APPLIED WELFARE ECONOMICS AND POLICY ANALAYSIS

■ Measurement of Welfare Change for an Individual

- applied welfare economics ultimately concerned with measuring changes in well-being of households, based on comparing different bundles of commodities

- principles of measurement for an individual household based on assumption that only information available is household's preference ordering

- ordering based on usual assumptions that preferences are complete, reflexive, transitive, increasing and continuous

- observation of preferences provides an *ordinal* ranking of alternatives, but we seek a money metric that provides a *cardinal* measure of utility

■ Basic Problem

- consider household that consumes goods x_1 and x_n subject to a budget constraint $m=p_1x_1 + p_nx_n$

- Figure 1 illustrates basic problem where prices and income may have changed, and distance between u_1 and u_2 is what has to be measured

- by assumption of homogeneity of degree zero, know that equi-proportionate changes in all prices and income will not affect equilibrium

- need to peg the price level, so treat x_n as numeraire commodity and set its price to one, and also treat x_n as a composite of all commodities other than x_1 (Hicks, 1939)

- in Figure 1, m^1 and m^2 are incomes in two situations, and slope of budget lines are $p^1{}_1$ and $p^2{}_1$

- Household chooses commodities to maximize:

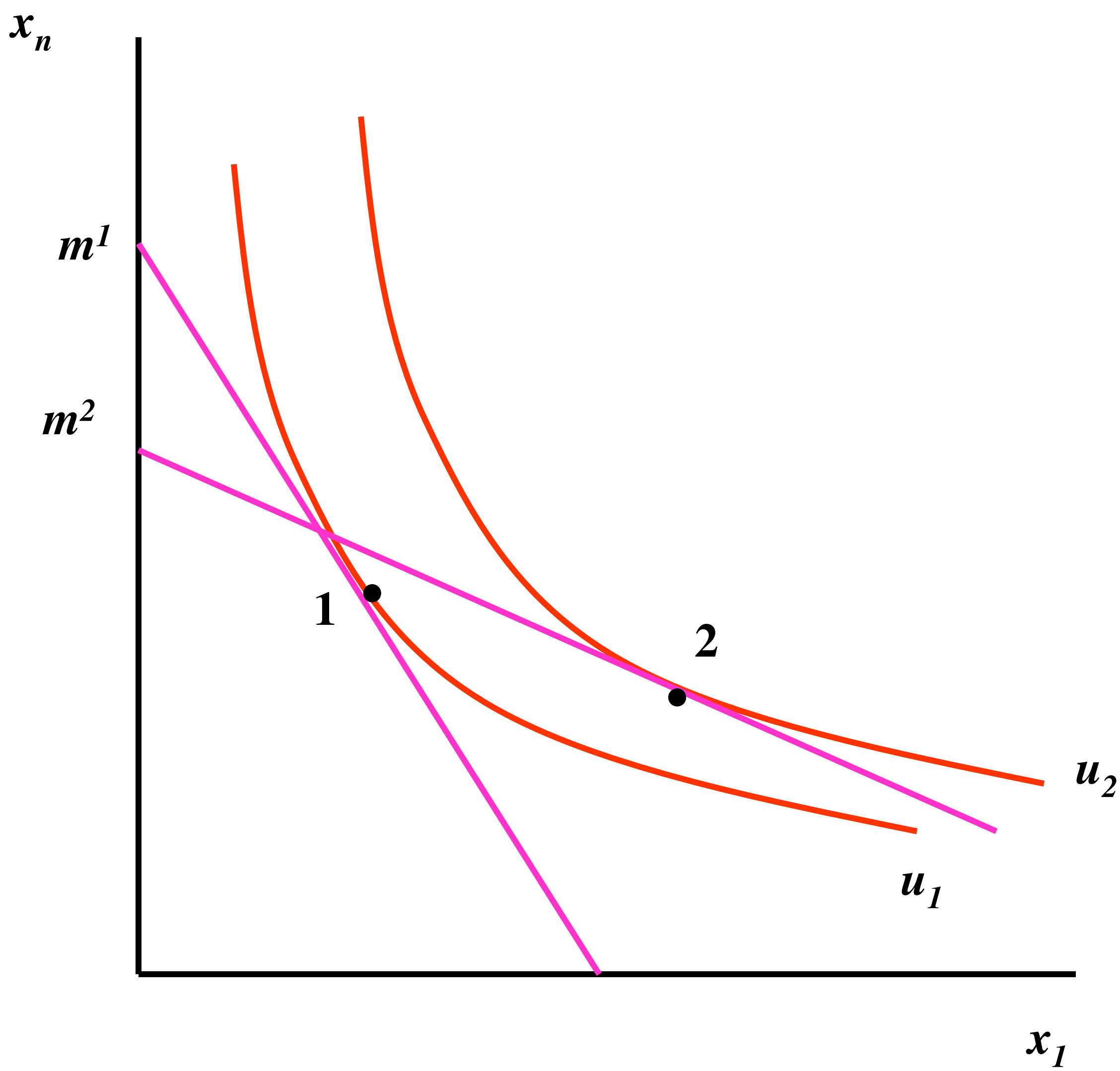
$$max_x u(x_1,...,x_n) \tag{1}$$

subject to:

$$\sum p_i x_i = m \tag{2}$$

- solution to this gives a set of demand functions $x_i(p,m)$ and a value for the Lagrange multiplier $\lambda(p,m)$, both of which depend on exogenous prices and income

Figure 1



- indirect utility function v obtained by substituting demand functions into utility function u:

$$v(p,m) = u[x_1(p,m),\ldots,x_n(p,m)] \quad \textbf{(3)}$$

- v(.) satisfies Roy's theroem:

$$x_i(p,m) = -\frac{\partial v(p,m)/\partial p_i}{\partial v(p,m)/\partial m} \quad \textbf{(4)}$$

which yields *Marshallian* demand functions by differentiation of v(.)

- consider a household faced with small change in prices and income, change in utility is derived as:

$$\begin{aligned} dv &= \sum (\partial v/\partial p_i)dp_i + (\partial v/\partial m)dm \\ &= -\lambda \sum x_i(p,m)dp_i + \lambda dm \end{aligned} \quad \textbf{(5)}$$

- dividing through by λ, get a monetary measure of welfare change measured in units of numeraire:

$$dW = dv/\lambda = -\sum x_i(p,m)dp_i + dm \quad \textbf{(6)}$$

**** see the appendix***

- all terms on right-hand side are measurable

- suppose prices and income change by some discrete amounts between situations 1 and 2, need to integrate differential welfare changes:

$$\Delta W = \int dW = -\sum \int_1^2 x_i(p,m)\,dp_i + \Delta m \qquad (7)$$

***** *see the appendix*

- (7) is a *line integral*, the sum of a series of integrals each of which depends on variables that are variables of integration in other integrals - depends on order in which prices and income are changed, and order of integration, i.e., integral is *path dependent*

- path dependency does not arise when only a single price or income is changed, e.g., if m changes, prices constant, $\Delta W = \Delta m$, i.e., vertical distance between budget lines in Figure 1 when Δm is measured in terms of numeraire

