatak

Before proceeding, we prove the following lemma, which establishes that the income $w(x, y)$ a farmer is willing to forgo to obtain a nonpecuniary gain in utility $x > 0$ is well-defined at any positive level of current income y. It further establishes that the income a farmer is willing to forgo is strictly increasing in the nonpecuniary gain and in current income.[5]

Lemma 1. *If $x \geq 0$ and $y > 0$, there exists an unique nonnegative $w(x, y) < y$, such that*

$$u(y - w) + x = u(y). \tag{2}$$

The function w is continuously differentiable and strictly increasing in x and y.

Let G denote the present value of expected *future* utility for a farmer who has accepted a joint liability group loan with another farmer of equal creditworthiness $\phi \geq 0$, with each assuming an individual debt obligation $L > 0$. If the group loan is repaid, the group credit arrangement survives and each farmer's present value of expected future utility is G. If the group loan is not repaid, the group defaults, both farmers are excluded from credit in the future, and each farmer's present value of expected future utility is $A - \phi$. As such, by the preceding lemma, a farmer with income y is willing to pay $w(G - A + \phi, y)$ to ensure that the group credit arrangement survives.

Suppose now that the two farmers realize incomes y_i and y_{-i}, respectively, and let $w_i = w(G - A + \phi, y_i)$ and $w_{-i} = w(G - A + \phi, y_{-i})$ denote the amounts they are willing to repay respectively to ensure the survival of the group credit arrangement. Then four mutually exclusive and exhaustive outcomes are possible:

1. If $w_i \geq L$ and $w_{-i} \geq L$, each farmer is willing to repay their personal debt obligation L, and does so, ensuring that the group credit arrangement survives.

2. If $w_i < L$ and $w_i + w_{-i} \geq 2L$, farmer i is unwilling to fully repay her personal debt obligation L, but farmer $-i$ is willing to repay his own debt obligation and cover farmer i's deficiency $L - w_i$, ensuring that the group credit arrangement survives.

(2000) and Van Tassel (1999), it is reasonable to assume the same penalty is suffered by both farmers if the group defaults.

[5] Formal proofs of all lemmas, propositions and corollaries are provided in the Appendix.

3. If $w_{-i} < L$ and $w_i + w_{-i} \geq 2L$, farmer $-i$ is unwilling to fully repay his personal debt obligation L, but farmer i is willing to repay her own debt obligation and cover farmer i's deficiency $L - w_{-i}$, ensuring that the group credit arrangement survives.

4. If $w_i + w_{-i} < 2L$, the two farmers collectively are unwilling to repay their joint debt obligation $2L$ so neither repays any amount, the group defaults, the group credit arrangement collapses, and both farmers are excluded from credit in perpetuity.

By Bellman's Principle of Optimality (Bellman, 1957), the present value of expected future utility for a farmer who has accepted a joint liability group loan G must satisfy the Bellman Equation

$$G = \delta E_{\tilde{y}_i, \tilde{y}_{-i}} V(\tilde{y}_i, \tilde{y}_{-i}; G) \tag{3}$$

where

$$V(y_i, y_{-i}; G) \equiv$$

$$\begin{cases} u(y_i) + A - \phi & w_i + w_{-i} < 2L \\ u(y_i - w_i) + G & w_i + w_{-i} \geq 2L,\ w_i < L \\ u(y_i - 2L + w_{-i}) + G & w_i + w_{-i} \geq 2L,\ w_{-i} < L \\ u(y_i - L) + G & w_i \geq L,\ w_{-i} \geq L \end{cases}$$

and where $w_i \equiv w(G - A + \phi, y_i)$ and $w_{-i} \equiv w(G - A + \phi, y_{-i})$.

The Bellman equation (4) characterizes G as a fixed-point of a continuous univariate map. The following proposition establishes that this mapping is a strong contraction with modulus $\delta < 1$, ensuring that G is well-defined for any $\phi \geq 0$. It further establishes that G is strictly decreasing in ϕ; that is, the value of accepting joint liability group credit declines with the farmers' creditworthiness.

Proposition 1. *For any $\phi \geq 0$, there exists an unique $G(\phi)$ that satisfies the Bellman Equation (4). $G(\phi)$ is continuously differentiable and strictly decreasing in $\phi \geq 0$.*

A farmer offered a joint liability group loan with someone of equal creditworthiness will accept it if, and only if, her expected future utility from doing so G exceeds her expected future utility from declining it A. We define a borrower's "willingness to pay" for access to group credit as the amount of income she would be willing to forgo, on average, to enter into a group credit arrangement rather than remaining autonomous and forgoing group credit:

$$W(\phi) \equiv E_{\tilde{y}} w(G(\phi) - A, \tilde{y}). \tag{4}$$

The following corollary establishes the continuity and monotonicity of willingness to pay. The subsequent corollary establishes that a farmer whose creditworthiness ϕ exceeds a well-defined critical level $\bar{\phi}$ will not benefit from entering into a joint liability group credit contract with another farmer of equal creditworthiness.

