Community Enforcement with Endogenous Information*

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Abstract

Many pairwise trading enviroments are vulnerable to risks of opportunistic exploitation of one trader by another (payment default, poor quality of supply etc.). Non-verifiability of these hold-ups by outsiders and the absence of public enforcement impair the efficiency of such environments. Sustenance of cooperative arrangements may then require mechanisms like social reputation based on reports made by traders. This paper explores the credibility and cost of reporting structures that define the nature of social reputation mechanisms. Assuming costless information flows, it is shown that cooperation can be sustained as a sequential equilibrium if agents are required to make simultaneous public reports. The result continues to hold with small costs of information transmission. The above result may not be robust to the presence of information processing costs, if the information collection process is unobservable. However, cooperation can be restored through the creation of endogenous equilibrium uncertainty.

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1 Introduction

Many pairwise trading environments are vulnerable to risks of opportunistic exploitation of one trader by another. Non-verifiability of these hold-ups by outsiders and the absence of public enforcement impair the efficiency of such environments. Informal credit transactions in developing countries and private exchange over the internet are arena where such issues are commonplace. In the context of electronic commerce, for example, internet fraud has grown rapidly over the last few years, particularly in online auctions. Sustenance of cooperative arrangements may then require mechanisms like social reputation based on reports made by traders. To this end, sites such *ebay* have started tracking seller reputations (through reports of previous partners).

This paper studies interactions between members in a community where players change partners over time and observability of a given player's action in any period is limited. It considers environments where the flow of relevant information is generated strategically by the players themselves. Within this context, the general questions addressed are: Can cooperative arrangements be sustained in such an environment through community enforcement and social reputations? What is the role of communication in sustaining cooperation? How are such cooperative possibilities affected by costs of information flow, i.e., costs of sending or receiving reports?

In credit markets in developed nations, some information about borrowers' past behaviour is available in the form of credit rating agencies etc. In developing countries in general, and in some instances in developed nations, centralised information flow is a rarity and there are usually no exogenous mechanisms for information flow. A key issue in such a context is the *credibility* of information. Most previous studies on cooperative possibilities under imperfect monitoring have assumed exogenous and costless information processes. This paper makes the natural assumption that information flow is *costly* and examines the implications of these two central features for long-run efficient arrangements.

To model infrequent interaction, random matching within the community is assumed. In such environments, the onus of maintaining cooperation through credible punishments shifts somewhat from the individual to the group. Following Kandori (1992) and Okuno-Fujiwara and Postlewaite (1995), the analysis is conducted with the help of *social norms*. A social norm may be roughly described as "...specification of desirable behaviour together with sanction rules in a community" and should preferably satisfy certain appealing criteria such as simplicity, local information processing, straightforwardness, global stability and independence of detail (Kandori (1992)).[1] Examples of environments where such issues are of critical importance can be found in the work of Greif (1993), Greif, Milgrom and Weingast (1994) etc. where the nature of information exchange and the role of the community in maintaining long-term efficient arrangements are explored in the context of mediaeval trading groups in Europe and Asia. Yet the theoretical literature has so far not considered issues related to endogenous and costly information flows.

The results are as follows. In the benchmark case with costless information flow, cooperation can be sustained if players are sufficiently patient. The problem of private information is resolved through communication and incentives to tell the truth are generated by punishment of incompatible messages.[2] However, messages do not need to be sent along the equilibrium path as the *threat* of sending messages is sufficient to induce cooperation. This result holds also in the presence of small costs of *sending* messages (with no costs of receiving information), through a suitable modification of the benchmark norm. The introduction of costs of *receiving* information can make a non-trivial difference: the equilibria described above are not robust to small costs of information processing. There are different ways of resolving this problem. Here, the following idea is used: players declare 'tags' for themselves each period with stage game actions along the equilibrium path being conditioned on the announced profile of tags. Incentives are thus generated to hear messages and players are induced to incur costs of processing information as long as these costs are sufficiently small. However, this also generates uncertainty in terms of the action to be played along the equilibrium path. It is this very uncertainty which enables cooperation to be restored in an environment with costly information gathering.

There is now a growing literature on imperfect monitoring games with private signals. Kandori (1992) and Okuno-Fujiwara and Postlewaite (1995) consider cooperation sustenance through social norms; however, they assume exogenous information flows. Compte (1998) and Kandori

[1] These criteria are discussed in greater detail below.

[2] This is based on ideas which first appeared in Postlewaite and Schmeidler (1986).

and Matsushima (1998) also look at environments in which all or some players receive potentially different exogenous signals about the actions or strategies of other players; the issues considered are whether different actions or deviations by different players can be statistically distinguished, how many people must receive certain signals and what kind of communication can help resolve coordination problems so that punishment threats are credible. Ben-Porath and Kahneman (1996) design communication norms when a player's actions are observable by a fixed subset of other players every period and information flows costlessly.[3] They also investigate the relationship between the degree of observability and the means by which long run payoffs are evaluated.[4] By contrast, in Ahn (1997), there is one player who can perfectly observe the actions of all other players whereas other players can observe the action of the perfect observer; the question considered is whether cooperation can be sustained through signalling behaviour of the perfect observer.

Ghosh and Ray (1996) explore imperfect monitoring games *without* information flows and consider equilibrium norms whereby players may endogenously form long-term relationships.[5] However, for their equilibrium to hold, it is crucial that there be a non-trivial proportion of completely myopic players in the community who cannot be distinguished *ex-ante* from 'rational' players. Sekiguchi (1997) also looks at imperfect monitoring games without information flows and derives approximate folk theorems where information is nearly perfect, though privately observed.

The rest of the paper is organised as follows. Section 2 outlines the basic model. Section 3 proves the benchmark cooperation result and section 4 introduces costs of information transmission. Section 5 considers the case where receiving messages is no longer costless. Section 6 concludes. Section 7 contains some proofs not found in other sections.

[3]See Mailath and Morris (1999) for a discussion of 'almost-public' monitoring.

[4]See also Ahn and Suominen (1999) for a discussion of a repeated game between a seller and many buyers with limited information and cooperation through 'word-of-mouth' communication.

[5]See also Datta (1996).

2 The model

Time is indexed by $t = 0, 1, 2, \ldots$ There is a finite set of players $\mathbf{N} = \{1, 2, ..., n\}$ with n even.[6] At the beginning of every date, *each* player is randomly matched with another player (thus at any date there are $\frac{n}{2}$ partnerships) and they play a one-shot game. The stage game is a 2-person game in normal form. The same one-shot game is played at all dates by all partnerships. There is a common finite action set $\mathbf{A}$ for the stage game and symmetric payoff functions for all players.

The stage game is described a payoff function $f : \mathbf{A} \times \mathbf{A} \rightarrow \mathsf{R}^2$. Payoffs are finite. Let $f_n(a_n, a_m)$ denote the payoff to player n in the stage game when he has chosen action $a_n \in \mathbf{A}$ while his opponent, player m has chosen action $a_m \in \mathbf{A}$. A minimax point $M^n = (M_n^n, M_m^n) \in \mathbf{A} \times \mathbf{A}$ for player n is:

$M_m^n \in \arg \min_{a_m \in A} (\max_{a_n \in A} f_n(a_n, a_m))$

and $M_n^n \in \arg \max_{a_n \in A} f_n(a_n, M_m^n)$.