- if only p_1 changes, *ceteris paribus*, welfare change is:

$$\Delta W = -\int_{p_1^1}^{p_1^2} x_1(p,m)\,dp_1 \qquad (8)$$

***** *see the appendix*

- Figure 2 shows p_1 falling from $p^1{}_1$ to $p^2{}_1$, integral in (8) being evaluated as move from 1 to 2 along price consumption line *PC*, each point on *PC* being a point on the Marshallian demand curve

- ΔW is the area $p^1{}_1 abp^2{}_1$, which is *Marshallian consumer surplus*

- while (8) appears to avoid path dependency problem, uniqueness is artificial, as there are several ways of getting from 1 to 2, each giving a different measure of welfare change, i.e., *compensating variation* (*CV*) and *equivalent variation* (*EV*)

■ Compensating Variation

- *CV* of move from 1 to 2 is that amount of income that could be taken away from a household in new situation, and leave them as well off as in the old

- Figure 3 shows situation where relative prices and income have changed, household being better off at 2, *CV* being $m^2 - e^1$, measured in terms of numeraire

Figure 2: Marshallian Consumer Surplus

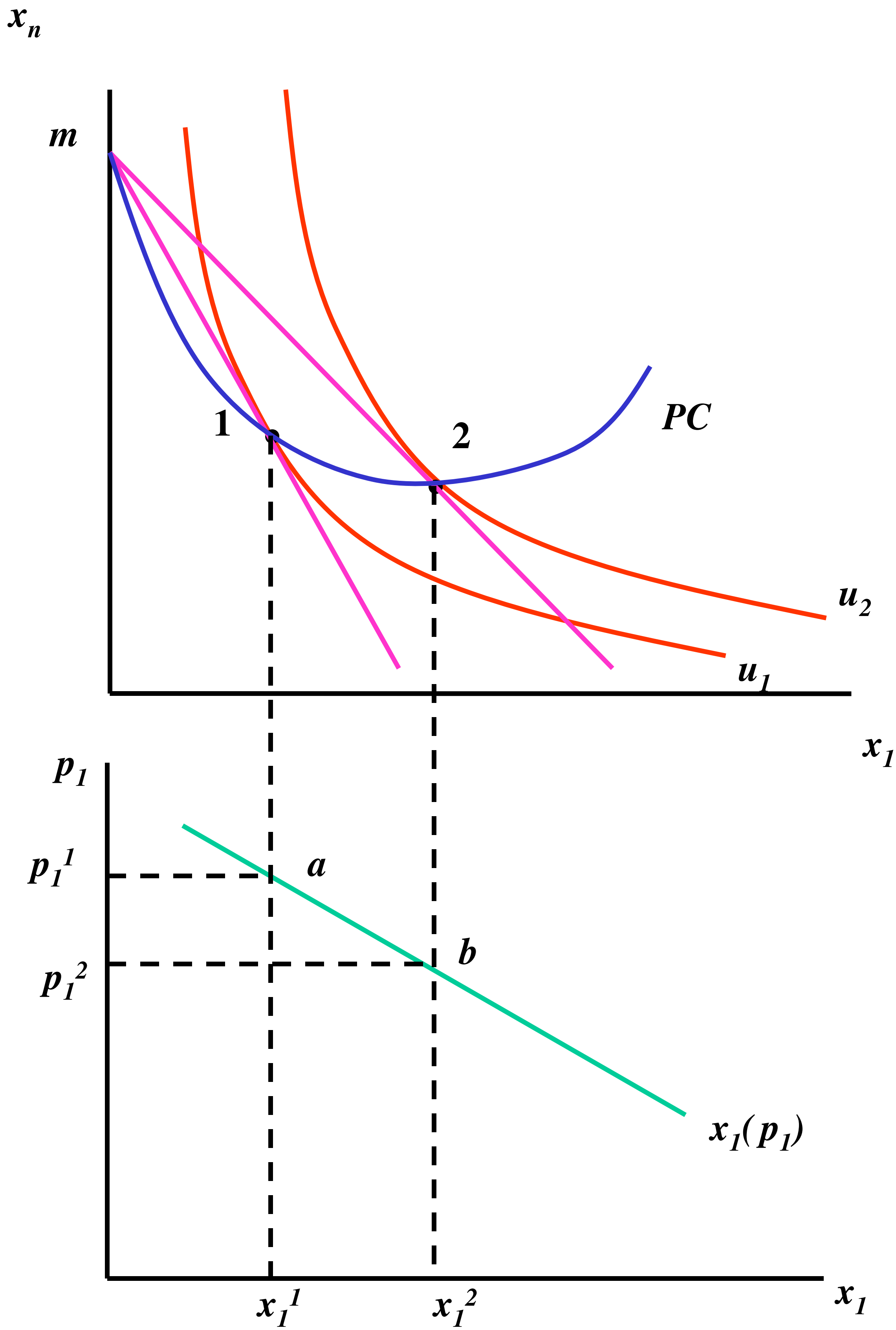
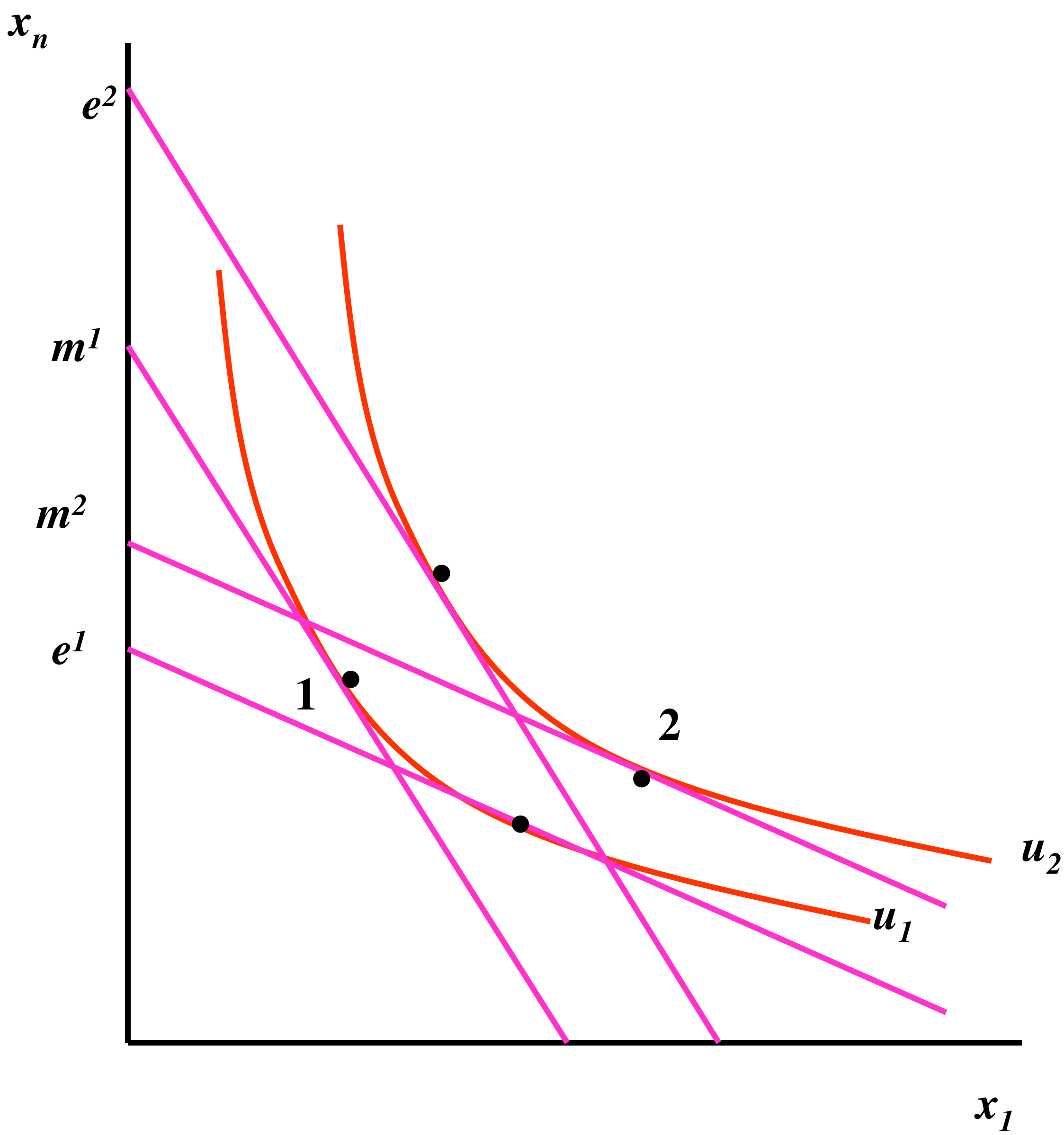