Corollary 1. *A farmer's willingness to pay $W(\phi)$ for access to a joint liability group loan with a partner of equal creditworthiness is a continuously differentiable strictly decreasing function of their common creditworthiness ϕ.*

Corollary 2. *There exists an unique $\bar{\phi} \geq 0$ such that a farmer will accept a joint liability group loan with another farmer of equal creditworthiness ϕ if, and only if, $\phi \leq \bar{\phi}$. The value $\bar{\phi}$, if positive, is characterized by $G(\bar{\phi}) = A$.*

A joint liability group loan, like any other unsecured loan, embodies a value deriving from the borrower's option to default when income is low and the utility forgone from repaying is unacceptably high. The option value depends on the borrower's creditworthiness. The more creditworthy the borrower, the greater the nonpecuniary penalty she suffers from shame, guilt, or loss of reputation if she defaults. As such, the more creditworthy the borrower, the lower the option value, and, thus, the lower the value the borrower places on a joint liability group loan. Indeed, a borrower of sufficiently high creditworthiness will decline joint liability group credit altogether to avoid assuming the perceived heavy responsibility of having to repay her loan, and possibly part of her partner's loan, in the event of a severe adverse income shocks.

3 Default

Let us now examine the sustainability of joint liability group credit. The following two lemmas characterize conditions under which the group will default on repaying their collective debt obligation, causing the joint liability group credit contract to collapse. They also allow us to derive the probability with which this will occur.

Lemma 2. *For any $\phi > 0$, there exist unique $y_1^*(\phi) > L$ and $y_2^*(\phi) > 2L$ such that $w(x, y_1^*) = L$ and $w(x, y_2^*) = 2L$, where $x \equiv G(\phi) - A + \phi$. The functions y_1^* and y_2^* are continuously differentiable and strictly increasing in ϕ.*

Lemma 3. *For any $\phi > 0$ and $y < y_2^*(\phi)$, there exists an unique positive $b(y; \phi) < y_2^*(\phi)$, such that $w(x, b) + w(x, y) = 2L$, where $x \equiv G(\phi) - A + \phi$. The function b is continuously differentiable and strictly decreasing in x and ϕ.*

Lemma 2 characterizes the level of income y_1^* at which the farmer is just willing to repay her own debt obligation and the level of income y_2^* at which the farmer is just willing to repay both her own and her partner's debt obligation. Lemma 3 characterizes the locus of incomes at which both farmers are just willing to repay their collective debt obligation $2L$.

The implications of the preceding two lemmas are illustrated in figure 1. The figure divides the (y_i, y_{-i}) plane of possible income outcomes for both farmers into four sections. For each (y_i, y_{-i}), let $w_i = w(G(\theta) - A + \phi, y_i)$ and $w_{-i} = w(G(\theta) - A + \phi, y_{-i})$ denote the amounts the farmers are willing to repay to ensure survival of the joint liability group credit contract, respectively. As seen in figure 1, for income pairs (y_i, y_{-i}) lying below the default boundary b, highlighted in yellow, $w_i + w_{-i} < 2L$; here, the two farmers collectively are unwilling to repay their joint debt obligation so neither repays any amount, the group defaults, and the group credit arrangement collapses. For income pairs (y_i, y_{-i}) lying in the northeast region, $w_i \geq L$ and $w_{-i} \geq L$; here, both farmers are willing to fully repay their own loans and do so, ensuring that the group credit arrangement survives. For income pairs (y_i, y_{-i}) lying in the northwest region, $w_i < L$ but $w_i + w_{-i} \geq 2L$; here, farmer i is unwilling to fully repay her loan, but farmer $-i$ is willing to repay his loan and cover farmer i's deficiency, ensuring that the group credit arrangement survives. For income pairs (y_i, y_{-i}) lying in the southeast region, $w_{-i} < L$ but $w_i + w_{-i} \geq 2L$; here, farmer $-i$ is unwilling to fully repay his loan, but farmer i is willing to repay her own loan and cover farmer $-i$'s deficiency, ensuring that the group credit arrangement survives.

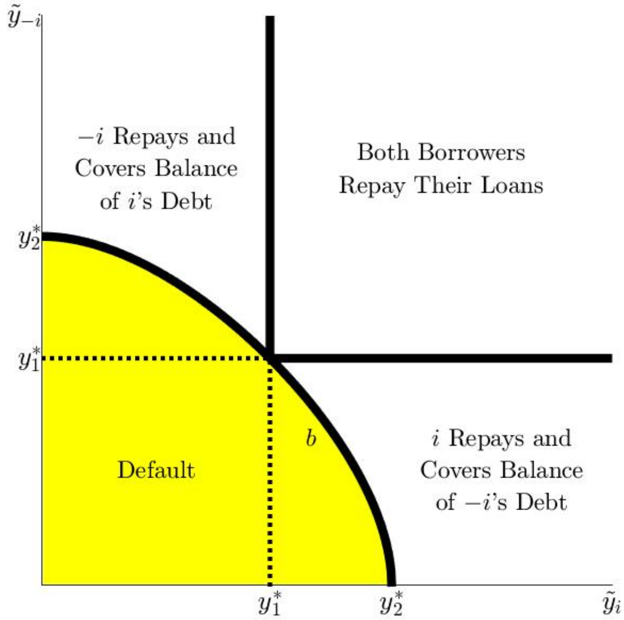

Figure 1: Loan Repayment Outcomes

Clearly, the probability that a group of creditworthiness ϕ will default on a joint liability group loan next period is

$$p(\phi) = Pr(w(x, \tilde{y}_i) + w(x, \tilde{y}_{-i}) < 2L), \tag{5}$$

where $x \equiv G(\phi) - A + \phi$. It follows that the lender's expected gross rate of return on a joint liability group loan extended to a pair of farmers of equal creditworthiness ϕ is

$$R(\phi) = (1 - p(\phi))(1 + r) - \bar{r}, \tag{6}$$

where r is the interest rate charged on the loan and $\bar{r}$ is the lender's cost of funds. For the joint liability group loan to the profitable to the lender, this rate of return must be positive.