$M^m \in \mathbf{A} \times \mathbf{A}$ for player m is defined similarly. The 'mutual minimax' profile is (M_n^m, M_m^n). Payoffs are normalised such that for any player n, $f_n(M^n) = 0$.

The set of feasible and individually rational payoffs is $\mathbf{V} = \{v \in cof(\mathbf{A} \times \mathbf{A}) | v > 0\}$, where co denotes the convex hull. Let $x_n = f_n(M_n^m, M_m^n)$ be the payoff to player n under mutual minimaxing and $\overline{v}_n(a_m) = \max_{a_n \in A} f_n(a_n, a_m)$. x_m and $\overline{v}_m$ are defined similarly. Also, let $\underline{v}_n(a_n) = \min_{a_m \in A} f_n(a_n, a_m)$. Define $\underline{v}_m$ similarly.

After the stage game, at the end of each period, each player n can send a message. Messages are public. Let $\mathbf{R}' = \{(0,0), (0,1), (1,0), (1,1)\}$. Define the common announcement set $\mathbf{R} = \mathbf{R}' \cup \varphi$, where a 0 means "no deviation" and a 1 means "deviation". For player n report (a, b) signifies that he did a while his partner did b.[7] φ indicated a null message, that is, no report was sent by the player concerning stage game behaviour.

Sending and receiving messages are potentially costly activities. Costs of sending and receiving messages are considered separately in this paper. Assume that every time a player sends a non-null message, he incurs a cost $c_t \geq 0$ and that if a player wishes to receive messages in any

[6] The extension to the case where n is odd is straightforward.

[7] Hence a report $(1, 0)$, for example, means that player n deviated while his partner conformed to the norm.

period, he has to incur a fixed cost $c_p \geq 0$ that period. Incurring the cost allows a player to hear all messages generated that period by other players.[8] In the benchmark case, these costs are set to 0.

The repeated game is as follows. Each period is divided into three phases. In the first phase, matches are formed. In the second phase, the one-shot game is played and players decide whether or not to incur the fixed cost of receiving messages to be sent by other players later in that period. In the third phase, players submit their reports. Partnerships then dissolve and the game goes to the next period. The assumptions related to timing are important for the results of section 5 and are discussed further below. The above assumptions ensure the decision to receive messages have to be made before a player knows whether messages are to be sent by others. As an example, think of having to decide whether to check email or not before knowing if there are any messages. Or, in the setting of a traditional community where social functions and institutions have an important role to play as a forum for information exchange, the (costly) decision to participate in such rituals may have to be made before knowing whether any benefits are forthcoming or not.

A strategy for player n, π^n in the repeated game is a sequence of functions $\pi^n = (\pi^n_{1,1}, \pi^n_{1,2}, ..., \pi^n_{t,1}, \pi^n_{t,2}, .$ where $\pi^n_{t,1}$ specifies an action as a function of observed actions and messages from previous periods and $\pi^n_{t,2}$ specifies an announcement or message as a function of observed actions and messages from previous periods and the observed actions in the current period.

A profile of strategies $\pi = (\pi_1, ..., \pi_n)$ determines a probability distribution over sequences of one-shot payoffs for each player. All players have a common discount factor $\delta \in (0, 1)$. Players are assumed to maximise the discounted sum of expected stage game payoffs over an infinite horizon. Throughout, only pure strategies are considered. All previous messages are perfectly remembered and the identity of any player's current partner is publicly known.

The characterised equilibria will be in terms of norms as discussed earlier. Since a player does not observe actions of all other players, subgame perfection is not an appropriate solution notion. Weak Perfect Bayesian Equilibrium is used as a solution concept. As will be seen in the

[8] Information processing costs are discussed further in section 5. Whether costs are fixed or are allowed to vary with the number of messages makes a quantitative, though not a qualitative difference to the arguments.

equilibria described here, beliefs are either unimportant or very simple.

3 Cooperation with truthful messages

This section shows that cooperation is sustainable even if information flows through endogenously and strategically generated reports of players. To be precise, it is shown that if players are sufficiently patient, any individually rational payoff for the stage game is sustainable as an equilibrium payoff every period for the repeated game. For the benchmark analysis, the results of which are reported in Proposition 1, information flows are assumed to be costless.

A norm or a collection of strategies which sustains cooperation as an equilibrium in the repeated game is presented below. First, a verbal description is given. However, the profile of strategies may seem a little involved, it is then described as a 'machine' which is like an automaton. Consider the following *benchmark norm*. The symmetric payoff $f(a,a) = v \in \mathbf{V}$, $(a,a) \in \mathbf{A} \times \mathbf{A}$ is to be sustained. Players may have one of two labels: 'innocent' or 'guilty'. At the beginning of any period, if (and only if) a player is guilty, then he is currently on a 'punishment path'. A punishment path is of finite length; two possible paths are considered of lengths T' and T respectively, with $T < T'$.[9] The meanings of the labels and the characteristics of the paths will be defined forthwith.

At the beginning of time all players are innocent. Every period the norm specifies the following behaviour in the stage-game. If two innocent players meet each other they each play a and get v. If at least one of the partners is guilty, they mutually minimax each other and get x.

The announcements are generated thus. After the stage game, if at least one of the partners in a partnership had the label 'guilty' at the beginning of that period, each is required to make a report from the set $\mathbf{R}'$. If both partners were innocent, they are both required to make a report (from $\mathbf{R}'$) if and only if at least one of them deviated at the stage game. Otherwise they are not required to send a report at all, i.e., they send the null announcement $\boldsymbol{\varphi}$.

If the two partners send different messages, then both become guilty for T' periods (i.e., goes onto a T'-period punishment path) starting in the next period. If they give the same report,

[9] The proof details the selection of the lengths of the punishment paths.

then if anyone is indicted by both parties, he becomes guilty starting the next period. If the incriminated player is currently already on a T'-period punishment path, then this path restarts for him. If not, he is going to be considered guilty for T periods starting in the next period, where $T < T'$. The other player in the match (in the event that only one of them is indicted) remains innocent (if previously innocent) or becomes innocent (if previously guilty). If the reports say both conformed then whoever is currently innocent remains innocent and whoever is guilty continues on the punishment path. If a player in a match with at least one guilty player does not send a report, he is sent onto a T'-period path, regardless of any other deviation that may have occurred.

If messages come in reporting deviations in different matches in the same period, they are all ignored. If a new deviation is reported, anyone currently on a punishment path is forgiven and the new deviator is punished. If a currently guilty player is consistently reported[10] to have conformed to the norm in any period, then the number of periods left on the punishment path reduces by 1, unless that period is the last for that person on the current punishment path, in which case the person becomes innocent from the next period onwards.

In the event of indifference, players are assumed to follow the norm. A single report coming in from a match between two innocent players is believed. Players' beliefs after the message stage are not important because actions in the next period are dependent only on the profile of reports which is publicly known. Finally, after the action stage, a players' belief is that any player not observed by him has not deviated.