Figure 3



- for case of several commodities, use household *expenditure* function, so household problem is:

$$min_x \sum p_i x_i \qquad (9)$$

subject to:

$$u(x_1,...,x_n) = u \qquad (10)$$

- first-order conditions for problem can be solved for x_i at various sets of prices p for a given utility u:

$$x_i = x_i(p,u), \quad i = 1,...,n \qquad (11)$$

- these are the *compensated demand* functions

- expenditure function derived by substituting (11) into objective function:

$$e(p,u) = \sum p_i x_i(p,u) \qquad (12)$$

where (12) shows income household needs to attain u at prices p, most important property is Shepard's lemma:

$$\partial e(p,u)/\partial p_i = x_i(p,u), \quad i = 1,...,n \qquad (13)$$

- i.e., compensated demand functions are derivatives of expenditure function w.r.t. prices; in Figure 3, e^1 is expenditure required for u_1 at prices p^2, so CV is:

$$CV = m^2 - e(p^2, u_1) \tag{14}$$

(14) is a key equation in applied welfare economics

- given $\Delta m = m^2 - m^1$, CV can be re-written:

$$\begin{aligned} CV &= m^1 - e(p^2, u_1) + \Delta m \\ &= e(p^1, u_1) - e(p^2, u_1) + \Delta m \end{aligned} \tag{15}$$

- as e is continuous in p, (15) can be re-written:

$$CV = \int_{p^2}^{p^1} \sum (\partial e(p, u_1)/\partial p_i) dp_i + \partial m \tag{16}$$

or using Shepard's lemma, and reversing order of integration:

$$CV = -\int_{p^1}^{P^2} \sum x_i(p, u_1) dp_i + \Delta m \tag{17}$$

- this is similar to (7), it is a line integral involving *compensated* demand functions rather than uncompensated functions

- line integral here is path independent as cross-partial derivatives of $x_i(p,u_1)$ are symmetric (follows from Shepard's lemma), integrability conditions met, i.e., value of line integral is unique

- suppose only price p_i changes:

$$CV = -\int x_i(p,u_1)\,dp_i \qquad (18)$$

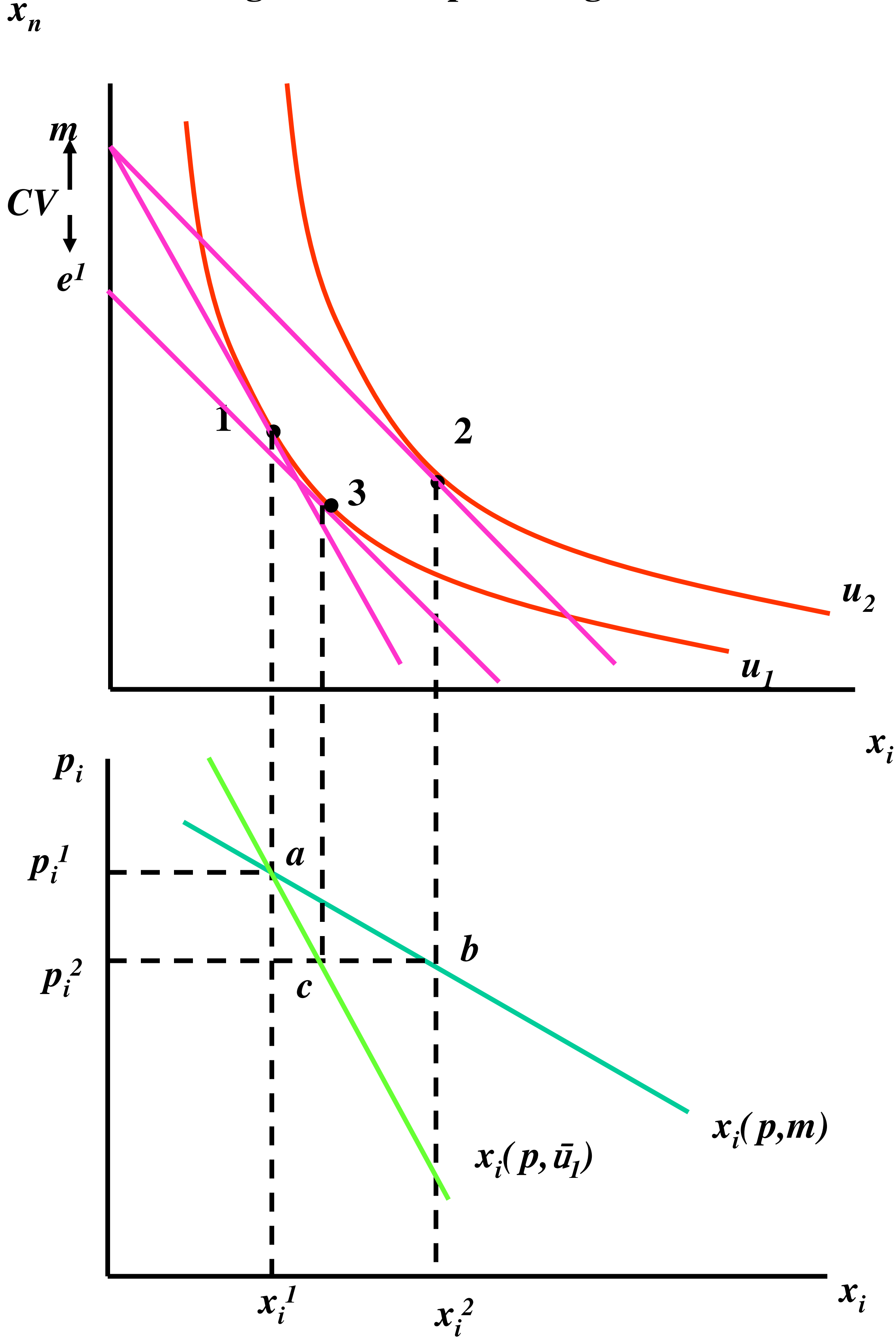
- this is shown in Figure 4, where CV is me^1, which is area to left of compensated demand curve $x_i(p,\hat{u}_1)$, or $p^1{}_i acp^2{}_i$, the Marshallian demand curve $x_i(p,m)$, so that $CV < \Delta W$