Proposition 2. *The probability of default $p(\phi)$ on a joint liability group loan extended to a pair of farmers of equal creditworthiness ϕ is a continuously differentiable and strictly decreasing function of ϕ.*

Corollary 3. *The lender's expected rate of return $R(\phi)$ on a joint liability group loan to a pair of farmers of equal creditworthiness ϕ is a continuously differentiable and strictly increasing function of ϕ.*

Corollary 4. *If the lender is risk-neutral and observes farmers' creditworthiness, then there exists an unique $\phi^* \geq 0$, possibly infinite, such that the lender will offer a loan to a pair of farmers of equal creditworthiness ϕ if, and only if, $\phi > \phi^*$.*

Thus, if the lender is risk neutral and observes farmers' creditworthiness, the lender will extend a joint liability group loan to a pair of farmers of equal creditworthiness ϕ, and *the farmers will accept the loan*, if, and only if, $\bar{\phi} \leq \phi \leq \phi^*$. It follows that the market for group credit will fail to exist if $\bar{\phi} > \phi^*$.

4 Model Parameterization

To develop a model that may be solved and simulated numerically, we make a number of assumptions. Without loss of generality, each farmer's expected income using a conventional technology is fixed at $E\tilde{y} = 1$, so that all values denominated in currency are expressed as proportions of expected average annual income without a joint liability group loan. For the baseline simulations, we further assume that:

- the farmers' utility function $u(y) \equiv (y^{1-\alpha} - 1)/(1-\alpha)$ exhibits constant relative risk aversion $\alpha = 2.0$;
- the farmers' subjective discount rate is $\rho = 0.05$;
- the nonpecuniary default utility penalty is $\phi = 0.1$, equivalent to approximately 10% of expected annual income;
- the cost of high-tech seed is $\kappa = 0.2$, or 20% of expected annual income;[6]
- the rate of return on high-tech seed investment is $r_h = 0.4$, or 40%;
- the interest rate on loans is $r = 0.2$, or 20%;

[6]Given our assumptions so far, the marginal utility of income at mean income 1 is precisely 1, so that, to a first order, the nonpecuniary utility may be measured in the same units as income.

- the lender's cost of funds is $\bar{r} = 0.05$, or 5%;
- the income volatility, that is, the standard deviation of log income, is $\sigma_y = 0.4$;
- the proportion of log income variance attributable to the systemic shock is $\eta = 0.5$, or 50%;[7]
- the systemic income shock is lognormally distributed with mean 1 and log variance $\sigma_z^2 = \eta\sigma_y^2$;
- the idiosyncratic income shock is lognormally distributed with mean 1 and log variance $\sigma_\epsilon^2 = (1-\eta)\sigma_y^2$; and
- the farmers' idiosyncratic income shocks and the systemic income shock are serially and mutually independent, and identically distributed over time.

Other model parameters may be derived from the primitive parameters. More specifically, the "hi-tech" seed productivity enhancement factor equals $\gamma = 1 + (1 + r_h)\kappa$ and the individual farmer's loan obligation equals $L = (1+r)\kappa$. The model parameters and their baseline values are summarized in table 1.

Table 1: Model Parameters and Baseline Values

Symbol	Value	Description
α	2.00	Coefficient of relative risk aversion
ρ	0.05	Subjective discount rate
ϕ	0.10	Borrower creditworthiness
σ_y	0.40	Income volatility
η	0.40	Borrower income correlation
κ	0.15	Cost of hi-tech seed
r_h	0.40	Expected return on hi-tech investment
r	0.20	Loan interest rate
$\bar{r}$	0.25	Lender cost of funds

[7]By construction, η is the correlation between the logs of the systemic shock and the farmer's income.

Throughout the remainder of the paper, parameters are varied individually, with others fixed at their baseline values. $G(\phi)$ is computed numerically using Broyden's method (Miranda and Fackler, 2002).

5 Systemic Risk

To assess impact of systemic risk on joint liability group credit, we solve the borrowers' joint Bellman equations numerically under the base case parameterization in table 1. We then compute five measures of joint liability group credit contract performance assuming different levels of correlation between borrowers' incomes, holding the expectation and variance of individual borrowers' incomes constant: Table 2 reports the probability of default, the lender's expected rate of return, and the farmers' willingness to pay to access joint liability group credit for varying borrower income correlations and creditworthiness; for comparison, it also reports the same for individual liability credit. Table 3 reports the minimum creditworthiness required by the lender to profitably offer a joint liability group loan and the maximum creditworthiness of a borrower willing to accept it with another borrower of equal creditworthiness; for comparison, it also reports the same for individual liability credit.