The profile of strategies is now presented as a machine: (S, s_0, T). S is a set of states, s_0 is an initial state and T is a transition function. Each $s \in S$ will specify a profile of actions. The actions have already been described as has the rule governing players' reports. The transition function specifies the state at the next period as a function of the state and the profile of reports in the current period.

The set of states is defined below:

[10]'Consistently reported' means that both partners gave the same message.

$$
\begin{aligned}
S &= S_1 \cup S_2 \cup S_3 \cup S_4 \cup \{0\} \\
S_1 &= \{\overline{(j,k,i)} | j,k \in N, 1 \le i \le T'\} \\
S_2 &= \{\overline{(j,i)} | j \in N, 1 \le i \le T'\} \\
S_3 &= \{(j,k,i) | j,k \in N, 1 \le i \le T\} \\
S_4 &= \{(j,i) | j \in N, 1 \le i \le T\}
\end{aligned}
$$

At the beginning of time all players are innocent, i.e., the initial state is 0. A state in S_1, for example means that two players j and k, are currently guilty and are in the i^{th} period of their T'-period punishment path. The transition function is defined in the following way (a 'deviation' means deviation from the action the current state and match specifies)

1. First, suppose player j, a partner in a match with at least one guilty player, does not send a report. All other players in partnerships with at least one guilty player send reports, and all players in partnerships with two innocent players send in null reports. Then switch to $\overline{(j,1)}$ irrespective of current state.

2. Next, suppose all partnerships with at least one guilty partner send in two reports. Suppose players in exactly one partnership (irrespective of the current labels of the partners) send in conflicting reports.[11] Let that partnership comprise of players k and j. Then switch to $\overline{(j,k,1)}$.

3. Next, suppose all partnerships with at least one guilty partner send in two reports and there are no partners sending in conflicting reports (irrespective of the current labels of the partners). Suppose player j is incriminated by himself and by his partner. Then

   a) If $s = \overline{(j,k,i)}, k \in N, 1 \le i \le T'$, or if $s = \overline{(j,i)}, 1 \le i \le T'$, switch to $\overline{(j,1)}$.

   b) Otherwise, switch to $(j,1)$.

4. Otherwise

   a) If $s = \overline{(j,k,i)}, 1 \le i \le T'-1$, switch to $\overline{(j,k,i+1)}$.

   b) If $s = \overline{(j,i)}, 1 \le i \le T'-1$, switch to $\overline{(j,i+1)}$.

   c) If $s = (j,k,i), 1 \le i \le T-1$, switch to $(j,k,i+1)$.

[11] For example, when n and m are matched, n reports $(0,1)$ while m reports $(0,1)$.

d) If $s = (j, i), 1 \leq i \leq T - 1$, switch to $(j, i + 1)$.

e) Otherwise, switch to or stay at 0.

The first result shows that cooperation can be sustained in the long run within the community by using the norm or collection of strategies described above. In addition, even though messages are strategic, when reports are sent, they are truthful. In equilibrium, players cooperate and messages are not sent. The intuition is simple. Innocent partners do not have an incentive to report anything when both have conformed. Since inconsistent messages are punished heavily, players will tell the truth, given simultaneous reporting.[12] Once it is shown that announcements will indeed be truthful, it is straightforward to show that if people are sufficiently patient, cooperation can be sustained. Hence players remain innocent and messages are never sent.

Proposition 1 *The norm described above constitutes an equilibrium and hence, any $v \in \mathbf{V}$ can be sustained as a long run equilibrium payoff as long as people are sufficiently patient, i.e., as long as $\delta \in (\delta_*, 1)$, for some $\delta_* \in (0, 1)$.*

Proof. See Section 7. ■

The main idea is that as long as deviation payoffs are bounded, one-shot gains don't matter in the long run if people are patient enough. And if punishments are sufficiently severe, people conform. This threat of renewed punishment is also sufficient to induce people to report truthfully, even though no one in the economy can monitor truthfulness. Hence the information structure of Kandori (1992) and Okuno-Fujiwara and Postlewaite (1995) is endogenised . Further, note that if there's only *one* other person in the economy who can observe the actions of a given player (such that the latter also observes the former), then even with random matching, community sanctions can work to maintain efficient agreements. The feasibility of the process rests on the ability to construct communication norms enabling maintenance of social reputations through credible information exchange.

Finally, this norm is evaluated in terms of the criteria laid down by Kandori (1992), which suggests that norms satisfying the following properties are the most appealing.

[12] Hence each player has the incentive to report truthfully as he believes the his partner will send truthful announcements.

- The norm should be simple. The definition of simplicity is of course subjective. In general, the equilibrium described above is about as simple as that of Kandori (1992) or Ben-Porath and Kahnemann (1996).

- There should be *local information processing*, where the only thing an agent knows when matched with someone is his own and his partner's state at the time, thus dispensing with the need for global information flow, obviously a troublesome assumption under many circumstances.

- The norm should be *straightforward*. Roughly speaking, straightforwardness implies that the amount of information available is sufficient (in the statistical sense) for the equilibrium to be maintained.

- The norm should be *globally stable*. Global stability for an equilibrium sustaining payoffs $v \in v(A)$ implies that given any finite history h, $\underset{t \to \infty}{Lt} E(v_n(t)/h) = v, \forall n \in N$. where $v_n(t)$ is player n's continuation payoff at t and $E(\cdot/h)$ is the conditional expectations operator. The finiteness of the punishment paths implies that the norm above is globally stable.

- The norm should be independent of details like the matching rule and the size of the population. As noted before, the above equilibrium is independent of population size. However, it is dependent on the assumption of uniform random matching. If, as in Kandori (1992), the existence of *repentance strategies* is assumed,[13] then the matching rule becomes irrelevant.

4 Costly information transmission

Consider now the benchmark environment augmented with costs of transmitting information. For the time being assume there are no costs of receiving messages, i.e., $c_p = 0$. Every time a

[13] The idea of repentance payoffs is simple and intuitive. Suppose n is a guilty agent and m is an innocent agent. Instead of mutually minimaxing each other when they are matched, m accepts a higher payoff while n is required to accept a lower payoff to show he is repenting. Actions sustaining such payoffs are assumed to exist.

player sends a report, he incurs a cost $c_t > 0$, which is small in comparison to the payoffs the community is trying to sustain in equilibrium. The question is, will the incentive to cooperate and to send truthful messages be preserved in this new environment? The problem is, costs of information generation creates a disincentive to send messages. Hence the strategy profile from the benchmark case needs to be modified as players may not send messages, as they expect their partners to do so. Thus there is a need to construct punishments when only one member of a partnership sends a report.

Notice that whether a player is sending a message or not is public information. Consider the following modification of the benchmark norm. Recall, players make announcements from the set $\mathbf{R}$. In matches between two innocent people, if a single report comes in, the player *not* sending a report is sent onto a punishment path of length $T'' > T'$, regardless of other deviations that may have occurred.