- if several prices change, surplus areas can be added up, order of price changes having no effect on result

■ Equivalent Variation

- EV defined as amount of income that must be added to initial income to give household utility level u_2, i.e., vertical distance e^2m^1 in Figure 3

Figure 4: Compensating Variation



- note, *EV* and *CV* related in that absolute value of *CV* for move from 1 to 2 corresponds to absolute value of *EV* of move from 2 to 1

- expenditure function can be used to obtain *EV*, using Figure 3, *EV* is e^2-m^1 or:

$$EV = e(p^1, u_2) - m^1 \tag{19}$$

- given $\Delta m = m^2 - m^1$, *EV* can be re-written:

$$\begin{aligned} EV &= e(p^1, u_2) - m^2 + \Delta m \\ &= e(p^1, u_2) - e(p^2, u_2) + \Delta m \end{aligned} \tag{20}$$

- as *e* is continuous in *p*, (20) can be re-written:

$$EV = \int_{p^2}^{p^1} \sum (\partial e(p, u_2)/\partial p_i) dp_i + \partial m \tag{21}$$

or using Shepard's lemma, and reversing order of integration:

$$EV = -\int_{p^1}^{p^2} \sum x_i(p, u_2) dp_i + \Delta m \tag{22}$$

- manner of expressing *EV* similar to *CV*, and again as (22) is a path independent line integral, it is unambiguously defined

- Figure 5 gives a graphical interpretation for reduction in price of good x_i in isolation, $e^2 - m$ is *EV* in terms of the numeraire, the compensated demand curve is $x_i(p, \hat{u}_2)$ and Marshallian demand curve is $x_i(p, m)$

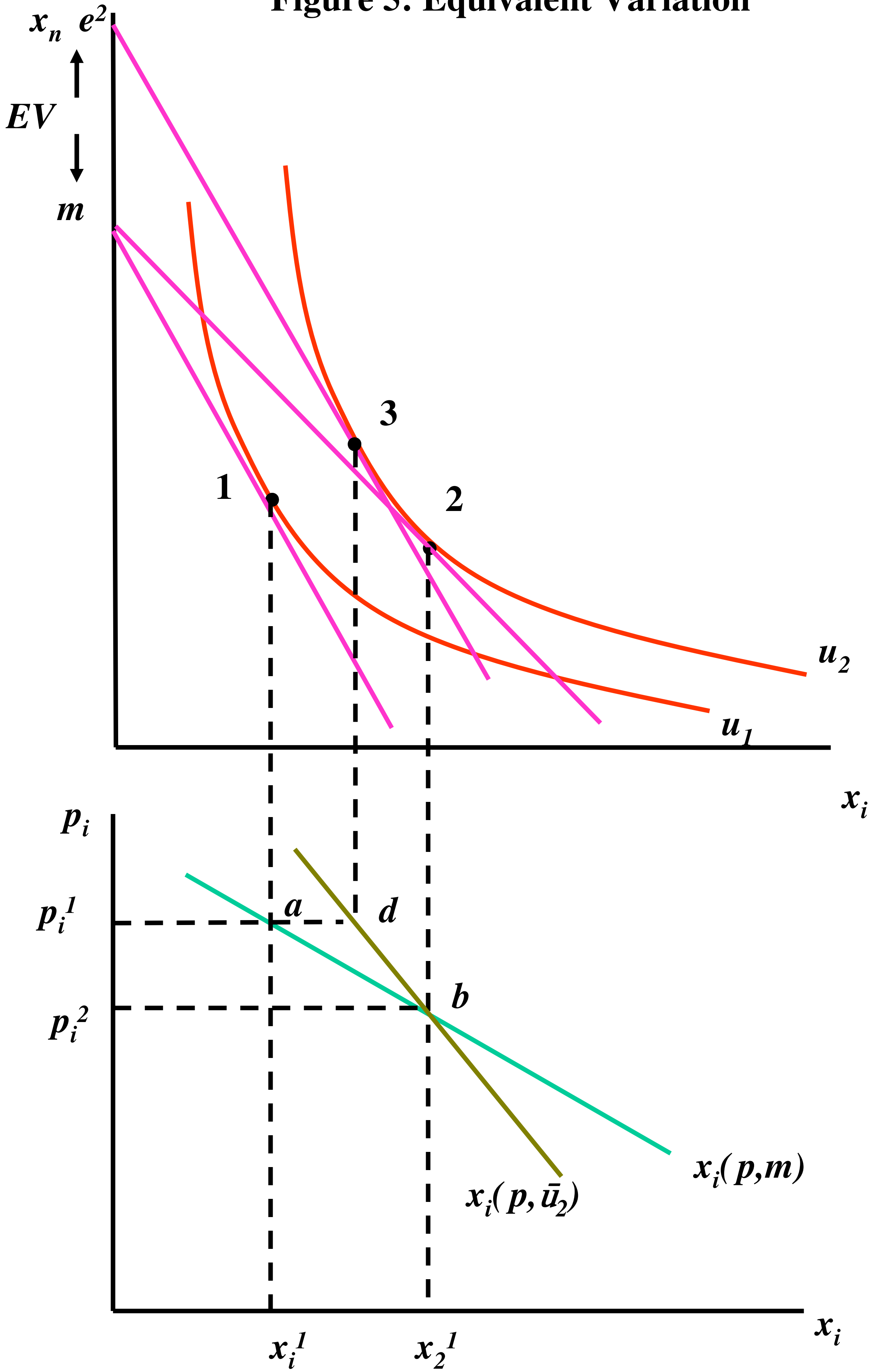
- *EV* is given by:

$$EV = -\int x_i(p, \hat{u}_2)\, dp_i \qquad (23)$$

this is area $p^1_i dbp^2_i$, which exceeds Marshallian consumer surplus $p^1_i abp^2_i$

- for normal goods, $EV > \Delta W > CV$ holds for single price changes, for multiple price changes there is a path dependency problem for ΔW, which means it can be greater or less than both *CV* and *EV*

Figure 5: Equivalent Variation



■ Deadweight Loss of a Commodity Tax

- Consider case of a commodity tax on x_i, and examine *CV* in absence of any other commodity taxes, and, assume, that tax revenue is returned to household as a lump-sum