Table 2: Performance of Individual and Joint Liability Group Credit, by Borrower Income Correlation and Creditworthiness.

	Joint Liability Group Credit					Individual Credit
	Borrower Income Correlation					
	0.00	0.25	0.50	0.75	1.00	
Creditworthiness 0.0						
Default Probability	0.148	0.179	0.205	0.226	0.238	0.238
Lender RoR on Loans	−0.028	−0.065	−0.095	−0.121	−0.135	−0.135
Farmer WTP for Credit	0.250	0.252	0.255	0.260	0.266	0.266
Creditworthiness 0.2						
Default Probability	0.114	0.143	0.169	0.190	0.208	0.208
Lender RoR on Loans	0.013	−0.022	−0.053	−0.078	−0.100	−0.100
Farmer WTP for Credit	0.130	0.133	0.137	0.144	0.155	0.155
Creditworthiness 0.4						
Default Probability	0.084	0.112	0.137	0.158	0.178	0.178
Lender RoR on Loans	0.050	0.016	−0.014	−0.040	−0.064	−0.064
Farmer WTP for Credit	−0.036	−0.033	−0.026	−0.016	0.002	0.002
Creditworthiness 0.6						
Default Probability	0.060	0.085	0.108	0.129	0.149	0.149
Lender RoR on Loans	0.077	0.048	0.020	−0.005	−0.028	−0.028
Farmer WTP for Credit	−0.290	−0.288	−0.278	−0.258	−0.223	−0.223

Table 3: Minimum Creditworthiness for Lender to Offer and Maximum Creditworthiness for Borrower to Accept Individual and Group Credit, by Borrower Income Correlation.

	Joint Liability Group Credit					Individual Credit
	Borrower Income Correlation					
	0.00	0.25	0.50	0.75	1.00	
Lender to Offer	0.126	0.312	0.480	0.628	0.755	0.755
Farmer to Accept	0.363	0.366	0.373	0.383	0.403	0.403

Joint liability group credit differs from individual credit in that the former embodies an implicit risk-sharing agreement among borrowers, but the latter does not. However, the potential for risk-sharing under joint liability group credit diminishes as the correlation between borrowers' incomes rises. Intuitively, if the borrowers' incomes are highly correlated, then when one borrower suffers a low income and is unable or unwilling to repay her entire debt obligation, the other borrower is likely to be in a similar state and be unable to cover his partner's deficit.

As seen in table 2, minimum rates of default and maximum lender rates of return on a joint liability group loan are achieved, for all levels of borrower creditworthiness, if the borrowers' incomes are perfectly independent. As the correlation between incomes rises, however, the probability of default rises and, accordingly, the lender's expected rate of return falls. In the polar extreme of perfectly correlated incomes, the performance of a joint liability group loan is identical to that of an individual liability loan, given that risk-sharing cannot occur. In general, rates of default are lower and lender rates of return are higher with joint liability group credit than with individual liability credit, regardless of the borrowers' creditworthiness.

As seen in table 3, under the base case parameterization, a joint liability group loan extended to borrowers whose incomes are uncorrelated is profitable for the lender only if borrowers' creditworthiness exceeds 0.126. However, if the borrowers' incomes exhibit a correlation of 0.5, their creditworthiness must exceed 0.480 for the loan to be profitable for the lender. An individual liability loan will be profitable to a lender only if the borrowers satisfies a more stringent creditworthiness 0.755.

The farmers' willingness to pay to access joint liability group credit generally rises with the correlation in borrowers' incomes. This challenges our intuition. However, farmer's will always prefer an individual liability loan over a joint liability group loan. And as the correlation between incomes rises, a joint liability group loan behaves increasingly like an individual liability loan. A joint liability group loan will occasionally allow the group credit arrangement to survive when one borrower is unwilling to repay her entire loan obligation but her partner is willing to cover the deficit. However, a borrower suffering a low income places limited value on her partner's actions under these circumstances, given that she nonetheless will repay more than she would have under an individual liability contract. Conversely, a joint liability group loan imposes a collateral obligation on the borrower to cover their partner's deficit when the situation is reversed. In any case, under a joint liability group loan, the borrower pays more than they would otherwise be willing to pay if the loan carried only individual liability.

As seen in table 3, the market viability of a joint liability group contract is highly sensitive to the correlation between the borrowers' incomes. If the borrowers' incomes are uncorrelated, a lender is willing to offer a joint liability group loan and borrowers are willing to accept it, only if the borrowers' creditworthiness lies between 0.126 and 0.363. As the correlation rises, the maximum creditworthiness of farmers willing to accept the contract rises, but the minimum creditworthiness required by the lender for profitability rises even faster. As such the range of creditworthiness over which a joint liability group loan will be both offered and accepted narrows. If the correlation rises to 0.50, the minimum creditworthiness required for a loan to be profitable is greater than the creditworthiness of any potential borrower willing to accept the loan. As such, the market for joint liability group credit fails to exist.