Consider matches such that at least one player is currently guilty. Suppose no player is on a maximal (T''-period) path. When reports come in, if a player has *not* made an announcement, he goes onto a punishment path of length $T'' > T'$ regardless of other deviations that may have occurred. If both partners in such a match are silent then both are sent onto a T''-period punishment path. On the other hand, suppose one of the players is already on a maximal punishment path. Then his partner is always required to make an announcement. Failure to do so results in him going onto a T''-period path, regardless of other deviations that may have occurred. The player on the maximal path is required to make an announcement unless he himself has deviated from the norm at the stage game. If people in different matches deviate from sending messages then all such deviations are ignored. If a player is currently on a T''-period path and if any punishment has to restart for him, then he goes on to a T''-period path again. The rest of the norm remains as before.

The profile of strategies described above are a little more complicated than the ones in the previous section and are once again described as a machine. The set of states is

$$
\begin{aligned}
S &= S_1 \cup S_2 \cup S_3 \cup S_4 \cup S_5 \cup S_6\{0\} \\
S_1 &= \{\overline{\overline{(j,k,i)}}|j,k \in N, 1 \leq i \leq T''\} \\
S_2 &= \{\overline{\overline{(j,i)}}|j \in N, 1 \leq i \leq T''\} \\
S_3 &= \{\overline{(j,k,i)}|j,k \in N, 1 \leq i \leq T'\} \\
S_4 &= \{\overline{(j,i)}|j \in N, 1 \leq i \leq T'\} \\
S_5 &= \{(j,k,i)|j,k \in N, 1 \leq i \leq T\} \\
S_6 &= \{(j,i)|j \in N, 1 \leq i \leq T\}
\end{aligned}
$$

The interpretation of the above can be found in the previous section. At the beginning of time all players are innocent, i.e., the initial state is 0. Every period the norm specifies the following behaviour in the stage-game. If two innocent players meet each other they each play a and get v. If at least one of the partners is guilty, they mutually minimax each other and get x.

The announcements are generated thus. Consider matches such that at least one player is currently guilty. If no player is on a maximal punishment path, each partner is required to make a report from the set $\mathbf{R}'$. If one of the players is on a maximal path, his partner is always required to make a report. The player on the maximal path is required to make an announcement unless he himself has deviated from the norm at the stage game. If both partners were innocent, they are both required to make a report (from $\mathbf{R}'$) if and only if at least one of them deviated at the stage game. Otherwise they are not required to send a report at all, i.e., they send the null announcement φ.

The transition function is defined in the following way

1. First, suppose player j in a partnership with two innocent players does not send a report while player k, his partner, does. All players in other partnerships with two innocent players send in null reports and all players in other partnerships send reports. Then irrespective of the current state, switch to $\overline{\overline{(j,1)}}$.

2. Next, suppose player j, a partner in a match with at least one guilty player, does not send a report. All other players in partnerships with at least one guilty player send reports, and all players in partnerships with two innocent players send in null reports. Then switch to $\overline{\overline{(j,1)}}$

irrespective of current state.

3. Next, suppose all partnerships with at least one guilty partner send in two reports. Suppose players in exactly one partnership (irrespective of the current labels of the partners) send in conflicting reports. Let that partnership comprise of players k and j. Then

a) If $s = \overline{\overline{(j,k,i)}}$ or if $s = \overline{\overline{(j,i)}}$ or if $s = \overline{\overline{(k,i)}}$, switch to $\overline{\overline{(j,k,1)}}$.

b) Otherwise, switch to $\overline{(j,k,1)}$.

4. Next, suppose all partnerships with at least one guilty partner send in two reports and there are no partners sending in conflicting reports (irrespective of the current labels of the partners). Suppose player j is incriminated by himself and by his partner. Then

a) If $s = \overline{\overline{(j,k,i)}}$ or if $s = \overline{\overline{(j,i)}}$, switch to $\overline{\overline{(j,1)}}$.

b) If $s = \overline{(j,k,i)}$ or if $s = \overline{(j,i)}$, switch to $\overline{(j,1)}$.

c) Otherwise, switch to $(j,1)$.

5. Otherwise

a) If $s = \overline{\overline{(j,k,i)}}, 1 \leq i \leq T'' - 1$, switch to $\overline{\overline{(j,k,i+1)}}$.

b) If $s = \overline{\overline{(j,i)}}, 1 \leq i \leq T'' - 1$, switch to $\overline{\overline{(j,i+1)}}$.

c) If $s = \overline{(j,k,i)}, 1 \leq i \leq T' - 1$, switch to $\overline{(j,k,i+1)}$.

d) If $s = \overline{(j,i)}, 1 \leq i \leq T' - 1$, switch to $\overline{(j,i+1)}$.

e) If $s = (j,k,i), 1 \leq i \leq T - 1$, switch to $(j,k,i+1)$.

f) If $s = (j,i), 1 \leq i \leq T - 1$, switch to $(j,i+1)$.

g) Otherwise, switch to or stay at 0.

Thus, a new tier has been added to the hierarchy of punishments described earlier. The relationship between the hierarchies and the prescribed behaviour at the stage game and the report stage remain the same as earlier. It is easy to see that this modified norm is capable of sustaining cooperation as in the benchmark case if c_t is small: in equilibrium, players cooperate at the stage game. Moreover, off the equilibrium path, truthful messages are always generated by players in matches where at least one partner is guilty. Similarly, in matches between two innocent players, if one of them deviates at the action stage, this is reported by both. However, in the event that two such players cooperate at the action stage, they have no incentives to send in any reports. The following proposition then obtains:

Proposition 2 *In an environment with small costs of information transmission c_t, the modified norm described above can sustain cooperation as an equilibrium as long as people are sufficiently patient. Moreover, this equilibrium is efficient in the sense that in equilibrium, messages are not sent and hence, players do not have to incur the cost c_t.*

Proof. See Section 7. ■

5 Costly information processing

Now consider the possibility of people having to expend resources to *receive* information. For simplicity, assume there are no costs of sending reports, i.e., $c_t = 0$. Recall, to hear messages, players have to incur a cost $c_p > 0$ in every period. c_p is small in compared to the equilibrium payoffs. Once the cost is incurred, all messages can be heard. The question is: are the norms described earlier robust to such small costs of information processing?

As discussed earlier, the results depend critically on the timing of the investment. It is clear that any profile of strategies detailing reporting requirements and stage game actions cannot sustain cooperation if players following such strategies do not listen to messages. This is simply because failure to listen to messages implies that deviations at the stage game will go unpunished (as messages reporting such deviations are not heard), and hence cooperation will break down.

Consider the benchmark norm described in section 4. Is it capable of sustaining cooperation in an environment with costly information gathering, or in other words, do players have the incentive to incur costs of information processing? Note that if any player's partner could perfectly observe his decision (to incur the cost or not) then by a suitable modification of messages and punishment contingencies, the modified norm can realise cooperation as self-enforcing behaviour. This can be achieved by requiring all players to simultaneously report, in addition to actions carried out during the stage game, whether he and his partner incurred the cost or not. Punishments based on incompatibility will ensure that one-shot savings of c_p will be wiped out if people are sufficiently patient. Similarly, earlier results go through if players could decide to incur the cost of processing information *after* they knew whether they had received messages or not. Since the receipt of an announcement indicates a deviation, this could give him the incentive to incur the cost in order to know the identity of the deviator, and hence the future path of play. However, if incurring the cost is a completely private activity and has to be undertaken *before* players know whether messages have come or not, the benchmark norm is then no longer an equilibrium, as shown below.