- all other commodities are aggregated as a composite x_n, which represents all other expenditure in money terms

- in Figure 6, household starts at 1 in absence of tax, and when commodity tax is imposed, relative price of x_i rises, new equilibrium being at 2

- income at 2 has risen to m^2, where $m^2 - m^1 = T$, where T is tax revenue returned as a lump-sum

- *CV* for move 1 to 2 is obtained by applying (14), i.e., $m^2 - e^1 < 0$, algebraically:

$$CV = m^2 - e(p^2, u_1)$$

$$= m^1 + T - e(p^2, u_1) \tag{24}$$

$$= e(p^1, u_1) - e(p^2, u_1) + T$$

$$= \int_{p_i^2}^{p_i^1} (\partial e(p,u_1)/\partial p_i)\, dp_i + T$$

$$= - \int_{p_i^1}^{p_i^2} x_i(p,u_1)\, dp_i + T \qquad (25)$$

- (25) gives exact measure of distance m^2-e^1, lower part of Figure 6 gives another interpretation

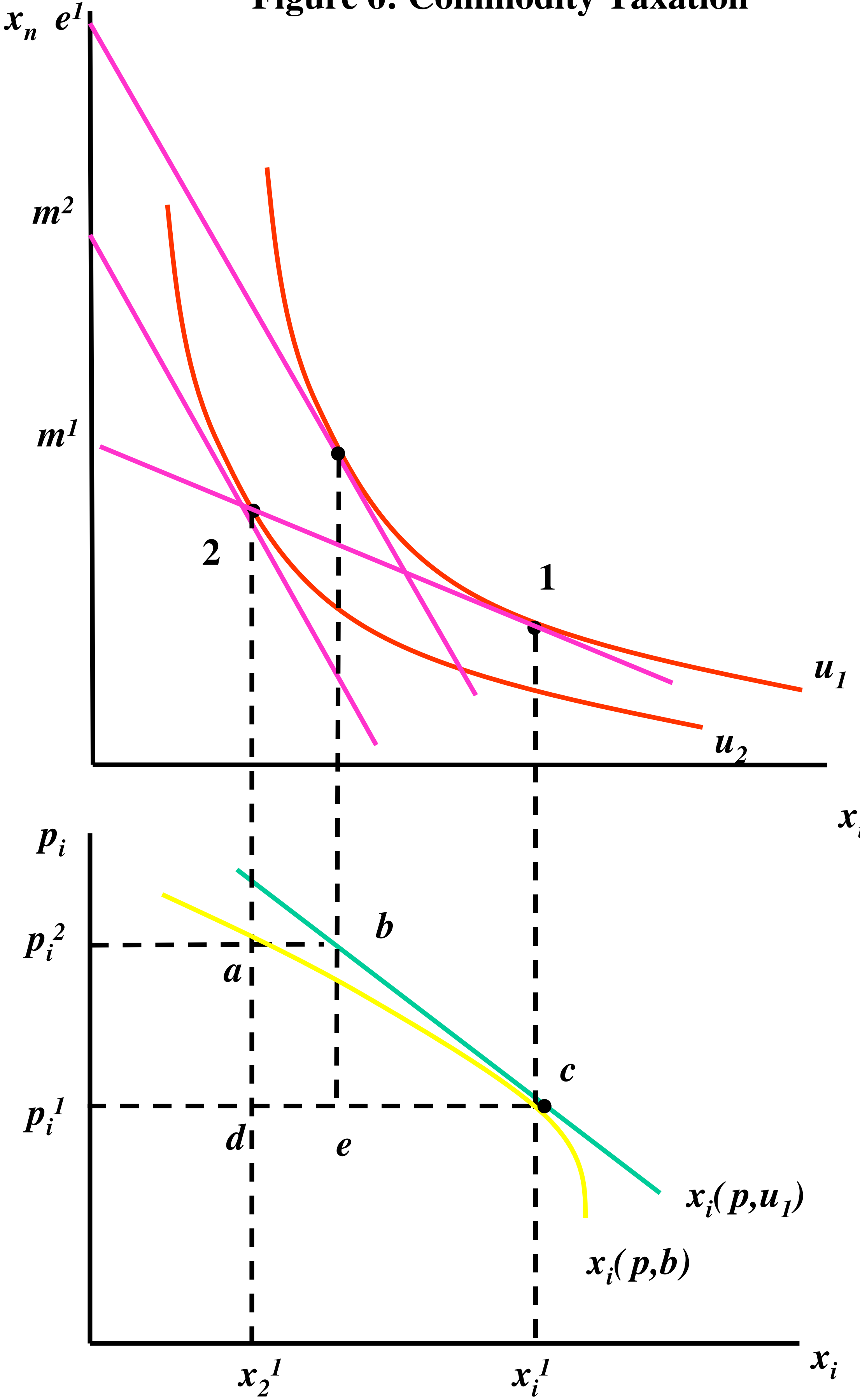
$x_i(p,u_1)$ is the compensated demand curve, and $x_i(p,b)$ is demand curve obtained by finding x_i for various prices, but constraining household to be on original budget line - or by varying commodity tax on x_i and returning revenue to consumer

$x_i(p,b)$ is tangent to compensated demand curve at $x^1{}_i$, and points 1 and 2 are on this demand curve

$\int x_i(p,u_1)\, dp_i$ **is area to left of**

compensated demand curve, and tax revenue is given by $p^2{}_i adp^1{}_i$, so exact value of CV is:

Figure 6: Commodity Taxation



$$CV = -p_i^2 bcp_i^1 + p_i^2 adp_i^1 = -abcd \qquad (26)$$

- *abcd* is the deadweight loss of the commodity tax, often approximated by area *bce*, the "Harberger triangle"

- *bce* is exact measure of deadweight loss if points *a* and *b* coincide, i.e., no income effects, otherwise it is an approximation, *abed* being the error of approximation

■ Approximations of Welfare Change

- information required to measure *CV* and *EV* is very demanding

- suppose only data available are prices and quantities consumed in situations 1 and 2, can it be determined a household is better off, and by how much?