The findings here expose a fundamental tension that exists in joint liability group credit design. For a joint liability group loan to significantly reduce rates of default and render the loan profitable for the lender, peer monitoring must be especially effective. In our model, this is reflected in a higher level of borrower creditworthiness. However, the greater the borrowers' creditworthiness the more likely that the borrower will not see value in joint liability group credit and decline it if is offered to them. Moreover, for peer monitoring to be effective requires social if not physical proximity of the borrowers, implying a higher correlation in their incomes due to the incidence of common systemic shocks such as droughts. This, in turn, reduces opportunities for risk sharing, raises rates of default, and reduces the lender's rate of return. In the end, borrowers who simultaneously benefit from joint liability group credit and promise a positive return to the lender may be few and far between.

6 Index Insurance

We now examine the effects of bundling index insurance with a joint liability group credit contract. Index insurance provides payouts based on the value of the observed systemic shock. We assume a simple yet fairly common form for the indemnity payout schedule:

$$I(z) = \max(0, z^* - z). \tag{7}$$

The payout is proportional to the negative deviation of the systemic factor from some target z^*. The more adverse the systemic shock, the greater the payout. If $z^* = 1$, the indemnity ensures that the farmer's expected income, conditional on the systemic shock, is no less than its mean of 1. Targets on the order of 85% of the mean are very common.

The premium π that must be paid to acquire the index insurance contract depends on the premium load θ, which is expressed as a proportion of the expected indemnity payout $E_{\tilde{z}} I(\tilde{z})$:

$$\pi = (1 + \theta) E_{\tilde{z}} I(\tilde{z}). \tag{8}$$

A premium load of 0 indicates that the insurance priced to be "actuarially fair", that is, so that the premium paid equals the expected indemnity. A premium load of -1 indicates that the insurance is fully subsidized and is thus freely available to farmers. A premium load greater than zero indicates that the insurance is not subsidized; this would be the case if the insurance is offered by private insurers, who must impose the load in order to cover administrative expenses and provide a market return on investment capital.

We consider two distinct provisions regarding the beneficiary of the index insurance indemnity payout bundled with the loan. With "farmer-insured" credit, the farmer receives the entire indemnity payout, regardless of whether he repays his loan. With "lender-insured" credit, the lender receives the payout, to be used as a matter of contract exclusively to cover part or all of the farmer's debt obligation, with any excess given to the farmer. In either case, purchase of the index insurance contract to be bundled with the loan is mandatory, and the cost is borne by or otherwise passed on to the farmer and incorporated into the loan. As such, the farmer's nominal debt obligation, before the lender's share of the indemnity is taken out, is

$$L = (1 + r)(\kappa + \pi). \tag{9}$$

For the purposes of solving and simulating the joint-liability group credit model, we assume baseline values of $z^* = 0.90$ and actuarially fair premiums, that is, $\theta = 0$. These values, however, are varied in subsequent parametric sensitivity analysis.

6.1 Coefficient of Relative Risk Aversion

Figure **??** presents the probability of default, the lender's expected rate of return, and the farmers' willingness to pay to access group credit, for uninsured, farmer insured, and lender insured loans, for varying levels of farmers' relative risk aversion. As can be seen in figure **??**, ...

Table 4: Performance of Joint Liability Group Credit versus Borrower Coefficient of Relative Risk Aversion, by Type of Index Insurance Coverage.

	Risk Aversion	Joint Liability Group Credit: Not Insured	Joint Liability Group Credit: Farmer Insured	Joint Liability Group Credit: Lender Insured	Individual Liability Credit
Default Probability	2	0.176	0.057	0.050	0.222
	3	0.144	0.036	0.033	0.200
	4	0.108	0.021	0.020	0.166
Lender RoR on Loans	2	−0.062	0.081	0.094	−0.117
	3	−0.023	0.107	0.114	−0.091
	4	0.020	0.125	0.129	−0.050
Farmer WTP for Credit	2	0.199	0.312	0.274	0.215
	3	0.291	0.409	0.373	0.306
	4	0.374	0.476	0.451	0.386

6.2 Creditworthiness

Figure **??** presents the probability of default, the lender's expected rate of return, and the farmers' willingness to pay to access group credit, for uninsured, farmer insured, and lender insured loans, for varying levels of farmers' creditworthiness. As can be seen in figure **??**, ...

Table 5: Performance of Joint Liability Group Credit versus Creditworthiness, by Type of Index Insurance Coverage.

	Credit-worthiness	Joint Liability Group Credit: Not Insured	Joint Liability Group Credit: Farmer Insured	Joint Liability Group Credit: Lender Insured	Individual Liability Credit
Default Probability	0.0	0.195	0.071	0.064	0.240
	0.3	0.143	0.035	0.029	0.189
	0.6	0.099	0.014	0.011	0.144
Lender RoR on Loans	0.0	−0.084	0.065	0.078	−0.138
	0.3	−0.021	0.108	0.117	−0.077
	0.6	0.031	0.133	0.138	−0.023
Farmer WTP for Credit	0.0	0.254	0.347	0.311	0.266
	0.3	0.061	0.236	0.191	0.085
	0.6	−0.283	0.106	0.056	−0.224

Table **??** presents the probability of default, the lender's expected rate of return, and the farmers' willingness to pay to access group credit, for uninsured, farmer insured, and lender insured loans, for varying levels of farmers' creditworthiness. As can be seen in table **??**, ...