An important question is how this model relates to the environments discussed earlier: anonymous exchange, and traditional communities. In traditional communities for example, an individual's access to information may be dependent on his degree of involvement in social ceremonies. Such involvement is of course built up slowly, and if the function of ceremonies were purely to exchange information about opportunistic behaviour, the incentive to involve oneself would be limited since none is expected. Similarly, in the context of anonymous exchange, there may be insufficient incentives to access and analyse traders' reputations in a world only with moral hazard.

5.1 A failure of cooperation

Continuing with the same stationarity and symmetry assumptions as before, let $\mathbf{U}$ denote the set of payoffs from the one-shot Nash equilibria of the stage game. Recall that $v \in \mathbf{V}$ is the target per period payoff. Without loss of generality, suppose $v \notin \mathbf{U}$. As before, at the report stage, players can either send null messages, or make announcements from the set $\mathbf{R}'$.

Recall, the benchmark norm constitutes an equilibrium of the repeated game Along the equilibrium path a player is required to play $a \in \mathbf{A}$ in the stage game each period, designed to yield gross payoff $v \in \mathbf{V}, v > c_p$, $v \notin \mathbf{U}$. This is common knowledge. Deviation by player n is reported immediately by himself and his partner. Reports are truthful, i.e., if n has not deviated, then he cannot be reported to have deviated. If a player is reported to have deviated, he is considered guilty and goes onto a punishment path. Multiple reports in at the same time are ignored. Cooperation is sustained by communication and stage-game strategies are conditioned *only* on reports received in the past and the initial history whereas message-game strategies are conditioned only on past reports, current observed behaviour and the initial history. If two partners in period t are both currently innocent in accordance with past reports both play a.

Consider a player n in period t. Suppose all $i \in \mathbf{N}$ are innocent at t and n and his partner played a, the prescribed action, in t. Suppose n is contemplating whether to incur the cost of processing information or not, given that everyone else is hearing messages. Since n is currently innocent and so is everyone else, and no deviations have occurred or are expected, everyone is expected to remain innocent in the future, in equilibrium.

A one-period deviation gives payoff $\sum_{\tau=t+1}^{\infty} \delta^{\tau}(v - c_p)$. Else, if he hears all messages in all periods including t, his payoff is $-c_p + \sum_{\tau=t+1}^{\infty} \delta^{\tau}(v - c_p)$.

Since $\delta < 1$ and $c_p > 0$ he does not incur the cost: by symmetry, neither does anyone else. Thus messages are not heard and deviations, even if reported, cannot be punished. Thus cooperation cannot be sustained by any such equilibrium. The following thus obtains:

Proposition 3 *If $c_p > 0$, the benchmark norm is no longer an equilibrium of the repeated game for any $\delta < 1$.*

Thus for $c_p = 0$, the benchmark equilibrium can sustain any payoff dominating the individually rational payoffs in the infinitely repeated game. But that is not true for *any* positive c_p. Hence, the set of payoffs under that profile of strategies is discontinuous at $c_p = 0$.

5.2 Restoring cooperation

Communication was introduced to facilitate the transfer of private information to the public domain. Hence it allowed coordination on strategies in the community by removing uncertainty. The previous result shows that communication, which resolved the uncertainty stemming from imperfect observability, removes all uncertainty in equilibrium. The reason cooperation breaks down is that, in equilibrium, messages lose their value. In the benchmark norm, along the equilibrium path, the future path of play is known to all players with probability 1. This holds trivially as all players are supposed to play the same action every period. However the failure of cooperation does not stem from this symmetry or stationarity (in terms of stage game actions) *per se*. What is critical is that every player knows what action to take in the future along the equilibrium path. Since deviations cannot occur in equilibrium, players lose the incentive to gather information. Therefore, to restore cooperation, players must be given the incentive to incur the cost of processing information every period. Clearly, this can only occur if future stage game actions depend on current information.

In an environment with costly information gathering, sustaining cooperation thus requires some uncertainty about future behaviour. There are different ways of constructing norms which have this property. Simple formulations are presented below based on extensions of the benchmark norm which restore cooperation. The model is augmented to incorporate the notion that uncertainty about the future path of play is necessary to sustain cooperation in this environment. All players are assumed to have a common finite 'tag' set $\mathbf{T} = \{\tau_1, ..., \tau_m\}$. Define the augmented common message set $\mathbf{M} = \mathbf{R}' \times \mathbf{T}$. If a player sends a message from $\mathbf{M}$, he not only sends an announcement about stage game behaviour but also declares a tag for himself.

First, assume first the norm requires the same payoff to be sustained every period along the equilibrium path. Let $v' = v - c_p > 0$, $v = f(a_v, a_v), a_v \in \mathbf{A}$. Define $\mathbf{A}_v = \{a_v | v = f(a_v, a_v)\}$. v is said to satisfy *Property A* if $\#\mathbf{A}_v \geq \#\mathbf{T} \times \mathbf{T}$.

To understand this, define $\mathbf{\Pi} = f(\mathbf{A} \times \mathbf{A})$. Consider $\pi > 0, \pi \in \mathbf{\Pi}$ such that π can be sustained by a symmetric action profile. The property ensures that there will be at least as many distinct symmetric action profiles sustaining π as the number of possible profiles of tags. Therefore, when two players are matched, there exists an appropriate symmetric action profile for them, given their profile of tags, sustaining any chosen π. Moreover, an action profile which is appropriate given a particular profile of tags will not be appropriate given a different profile of tags.[14]

[14] Property A is admittedly a little strong. However, as discussed earlier, the action profile along the equilibrium path cannot be stationary and known in advance with probability 1. Hence some device is necessary to coordinate actions. Introduction of tags and enrichment of the action and message spaces provide such a device. Dropping the assumption that the *payoff* along the equilibrium path is stationary would allow a considerable weakening of Assumption A. This is discussed further below.

The following example illustrates. Let $\mathrm{T} = \{\tau_1, \tau_2\}$, $\mathrm{A} = \{c_1, .., c_4, d_1, .., d_4\}$. $f_1(c_i, c_i) = f_2(c_i, c_i) = 1$, $f_1(d_i, d_i) = f_2(d_i, d_i) = 0, \forall i$. $f_1(c_i, d_{i'}) = f_2(d_j, c_{j'}) = -l, l > 0$, $f_1(d_i, c_{i'}) = f_2(c_j, d_{j'}) = 1 + g, g > 0$.

Consider the benchmark norm with the following modifications. The symmetric payoff $v \in \mathbf{V}$ is to be sustained every period such that v satisfies Property A. As before, players may have one of two labels, innocent or guilty. At the beginning of time all players are innocent and each player has a random tag attached to him. Before detailing the actions to be played the punishment contingencies and the report stage prescriptions are described.