- common to construct quantity indices such as the *Laspeyres* and *Paasche* indices, Q_L and P_L, defined as:

$$Q_L = \frac{\sum p_i^1 x_i^2}{\sum p_i^1 x_i^1} \quad and \quad Q_P = \frac{\sum p_i^2 x_i^2}{\sum p_i^2 x_i^1} \qquad (27)$$

- both indices are the weighted ratios of quantities consumed in the two periods, where for Laspeyres, the weights are initial prices, and for Paasche the weights are new prices

- in Figure 7, indices illustrated for two commodity case, prices being measured in terms of composite commodity x_n

Q_L is given by ratio ℓ/m^1, which compares to the ratio implied by the *EV* of e^2/m^1, so Laspeyres index is an overestimate of the true quantity index

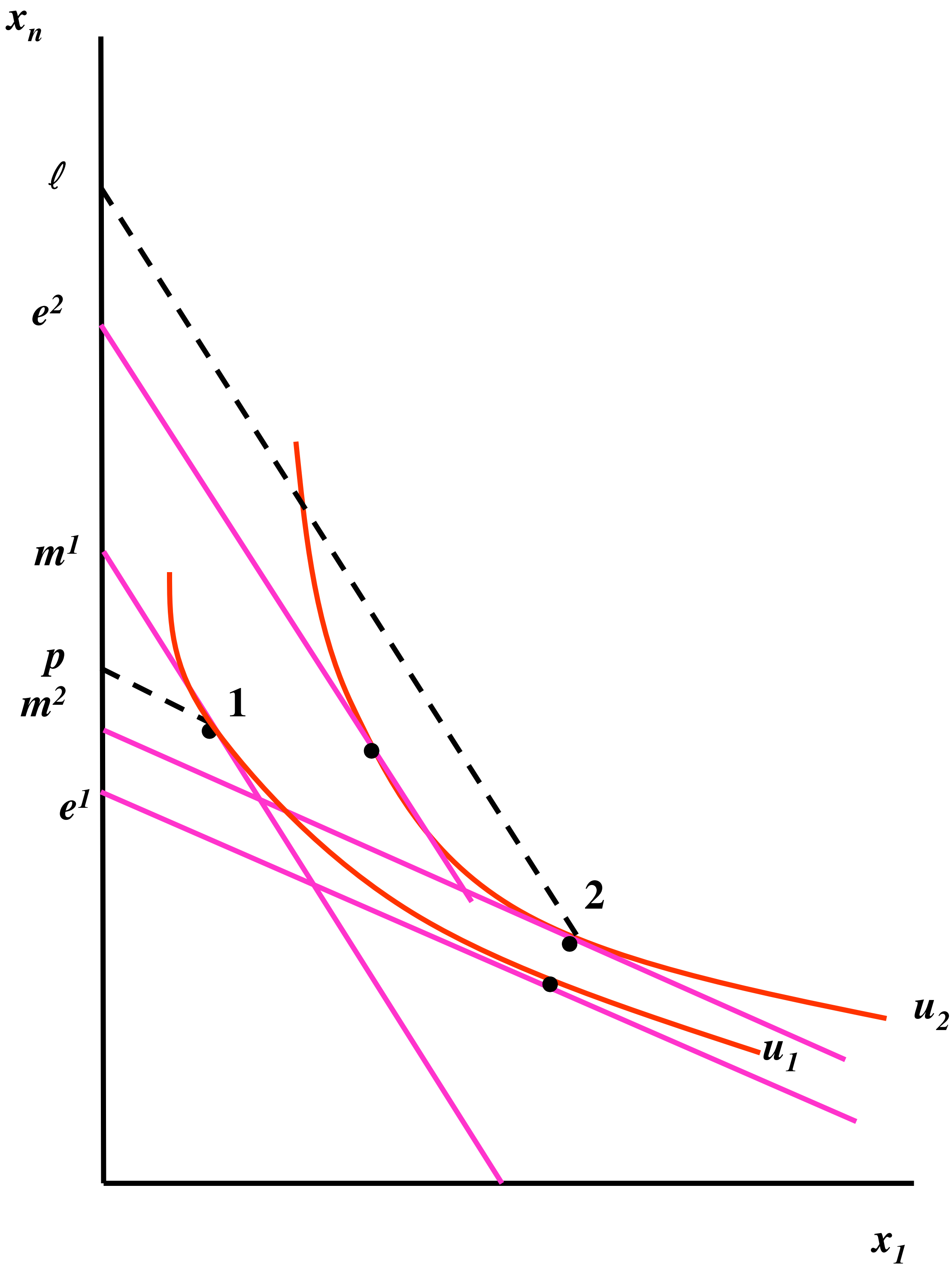
Q_P is the ratio m^2 / p, which compares to true quantity index implied by *CV* of m^2 / e^1, so Paasche index is an underestimate

- indices can also be written in level form:

$$\ell - m^1 = \sum p_i^1 x_i^2 - \sum p_i^1 x_i^1 = \sum p_i^1 \Delta x_i$$

$$m^2 - p = \sum p_i^2 \Delta x_i$$

Figure 7



- **indices are *first-order* approximations of *EV* and *CV*; taking a Taylor series expansion of the expenditure function around final situation:**

$$e(p^1,u_2) = e(p^2,u_2) - \sum x_i(p^2,u_2)\partial p_i + R,$$

$$\partial e/\partial p_i(p^2,u_2) = x_i(p^2,u_2), \partial p_i = p_i^2 - p_i^1 \qquad (28)$$

***R* is the sum of terms higher than first order**

**** see the appendix***

- **Taylor series can be substituted into definition of *EV* in (20):**

$$EV = -\sum x_i(p^2,u_2)\Delta p_i + \Delta m + R \qquad (29)$$

- **adding and subtracting $\Sigma p^1_i x^2_i$ to Δm:**

$$\Delta m = \sum x_i^2(p_i^2 - p_i^1) + \sum p_i^1(x_i^2 - x_i^1)$$

$$= \sum x_i^2 \Delta p_i + \sum p_i^1 \Delta x_i$$

- **substituting into (29):**

$$EV = \sum p_i^1 \Delta x_i + R \qquad (30)$$

ignoring *R* shows the Laspeyres index is a first-order approximation of *EV*

- in order to evaluate higher order terms in *R*, more information other than initial and final prices and quantities

- for example, we can write *CV* as:

$$CV = e(p^1,u_1) - e(p^2,u_1) + \Delta m$$

a Taylor series expansion can be applied around initial value of expenditure function:

$$e(p^2,u_1) = e(p^1,u_1) + \sum_i \frac{\partial e(p^1,u_1)}{\partial p_i}\partial p_i$$
$$+ \frac{1}{2}\sum_i \sum_j \frac{\partial^2 e(p^1,u_1)}{\partial p_i \partial p_j}\partial p_i \partial p_j + R \qquad (31)$$

where *R* are terms higher than second order

- from Hotelling's lemma:

$$\partial e(p,u_1)/\partial p_i = x_i(p,u_1),$$

$$and \ \partial^2 e(p,u_1)/\partial p_i \partial p_j = \partial x_i(p,u_1)/\partial p_j$$

latter term being the compensated price derivative or substitution effect, S_{ij}, so Taylor expansion is:

$$e(p^1,u_1) - e(p^2,u_1)$$

$$= -\sum x_i(p^1,u_1)\Delta p_i - \frac{1}{2}\sum\sum S_{ij}\Delta p_i \Delta p_j - R \quad (32)$$

where S_{ij} is evaluated at u_1 and p^1

- by using higher order terms, a closer approximation to *CV* is obtained, and usually the term *R* is ignored:

$$CV = -\sum x_i^1 \Delta p_i - \frac{1}{2}\sum\sum S_{ij}\Delta p_i \Delta p_j + \Delta m \quad (33)$$

** see the appendix*

- information is needed on the substitution effects, which can be determined from the Slutsky equation derived from an empirical estimate of the demand function