Table 6 presents the minimum farmer creditworthiness for which a group loan is profitable for the lender and the maximum farmer creditworthiness for which the group loan benefits the farmers, for uninsured, farmer insured, and lender insured loans. As can be seen in table 6, ...

Table 6: Minimum Creditworthiness for Lender to Offer and Maximum Creditworthiness for Borrower to Accept Credit, by Type of Credit

	Group Credit			
	Uninsured	Farmer Insured	Lender Insured	Individual Credit
Lender Offer	0.321	0.000	0.000	0.630
Farmer Accept	0.468	0.754	0.634	0.507

6.3 Income Volatility

Figure **??** presents the probability of default, the lender's expected rate of return, and the farmers' willingness to pay to access group credit, for uninsured, farmer insured, and lender insured loans, for varying levels of farmers' income volatility. As can be seen in figure **??**, ...

Table 7: Performance of Joint Liability Group Credit versus Income Volatility, by Type of Index Insurance Coverage.

		Joint Liability Group Credit			Individual
	Income Volatility	Not Insured	Farmer Insured	Lender Insured	Liability Credit
Default Probability	20%	0.004	0.000	0.000	0.015
	40%	0.176	0.057	0.050	0.222
	60%	0.054	0.069	0.066	0.070
Lender RoR on Loans	20%	0.145	0.150	0.150	0.132
	40%	−0.062	0.081	0.094	−0.117
	60%	0.086	0.067	0.079	0.066
Farmer WTP for Credit	20%	0.231	0.278	0.278	0.235
	40%	0.199	0.312	0.274	0.215
	60%	0.576	0.495	0.432	0.658

6.4 Borrower Income Correlation

Figure **??** presents the probability of default, the lender's expected rate of return, and the farmers' willingness to pay to access group credit, for uninsured, farmer insured, and lender insured loans, for varying percentages of systemic risk. As can be seen in figure **??**, ...

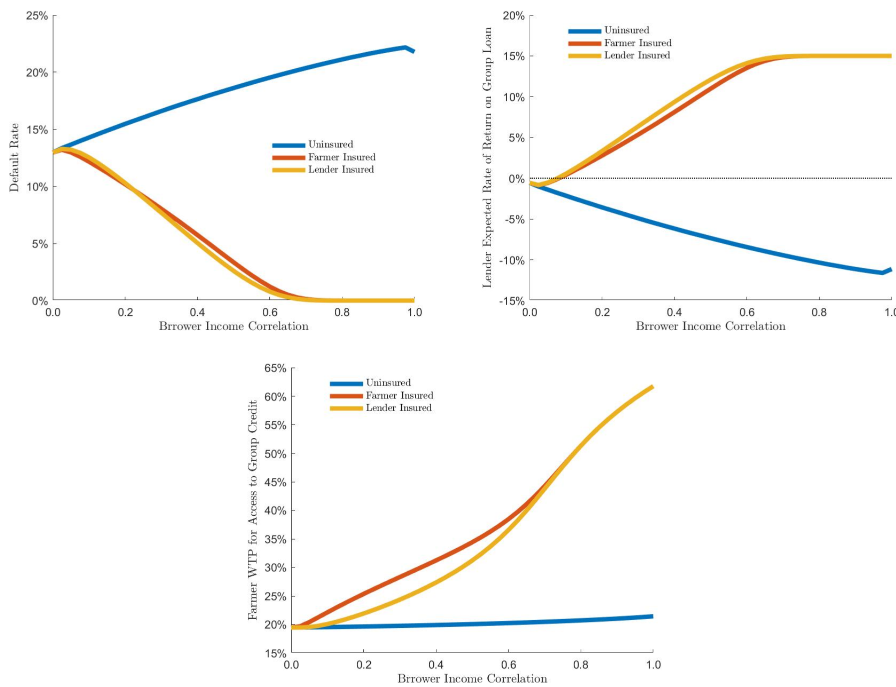

Figure 2: Performance of Group Credit Contract versus Percentage Systemic Risk, by Type of Credit

Table 8: Performance of Joint Liability Group Credit versus Borrower Income Correlation, by Type of Index Insurance Coverage.

	Corre-lation	Joint Liability Group Credit: Not Insured	Joint Liability Group Credit: Farmer Insured	Joint Liability Group Credit: Lender Insured	Individual Liability Credit
Default Probability	0.0	0.130	0.130	0.130	0.218
	0.5	0.186	0.034	0.026	0.223
	1.0	0.218	0.000	0.000	0.218
Lender RoR on Loans	0.0	−0.006	−0.006	−0.006	−0.111
	0.5	−0.073	0.110	0.121	−0.118
	1.0	−0.111	0.150	0.150	−0.111
Farmer WTP for Credit	0.0	0.195	0.195	0.195	0.214
	0.5	0.201	0.344	0.312	0.215
	1.0	0.214	0.618	0.618	0.214

6.5 Cost of Hi-Tech Seed

Figure **??** presents the probability of default, the lender's expected rate of return, and the farmers' willingness to pay to access group credit, for uninsured, farmer insured, and lender insured loans, for varying hi-tech seed costs. As can be seen in figure **??**, ...