Every period, after the stage game, all players have to send a message drawn from the set $\mathbf{M} = \mathbf{R}' \times \mathbf{T}$.[15] Players are assumed to make a selection τ_i from $\mathbf{T}$ with $\Pr(\tau_i) = p_i > 0 \forall i$ such that selections are independent across players and periods. A player not sending a message is sent onto a T'-period punishment path regardless of other deviations. All other punishment contingencies are as in the benchmark norm.

If two players are matched such that at least one is guilty, they mutually minimax each other and get x. Finally, consider a partnership between two innocent players n and m in any period. Suppose in the previous period they announced the tags τ_i and τ_j respectively for themselves. Then the norm requires them to play the action a_{ijv} such that $f(a_{ijv}, a_{ijv}) = v$. Since v satisfies Property A, the existence of such actions is guaranteed. The rest of the norm remains unchanged.

The profile is presented once again as a machine. The set of states is defined below:

$f_1(c_i, c_k) = f_1(c_i, d_{i'}), i < k$, $f_1(c_i, c_k) = f_1(d_j, c_k), i > k$. $f_2(c_i, c_k) = f_2(c_i, d_{i'}), i < k$, $f_2(c_i, c_k) = f_2(d_j, c_k), i > k$. $f_1(d_i, d_k) = f_1(c_j, d_{j'}), i < k$, $f_1(d_i, d_k) = f_1(d_j, c_{j'}), i > k$. $f_2(d_i, d_k) = f_2(c_j, d_{j'}), i < k$, $f_2(d_i, d_k) = f_2(d_j, c_{j'}), i > k$.

This game is similar to the Prisoner's Dilemma inasmuch as that all defection strategies either weakly or stongly dominate all cooperation strategies.

[15] Recall, in the benchmark norm, players are required to send messages from the set $\mathsf{R} = \mathsf{R}' \cup \varphi$. This is because allowing players to announce different tags makes no difference to the results. Here, however, the announced tags make a crucial difference.

$$
\begin{aligned}
S &= \widehat{S} \times \widehat{T} \\
\widehat{T} &= \prod^{n} \mathbf{T} \\
\widehat{S} &= S_1 \cup S_2 \cup S_3 \cup S_4 \cup \{0\} \\
S_1 &= \{\overline{(j,k,i)} | j,k \in N, 1 \le i \le T'\} \\
S_2 &= \{\overline{(j,i)} | j \in N, 1 \le i \le T'\} \\
S_3 &= \{(j,k,i) | j,k \in N, 1 \le i \le T\} \\
S_4 &= \{(j,i) | j \in N, 1 \le i \le T\}
\end{aligned}
$$

At the beginning of time all players are innocent, i.e., the initial state is 0, together with some initial profile of tags. Stage game actions and the announcement rules have already been detailed. The transition function is described below and describes the transition purely in terms of $\widehat{S}$, as players are assumed to announce tags randomly and without reference to occurrences currently or in the past.

1. First, suppose player j has not sent a report from $\mathbf{M}$ while all other players have. Then switch to $\overline{(j,1)}$ irrespective of current state.

2. Next, suppose all players have sent reports and players in exactly one partnership (irrespective of the current labels of the partners) send in conflicting reports. Let that partnership comprise of players k and j. Then switch to $\overline{(j,k,1)}$.

3. Next, suppose all players have sent reports and there are no conflicting reports. Suppose player j is incriminated by himself and by his partner. Then

    a) If $\widehat{s} = \overline{(j,k,i)}, k \in N, 1 \le i \le T'$, or if $\widehat{s} = \overline{(j,i)}, 1 \le i \le T'$, switch to $\overline{(j,1)}$.

    b) Otherwise, switch to $(j,1)$.

4. Otherwise

    a) If $\widehat{s} = \overline{(j,k,i)}, 1 \le i \le T' - 1$, switch to $\overline{(j,k,i+1)}$.

    b) If $\widehat{s} = \overline{(j,i)}, 1 \le i \le T' - 1$, switch to $\overline{(j,i+1)}$.

    c) If $\widehat{s} = (j,k,i), 1 \le i \le T - 1$, switch to $(j,k,i+1)$.

    d) If $\widehat{s} = (j,i), 1 \le i \le T - 1$, switch to $(j,i+1)$.

    e) Otherwise, switch to or stay at 0.

The following proposition shows that the above norm is capable of sustaining cooperation in the long run within the community. In addition, even though messages are non-verifiable, when reports are sent, they are truthful. Along the equilibrium path, players cooperate and incur the costs of processing information. Hence there is an 'inefficiency' in the presence of information gathering costs. The results also extend, with suitable modifications of the norm, to the case where sending information is costly.

Proposition 4 *Any $v' = v - c_p \in \mathbf{V}$ such that v satisfies Property A can be sustained as a long run equilibrium payoff by the above collection of strategies as long as people are sufficiently patient, i.e., as long as $\delta \in (\delta^*, 1)$, for some $\delta^* \in (0, 1)$.*

Proof. See Section 7. ■

As mentioned earlier, the need for a feature such as property A arises due to the requirement of a stationary payoff every period. The logic is simple: the same payoff is to be generated every period while different actions have to be played. Hence there must be multiple actions giving the same payoff. Relaxing the requirement of stationary payoffs allows the relaxation of the property, as shown below. A formal description is eschewed in favour of a sketch of the argument as the details follow from straightforward modifications of past results.

Recall, $\mathbf{T} = \{\tau_1, ..., \tau_m\}$. Define $\overline{\mathbf{A}} = \{a \in \mathbf{A} | f(a, a) > c_p\}$. Suppose *Assumption B* holds: $\#\overline{\mathbf{A}} \geq \#\mathbf{T} \times \mathbf{T}$.

Hence there exists $1-1$ map $g : \mathbf{T} \times \mathbf{T} \to \overline{\mathbf{A}}$. Let $g(\tau_i, \tau_j) = \overline{a}_{ij}$ and $f(\overline{a}_{ij}, \overline{a}_{ij}) = \overline{v}_{ij}$. The following lemma thus obtains:

Lemma 1 *Under Assumption B, there exists a $1-1$ map $g : \mathbf{T} \times \mathbf{T} \to \overline{\mathbf{A}}$ with $g(\tau_i, \tau_j) = \overline{a}_{ij}$ and $f(\overline{a}_{ij}, \overline{a}_{ij}) = \overline{v}_{ij}$ and a probability vector $p = (p_1, ..., p_m) > 0$ such that $\sum p_i p_j \overline{v}_{ij} = v$ for any v in the interior of $\mathbf{V}$.*

Proof. Since v in the interior of $\mathbf{V}$, the proof follows from assumption B and the convexity of $\mathbf{V}$. ■

Constructing the strategy profile is now straightforward. Suppose the community has a target net per-period expected payoff of $v' = v - c_p > 0$ with v in the interior of $\mathbf{V}$. Under assumption B, any such target payoff can be sustained by essentially the same norm as above. Players are assumed to make a selection τ_i from $\mathbf{T}$ with $\Pr(\tau_i) = p_i > 0 \forall i$ such that selections are independent across players and periods. Off the equilibrium path, players are punished through mutual minimaxing. In equilibrium, players get v' every period in expected terms. Modification of the arguments used in the proof of the first and fourth propositions the above lemma immediately show that such a profile of strategies would constitute an equilibrium.