- Example: (33) can be applied to the deadweight cost of taxation problem for a single commodity tax:

$$CV = -x_i^1 t_i - \frac{1}{2} S_{ii} t_i^2 + T \qquad (34)$$

t_i is the tax per unit of commodity *i* consumed

- as $x^1_i t_i \approx T$, measure reduces to $1/2 S_{ii} t^2 i$, which is an approximate measure of the Harberger triangle

- an alternative way of writing this is in terms of elasticities; define compensated elasticity of demand ε_{ii} as $S_{ii}\, p_i \,/\, x_i$, then the Harberger triangle becomes:

$$-\varepsilon_{ii} t_i^2 x_i / (2 p_i) \approx \varepsilon_{ii} \tau_i T/2 \qquad (35)$$

**** see the appendix***

where $\tau_i = t_i \,/\, p_i$ is the *ad valorem* rate of tax

■ Use of Marshallian Consumer Surplus

- if income effects are zero, *CV*, *EV* and Marshallian (*S*) consumer surplus are identical

- *S* has some appeal as an approximate measure of welfare change:

(i) it is directly observable from estimation of demand functions

(ii) estimation is inexact, so errors of measurement may outweigh any theoretical differences between *CV*, *EV*, and *S*

(iii) for a single price change, *S* has as much claim to legitimacy as an exact measure of welfare change as do *CV* and *EV*

- Willig (1976) has argued that in most cases that *S* will be a close approximation for *CV* and *EV*, and well within errors of empirical measurement

- Willig establishes following result for a single price change; suppose η is income elasticity of demand, provided $\eta S / 2m < 0.05$, and $|S| / m < 0.9$, following inequalities hold:

$$\frac{S - CV}{|S|} < \frac{\eta |S|}{2m} < 0.05$$
$$\frac{EV - S}{|S|} < \frac{\eta |S|}{2m} < 0.05 \qquad (36)$$

- error involved in using Marshallian consumer surplus is less than 5 percent under stated conditions (see Willig for derivation)

- in Figure 8, compensated and Marshallian demand curves drawn for fall in price x_i, where difference between *S* and *CV* is area adc, where *dc* is income effect associated with *CV* at new price p^2_i

- assuming demand is approximately linear, *adc* = 1/2*dc*(p^1_i - p^2_i), and as *dc* is solely from income effect, approximated as:

$$dc \approx \eta S x_i / m \qquad (37)$$

so difference between *S* and *CV* is:

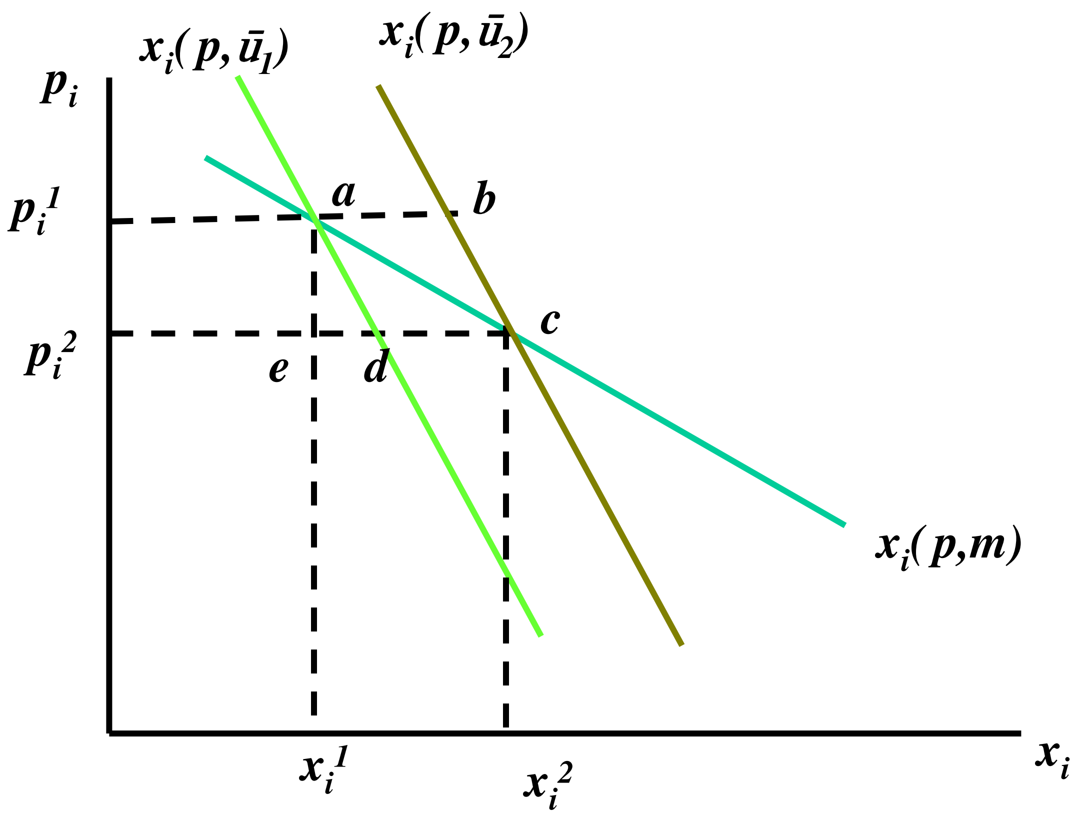
$$S - CV \approx \frac{1}{2}\frac{\eta S}{m} x_i (p_i^1 - p_i^2) \qquad (38)$$

- since $x_i(p^1_i - p^2_i)$ is approximately equal to *S*:

$$S - CV \approx \frac{1}{2}\frac{\eta S^2}{m} \qquad (39)$$

which corresponds to (36)

Figure 8



- approximation in (36) only valid if $|S| / m < 0.9$, i.e. variations in expenditure on good cannot be too large relative to consumer income

- approximation may not be useful if aim is to measure Harberger triangle due to tax changes, i.e., *ade*, or *abce*, if income effects are not zero, Hausman (1981)

- Lavergne, Requillart and Simioni (2001) use method developed by Breslaw and Smith (1995) to approximate *EV* from knowledge of Marshallian demand function

- apply model to calculate deadweight losses of market power in French food industry using 1987 data

- establish Hicksian deadweight loss is lower (greater) than Marshallian deadweight loss when income elasticity of demand is positive (negative)

- suggests it is advisable to use Hicksian measure to evaluate welfare loss due to market power

■ Welfare Change in Single-Consumer Economies

- conventional to proceed as if households can be aggregated into a single representative household

- implies certain assumptions:

(i) aggregate demand has same properties as individual demand functions - i.e., represent an aggregate preference ordering, so that aggregate welfare measures can be constructed

(ii) aggregate welfare change must have normative significance, i.e., if aggregate *CV* rises, society is better off

- well-known that, in aggregate, demand cannot be written as a unique function of prices and income, as income distribution influences aggregate demand (see the appendix*)**

- Samuelson (1956) pointed out two circumstances when this can be avoided:

(a) If government ensures income is distributed optimally using lump-sum taxes, i.e., marginal social utility of income is identical - implies a unique aggregate demand function