Table 9: Performance of Joint Liability Group Credit versus Cost of Hi-Tech Seed, by Type of Index Insurance Coverage.

		Joint Liability Group Credit			Individual
	Hi-Tech Seed Cost	Not Insured	Farmer Insured	Lender Insured	Liability Credit
Default Probability	0.1	0.163	0.025	0.020	0.210
	0.2	0.186	0.086	0.079	0.231
	0.3	0.192	0.129	0.126	0.182
Lender RoR on Loans	0.1	−0.045	0.120	0.127	−0.102
	0.2	−0.073	0.046	0.063	−0.128
	0.3	−0.080	−0.005	0.016	−0.069
Farmer WTP for Credit	0.1	0.130	0.293	0.265	0.143
	0.2	0.252	0.336	0.292	0.268
	0.3	0.333	0.378	0.334	0.404

6.6 Rate of Return on Hi-Tech Seed

Figure **??** presents the probability of default, the lender's expected rate of return, and the farmers' willingness to pay to access group credit, for uninsured, farmer insured, and lender insured loans, for varying levels of expected return on the hi-tech seed investment. As can be seen in figure **??**, ...

Table 10: Performance of Joint Liability Group Credit versus Rate of Return on Hi-Tech Seed, by Type of Index Insurance Coverage.

		Joint Liability Group Credit			Individual
	Hi Tech Seed Rate of Return	Not Insured	Farmer Insured	Lender Insured	Liability Credit
Default Probability	20%	0.321	0.183	0.183	0.358
	40%	0.176	0.057	0.050	0.222
	60%	0.072	0.006	0.004	0.114
Lender RoR on Loans	20%	−0.235	−0.070	−0.052	−0.280
	40%	−0.062	0.081	0.094	−0.117
	60%	0.064	0.143	0.145	0.013
Farmer WTP for Credit	20%	0.135	0.237	0.180	0.148
	40%	0.199	0.312	0.274	0.215
	60%	0.286	0.431	0.418	0.302

6.7 Loan Interest Rate

Figure **??** presents the probability of default, the lender's expected rate of return, and the farmers' willingness to pay to access group credit, for uninsured, farmer insured, and lender insured loans, for varying loan interest rates. As can be seen in figure **??**, ...

Table 11: Performance of Joint Liability Group Credit versus Loan Interest Rate, by Type of Index Insurance Coverage.

		Joint Liability Group Credit			Individual
	Interest Rate	Not Insured	Farmer Insured	Lender Insured	Liability Credit
Default Probability	20%	0.176	0.057	0.050	0.222
	40%	0.328	0.266	0.279	0.364
	60%	0.460	0.449	0.502	0.481
Lender RoR on Loans	20%	−0.062	0.081	0.094	−0.117
	40%	−0.109	−0.022	−0.003	−0.159
	60%	−0.186	−0.168	−0.161	−0.219
Farmer WTP for Credit	20%	0.199	0.312	0.274	0.215
	40%	0.160	0.237	0.175	0.174
	60%	0.140	0.209	0.139	0.151

6.8 Insurance Coverage Level

Figure 4 presents the probability of default, the lender's expected rate of return, and the farmers' willingness to pay to access group credit, for uninsured, farmer insured, and lender insured loans, for varying index insurance coverage levels. As can be seen in figure 4, ...

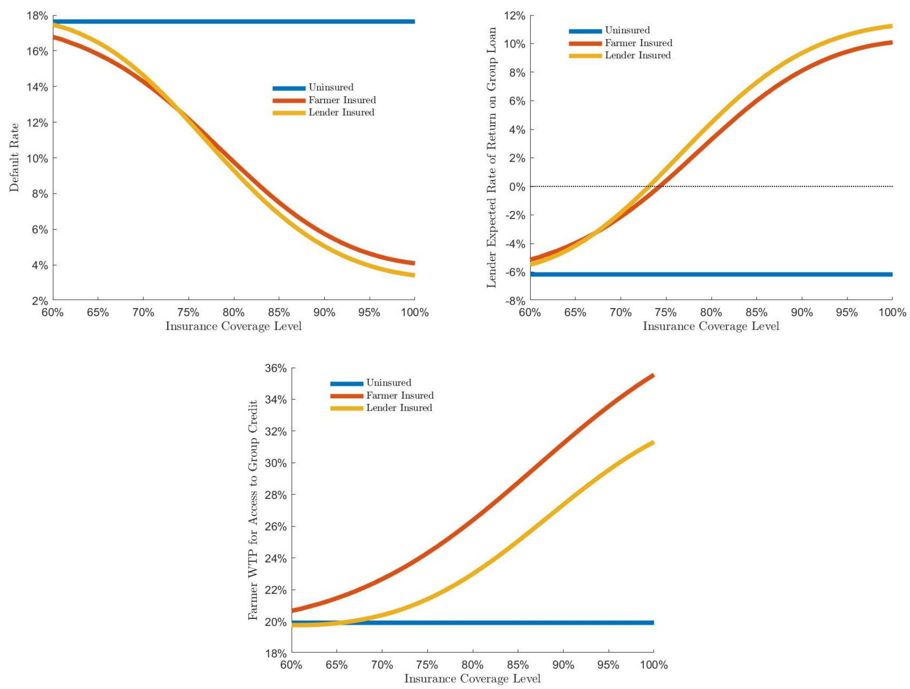

Figure 3: Performance of Group Credit Contract versus Insurance Coverage Level, by Type of Credit

Table 12: Performance of Joint Liability Group Credit versus Insurance Coverage Level, by Type of Index Insurance Coverage.