Proposition 5 *Under Assumption B, any $v' = v - c_p \in \mathbf{V}$ such that v is in the interior of $\mathbf{V}$ can be supported as a long run equilibrium payoff in expected terms as long as $\delta \in (\delta^*, 1)$, for some $\delta^* \in (0, 1)$.*

Proof. The proof follows from the above lemma and a straightforward extension of the arguments used in the proofs of Propositions 1 and 4. ■

The creation of tags and sending of messages designed to generate uncertainty about actions to be played along the equilibrium path may be criticised as a somewhat artificial construct. The need to do this arises due to the nature of opportunism, which is solely due to moral hazard. The possibility of adverse selection could restore individual incentives to gather information. This may then be a better model for the world of anonymous exchange, as social ceremonies in traditional communities tend to be complex institutions and can perform many different functions.

In contrast to the benchmark case, however, messages have to be sent every period. There is thus an added inefficiency if transmission of information is costly. Interestingly the presence of transmission costs does not create inefficiencies (see Section 4) if gathering information is costless. However, in the presence of processing costs, since messages have to be sent every period, there can be multiple inefficiencies, stemming from the costs of sending as well as receiving information.

6 Conclusion

The paper studied the maintenance of cooperation in communities with infrequent pairwise interactions and showed that folk theorems can be proved even if there is limited monitoring in the economy and information flows are endogenous and non-verifiable. The role of communication turns out to be crucial in devising methods to ensure that deviations from social norms are punished. Moreover, it was shown cooperation can be sustained, under some circumstances, even if gathering or sending information are costly and private activities.

It has been observed that size of a group and the ease of information flow within a group are related. The argument is that information should flow more smoothly in smaller communities. Of course, in smaller communities each individual has more to lose from punishing deviators. In the model, information flows are meagre but this difficulty is not related to group size. In this context, it is useful to compare the results here with the *contagion equilibria* of Kandori (1992).[16] A nice feature of such strategies is that they do not require communication. However, they require some restrictions on the stage game payoffs and are not very useful with large populations. Also, in general, there is a lack of robustness to noise.[17] With uncertainty, it is difficult to sustain cooperation. In both mechanisms, along the equilibrium path, cooperation always occurs and messages are never sent. There is thus some observational equivalence between them. However, it can be easily shown that for the same game, the equilibrium strategies considered in this paper can sustain cooperation with a smaller discount factor. To an extent this results from the assumption that all messages can be heard publicly. A possible area of future research would be to investigate how contagion equilibria and communications based equilibria compare when messages are no longer fully public, as in Ahn and Suominen (1999).

Finally, in this paper, costly information processing led to a failure of cooperation as there was no value to hearing messages in equilibrium. The creation of endogenous equilibrium uncertainty helped resolve the problem. It would be interesting to see whether such problems could be tackled by the presence of exogenous uncertainty.

[16] See also Ellison (1994) and Harrington (1995).

[17] Ellison (1994) shows that contagion equilibria are not too fragile. However, the equilibrium he constructs is not globally stable.

7 Proofs

Proof of Proposition 1. First of all, if everyone always follows the norm, clearly payoff v can be sustained. The question therefore is whether it is in the interest of everyone to follow the norm. Since the equilibrium concept is Nash, it is sufficient to check if it is in the interest of any arbitrary player to follow the norm, given that everyone else is following it. For simplicity, shall assume that the number of players is large[18].

The first step is to show that, given any set of actions during the stage game, a player sends truthful messages. Suppose n and m, two arbitrary players, have been matched in some period t.

A) n is innocent while m is currently being punished. The case where both are currently guilty is similar.

i) Suppose they both follow the norm during the stage game.

Given m's strategy of reporting of $(0,0)$, if n says anything else, a T'-period punishment starts for him. Hence he prefers to report $(0,0)$. Similarly, m he prefers to say $(0,0)$, as his punishment path then reduces by a period.

ii) Suppose n has deviated during the stage game, while m has not.

Given m's report of $(0,1)$, if n reports $(1,0)$ a T-period punishment path starts for him. If he reports anything else, a T'-period punishment path starts. Similarly, if m prefers to report $(0,1)$, as he is then forgiven.[19]

iii) Suppose m has deviated during the stage game, while n has not.

Given m's strategy of reporting $(1,0)$, if n prefers to reports $(0,1)$, as he then remains innocent. If m is currently on a T'-period punishment path, he reports $(1,0)$ by indifference. If he is on a T-period punishment path, he also prefers to report $(1,0)$ as otherwise he goes onto a T'-period path.

[18] This assumption simplifies calculations and is otherwise unimportant.

[19] This way of generating strict incentives is new. However, it does make punishments (somewhat) history-dependent. Also, sanctions are not inefficient in equilibrium. For alternate approaches, see Kandori and Matsushima (1998) and the discussion of revelation constraints and potential inefficiencies in Compte (1998). Their approaches are in general not relevant for matching models.

B) Both n and m are currently innocent.

i) Suppose they both follow the norm during the stage game. Clearly, neither has any individual incentive to send a report.

ii) Suppose n has deviated during the stage game, while m has not. The case where m deviated while n did not is symmetric.

Given m's strategy of reporting $(0,1)$, if n reports $(1,0)$, a T-period punishment path starts for him. If he reports anything else, a T'-period punishment path starts. Hence, he reports $(1,0)$. Similar arguments show that m also prefers to report $(0,1)$.

Hence, irrespective of the history, messages are generated truthfully. The next step is to check that, given that messages will be truthful, the prescribed stage game actions are indeed incentive-compatible.

Suppose player n is currently guilty. Suppose also player n is currently on a T'-period punishment path.

If he conforms, he gets at least $x + \delta x + ... + \delta^{T'-1}x + \frac{\delta^{T'}v}{1-\delta}$.

If he deviates, he gets at most $0 + \delta x + ... + \delta^{T'}x + \frac{\delta^{T'+1}v}{1-\delta}$.

Choose T' such that $x\frac{1-\delta^{T'}}{1-\delta} + \frac{\delta^{T'}v}{1-\delta} > 0$. This is always possible for δ sufficiently close to 1.

Clearly, he is strictly better off conforming for any $\delta < 1$.

Also, the same argument holds if n is currently on a T-period punishment path as the payoff difference between starting a T-path and T'-path is $\delta^{T+1}(1 + ... + \delta^{T'-T-1})(v - x) > 0$.

If player n is currently innocent, by conforming, he gets $\frac{v}{1-\delta}$.

By deviating, he gets at most $\overline{v}_n + \delta x + ... + \delta^{T}x + \frac{\delta^{T+1}v}{1-\delta}$.

Therefore, in order for the norm to be an equilibrium, it has to be the case that

$$\delta(1 - \delta^T)x + \delta^{T+1}v + (1 - \delta)\overline{v}_n \leq v$$

As $\delta \to 1$, holding δ^T constant such that $\delta^T < 1$, the L.H.S. becomes $0 < (1 - \delta^T)x + \delta^T v < v$.