Government has to continually redistribute income optimally - if not, aggregate demand, and estimates of welfare will not correspond to social indifference curves

(b) A sufficient condition (Deaton and Muellbauer, 1980) is that all households have identical and homothetic preferences - preference map is independent of distribution of income

(** see the appendix*)

■ Welfare Cost of Commodity Taxation

- consider imposition of a commodity tax in an otherwise undistorted economy, if production possibilities are linear, analysis of Figure 6 applies with minor reinterpretation

initial budget constraint is now production possibility curve in two-good case, slope of which is marginal rate of transformation - in this case relative supply prices are fixed for all output combinations

- in lower part of Figure 6, demand curve is that obtained by facing household with different relative prices in consumption, but constraining them to be on production frontier

- constructed by taking slope of indifference curve at all points along production frontier - this is known as the *general equilibrium demand curve*, or the *Bailey demand curve*

- in this case, *CV* measures are same as for single-consumer case, with tax revenue returned to households in lump-sum form

- general equilibrium considerations only matter once relative supply prices change, as in Figure 8, where it is assumed that factor supplies are fixed, so utility changes arise only from consumption effects

- imposition of tax on x_i causes economy to move from equilibrium 1 to 2, with utility levels u_1 and u_2:

> **(i) at 1, value of output is m^1 in terms of numeraire,**

(ii) at 2, value of output measured at factor cost is Y^2 and consumer income is m^2, where m^2 - Y^2 is tax revenue $(q^2_i - p^2_i)x^2_i$

- CV associated with tax is $m^2 - e^1$

- in lower part of Figure 8, $x_i(q_i,u_1)$ is the compensated demand curve, and $x_i(q_i,P)$ is general equilibrium demand curve

- $S_i(p_i)$ is general equilibrium supply curve for various supply prices p_i

- geometric interpretation of CV given by:

$$CV = m^2 - e(q^2,u_1)$$

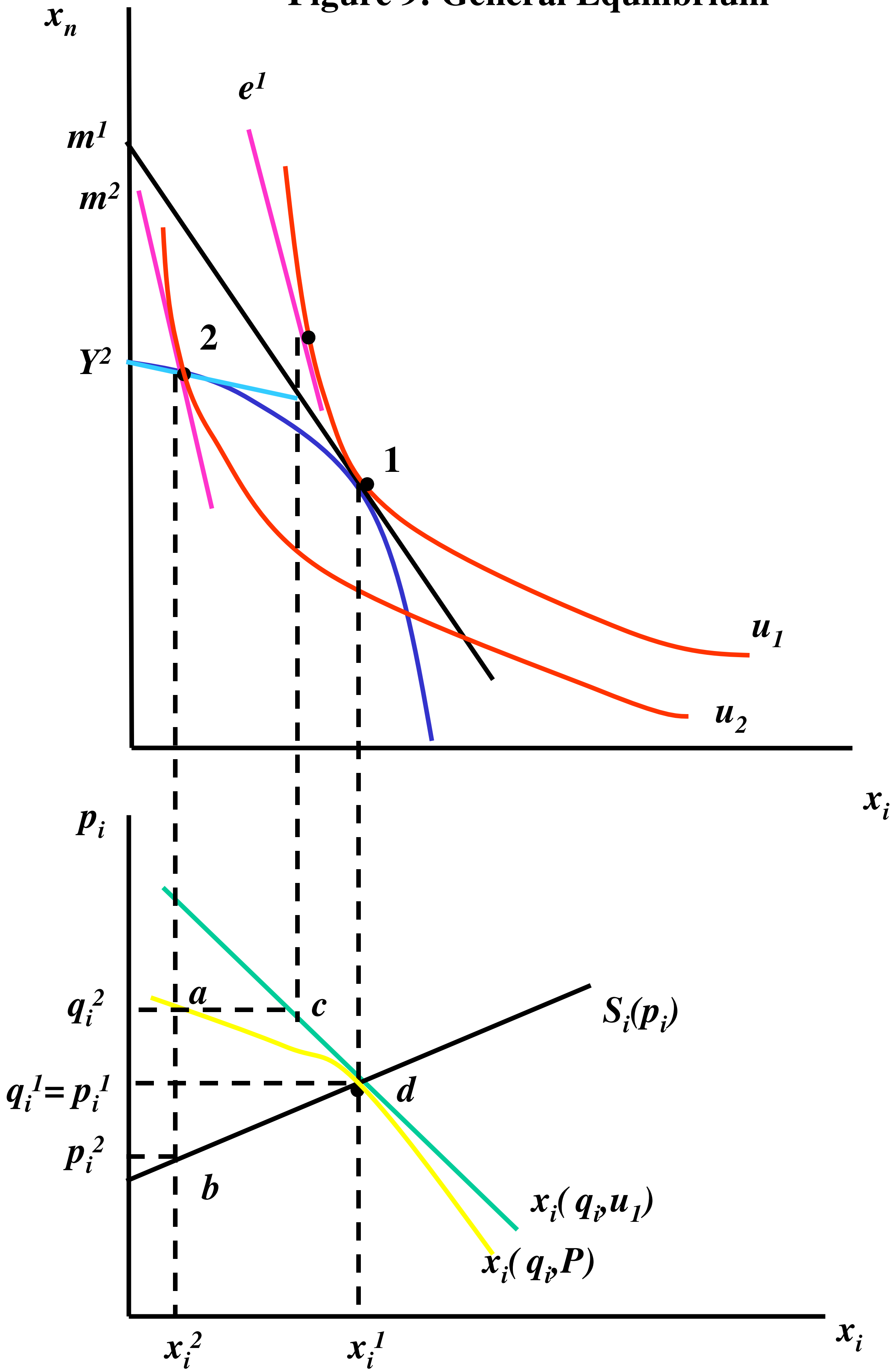
$$= m^1 - (m^1 - Y^2) + (m^2 - Y^2) - e(q^2,u_1) \qquad (1)$$

$$= e(q^1,u_1) - e(q^2,u_1) - (m^1 - Y^2) + (m^2 - Y^2)$$

where $m_1 = e(q^1,u_1)$, and $m^2 - Y^2 = T$, and $m^1 - Y^2$ is equal to:

$$\int_{p_i^2}^{p_i^1} S_i(p_i)\,dp_i \qquad (2)$$

Figure 9: General Equilibrium



CV **becomes:**

$$CV = -\int_{q_i^1}^{q_i^2} x_i(q,u_1)\,dq_i + \int_{p_i^1}^{p_i^2} S_i(p_i)\,dp_i + T \tag{3}$$

$$= -\, q_i^2 c d q_i^1 - p_i^1 d b p_i^2 + q_i^2 a b p_i^2$$

$$= -\, acdb$$

- if income effects are negligible, *c* coincides with *a*, and deadweight loss is *adb*

- otherwise this is only an approximation, i.e., general equilibrium demand curve contains income compensation due to return of tax revenue, but there are still income effects

- in empirical work, general equilibrium demand curve may be no easier to observe than compensated demand curve, so Marshallian demand curve is best approximation to *CV*