	Coverage Level	Joint Liability Group Credit: Not Insured	Joint Liability Group Credit: Farmer Insured	Joint Liability Group Credit: Lender Insured	Individual Liability Credit
Default Probability	60&	0.176	0.168	0.175	0.222
	80%	0.176	0.098	0.093	0.222
	100%	0.176	0.041	0.034	0.222
Lender RoR on Loans	60&	−0.062	−0.051	−0.055	−0.117
	80%	−0.062	0.033	0.045	−0.117
	100%	−0.062	0.101	0.112	−0.117
Farmer WTP for Credit	60&	0.199	0.207	0.198	0.215
	80%	0.199	0.264	0.230	0.215
	100%	0.199	0.356	0.313	0.215

6.9 Insurance Premium Load

Figure **??** presents the probability of default, the lender's expected rate of return, the farmers' willingness to pay to access group credit, for uninsured, farmer insured, and lender insured loans, for varying index insurance premium loads. As can be seen in figure **??**, ...

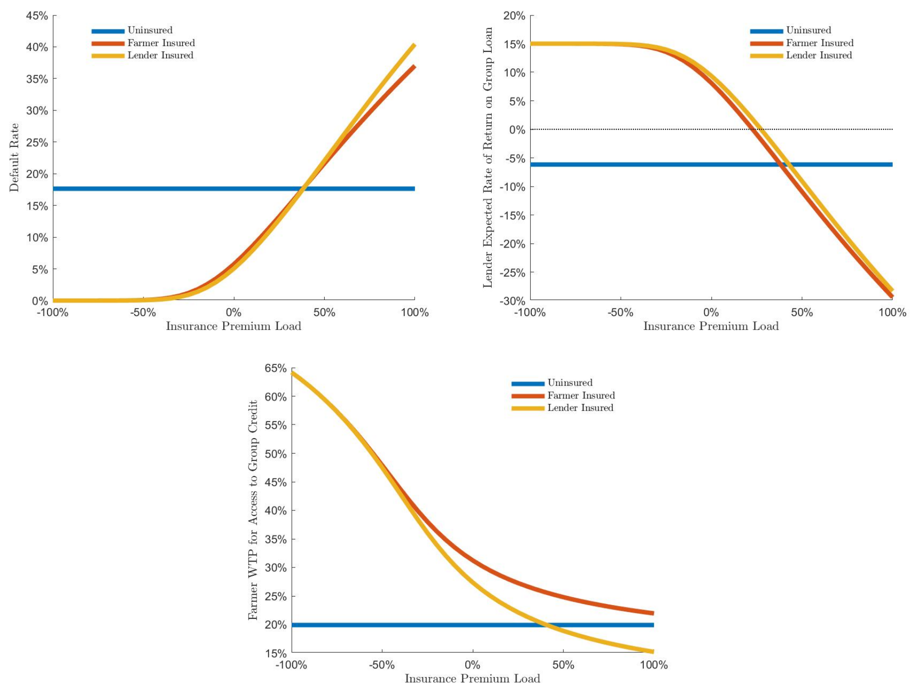

Figure 4: Performance of Group Credit Contract versus Insurance Premium Load, by Type of Credit

Table 13: Performance of Joint Liability Group Credit versus Insurance Premium Load, by Type of Index Insurance Coverage.

	Premium Load	Joint Liability Group Credit: Not Insured	Joint Liability Group Credit: Farmer Insured	Joint Liability Group Credit: Lender Insured	Individual Liability Credit
Default Probability	−100%	0.176	0.000	0.000	0.222
	0%	0.176	0.057	0.050	0.222
	100%	0.176	0.370	0.404	0.222
Lender RoR on Loans	−100%	−0.062	0.150	0.150	−0.117
	0%	−0.062	0.081	0.094	−0.117
	100%	−0.062	−0.294	−0.283	−0.117
Farmer WTP for Credit	−100%	0.199	0.642	0.642	0.215
	0%	0.199	0.312	0.274	0.215
	100%	0.199	0.219	0.152	0.215

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A Proofs of Propositions

A.1 Proof of Lemma 1

Proof. If $x = 0$, then $w(x, y) = 0$. Assume then, that $x > 0$ and let $f(w) \equiv u(y - w) - u(y) + x$. Then $f'(w) = -u'(y - w) < 0$ for $w <\in (0, y)$, $\lim_{w \to 0} f(w) = x > 0$, and $\lim_{w \to y} f(w) = -\infty$, so that f must posses an unique root on the interval $(0, y)$. The continuous differentiability and monotonicity properties of w follow directly from the Implicit Function Theorem. □

A.2 Proof of Corollary 2

Proof. If $G(0) \leq A$, then $\bar{\phi} = 0$; if $\lim_{\phi \to \infty} G(\phi) \geq A$, then $\bar{\phi} = \infty$; otherwise, let $\bar{\phi} > 0$ be the unique root of $G(\phi) - A$. □