Hence, n is strictly better off conforming if δ is high enough.

Thus the norm is an equilibrium. Q.E.D. ■

Proof of Proposition 2. Most of the proof is identical to that of the previous proposition and hence the details are omitted. The main thing to check is whether players have incentives to send messages, given that transmission of information is now costly.

Consider a match between two players, such that none of them are on a maximal punishment path. By the arguments of the previous proposition, if the players send messages, they are going to be truthful. At the same time, since $T'' > T' > T$, they prefer to send messages as long as c_t is not too large.

Consider a match with one player on a maximal punishment path. His partner clearly always has the incentive to send a report, and tell the truth, by the same argument as earlier. The player on the maximal path has the incentive to report and tell the truth as long as he himself has not deviated at the stage game. If he deviates at the stage game, then given his partner is telling the truth, a T''-period path restarts for him, regardless of what he says. Hence he prefers not to report.

Finally, by the same arguments as above, T'', T' and T can always be chosen such that as long as c_t is sufficiently small and δ sufficiently close to 1, players will conform to the norm. Q.E.D. ■

Proof of Proposition 4. The proof is similar to that of Proposition 1 and proceeds in three basic steps. First, it will be shown that players have the incentive to send truthful messages in stage 3, i.e., after the stage game is over and players have decided whether or not to incur processing costs. Next, players' incentives to incur the costs of receiving information will be investigated, given any profile of actions at the stage game. Finally, it will be shown that players have the incentive to conform at the stage game. For simplicity, assume that $\mathbf{T}=\{\tau_1, \tau_2\}$.[20]

Recall, the norm reserves the heaviest punishment for players who either do not send messages, or send inconsistent messages. Adapting the proof of Proposition 1, it is immediate that players will always send messages, and indeed will send truthful messages. This result is true irrespective of the profile of actions played at the stage game, i.e., irrespective of whether players conformed or deviated at the stage game. Also, by the same logic, messages will be truthful irrespective of whether players have incurred the cost of processing information or not. This is simply because punishment for not sending messages, or sending inconsistent messages cannot be conditioned on whether or not the player has incurred the cost of gathering information, as that activity is

[20] Allowing $\mathbf{T}$ to have more than two elements strengthens the results.

completely private.

Given that at the report stage, stage game actions will become fully revealed, the next step is to check whether players have the incentive to invest in hearing messages. Define V_I to be the lifetime expected payoff of a currently innocent player and $V_{\overline{T}}(V'_{\overline{T}})$ to be the lifetime expected payoff of a player on a $T(T')$-period punishment path who is currently on the $\overline{T}^{\text{th}}$ period.

Consider a player in a partnership such that both partners conform at the stage game. Suppose he is currently innocent. If he incurs the cost, he will know the appropriate action to play the next period. Hence his payoff is $\delta V_I - c_p$. If he does not, with some probability, he will play the wrong action.[21] Hence his maximum payoff will be $\delta[p_i\{(v - c_p) + \delta V_I\} + (1 - p_i)\{(\overline{v} - c_p) + \delta V_1\}]$. Since $\overline{v}$ is bounded and c_p is small, he will incur the cost.

Notice, the same argument holds for a player who is currently guilty and is on the last period of his punishment path as given that he has conformed, he is innocent the next period onwards. On the other hand, if he is currently guilty, and is not on the last period of his punishment path, he has no incentive to incur the cost. This arises as his prescribed action on the punishment path is the mutual minimax action, regardless of whether he has incurred the cost or not.

Now consider a player who has deviated at the stage game. Since messages will fully reveal stage game actions, it is clear that such a player will become guilty the next period onward, and hence will be required to play the mutual minimax action for the duration of his punishment path. Hence he will not incur the cost.

Conversely, a player whose partner deviates at the stage game will become innocent or remain innocent from the next period onwards. If he incurs the cost, he will know the tag of his partner the next period, whoever that partner may be. But if he does not, any action he chooses to play against his partner the next period will have positive probability of being wrong. If he plays the wrong action, then with probability 1, he will be guilty from the period after. Clearly, if c_p is small, the one-shot saving will be outweighed by the prospect of becoming guilty.

The final step is to show that players have an incentive to conform to the norm at the stage game. This follows easily from the proof of Proposition 1 as c_p is small. Hence, all players who are currently guilty will eventually become innocent again. Moreover, all innocent players will

[21] If T has more than 2 elements, the probability of playing the wrong action will be higher.

remain innocent and will incur the cost of processing information every period. Since everyone is innocent in the initial history, cooperation will be sustained. Q.E.D. ■

8 References

1. Ahn, I. (1997): "Imperfect Information Repeated Games with a Single Perfect Observer," mimeo.

2. Ahn, I. and M. Suominen (1999): "Word-of Mouth Communication and Community Enforcement," mimeo.

3. Ben-Porath, E. and M. Kahneman (1996): "Communication in Repeated Games with Private Monitoring," *Journal of Economic Theory*, **70**, p. 281-97.

4. Compte, O. (1998): "Communication in Repeated Games with Imperfect Private Monitoring," *Econometrica*, **66**, p. 597-626.

5. Datta, S. (1996): "Building Trust," STICERD Discussion Paper # TE/96/305, LSE.

6. Ellison, G. (1994): "Cooperation in the Prisoners' Dilemma with Anonymous Random Matching," *Review of Economic Studies*, **61**, p. 567-88.

7. Ghosh, P. and D. Ray (1996): "Cooperation in Community Interaction without Information Flows," *Review of Economic Studies*, **63**, p. 491-519.

8. Greif, A. (1993): "Contract Enforceability and Economic Institutions in Early Trade: the Maghribi Traders' Coalition," *American Economic Review*, **83**, p. 525-48.

9. Greif, A., P. Milgrom and B. Weingast (1994): "Coordination, Commitment and Enforcement: the Case of the Merchant Guild," *Journal of Political Economy*, **104**, p. 745-76.

10. Harrington, J. E., Jr. (1995): "Cooperation in a One-shot Prisoners' Dilemma," *Games and Economic Behavior*, **8**, p. 364-77.

11. Kandori, M. (1992): "Social norms and Community Enforcement," *Review of Economic Studies*, **59**, p. 63-80.

12. Kandori, M. and H. Matsushima (1998): "Private Observation, Communication and Collusion," *Econometrica*, **66**, p. 627-52.

13. Mailath, G. J. and S. Morris (1999): "Repeated Games with Almost-Public Monitoring," CARESS Working Paper # 99-09, University of Pennsylvania.

14. Okuno-Fujiwara, M. and A. Postlewaite (1995): "Social Norms and Random Matching Games," *Games and Economic Behaviour*, **9**, p. 79-109.

15. Postlewaite, A. and D. Schmeidler (1986): "Implementation in Differential Information Economies,", *Journal of Economic Theory,* **39**, p. 14-33.

16. Sekiguchi, T. (1997): "Efficiency in Repeated Prisoner's Dilemma with Private Monitoring," *Journal of Economic Theory*, **76**, p. 345-61.