■ Commodity Taxation

Basic problem in commodity taxation: if a social welfare function is assumed, what is the choice of commodity tax rates that will maximize social welfare subject to a revenue constraint?

Ramsey (1927) developed first solution to problem - result overlooked until rediscovered by Samuelson in 1951; modern theory developed by Diamond and Mirlees (1971)

■ The Ramsey Rule

(i) Basic Assumptions:

- competitive economy with *n* consumption goods, and a single form of labor which is only input

- there is a single household, whose preferences are represented as an indirect utility function (there could be a population of identical households)

- given constant returns to scale, competition ensures that pre-tax price of good p_i:

$$p_i = c^i w, \qquad i = 1,\ldots,n \tag{1}$$

c^i is a technical coefficient describing amount of labor required to produce one unit of good i, and w is wage rate

- labor, which is untaxed, is chosen as the *numeraire*, and w is fixed at a constant value, implying a set of fixed pre-tax prices for consumption goods

- consumer prices q_i, are pre-tax prices plus taxes:

$$q_i = p_i + t_i, \qquad i = 1,\ldots,n \tag{2}$$

- the revenue constraint is:

$$R = \sum_{i=1}^{n} t_i x_i \tag{3}$$

where x_i is consumption of good i; to ensure markets clear, R is used to purchase labor which then produces nothing that is traded

- preferences of the single household given by:

$$U = V(q_1,...,q_n,w,I) \qquad (4)$$

(for the prices not to be distorted by the taxes, $\sum t_{ii} x_{ii}$ worth of labor must be wasted)

household consumes goods, and supplies labor used in production and by the government; because of the competitive assumption and constant returns, there is no profit income, and, so lump-sum income *I* is zero

(ii) Derivation of Ramsey Rule

- optimal tax problem is given by maximization problem:

$$\max_{\{t_1,...,t_n\}} V(q_1,...q_n,w,I) \ s.t. \ R = \sum_{i=1}^{n} t_i x_i \qquad (5)$$

The Lagrangean is:

$$\mathfrak{L} = V(q_1,...,q_n,w,I) + \lambda[\sum_{i=1}^{n} t_i x_i - R] \qquad (6)$$

- the first-order condition for choice of tax rate on good *k* is:

$$\frac{\partial \mathcal{L}}{\partial t_k} \equiv \frac{\partial V}{\partial t_k} + \lambda [x_k + \sum_{i=1}^{n} t_i \frac{\partial x_i}{\partial q_k}] = 0$$

$$\text{where } \frac{\partial V}{\partial q_k} \equiv \frac{\partial V}{\partial t_k}, \quad \frac{\partial x_i}{\partial q_k} \equiv \frac{\partial x_i}{\partial t_k} \tag{7}$$

Re-arranging (7):

$$\frac{\partial V}{\partial t_k} = -\lambda [x_k + \sum_{i=1}^{n} t_i \frac{\partial x_i}{\partial q_k}] \tag{8}$$

Such a condition has to hold for all *n* goods, i.e., utility cost of raising tax revenue should be in same proportion to marginal revenue raised by the tax

Using Roy's identity:

$$\frac{\partial V}{\partial q_k} = -\frac{\partial V}{\partial I} x_k = -\alpha x_k \tag{9}$$

***I* is lump sum income, and *α* is marginal utility of income, evaluated at *I*=0**

- substituting (9) into (8):

$$\alpha x_k = \lambda[x_k + \sum_{i=1}^{n} t_i \frac{\partial x_i}{\partial q_k}] \qquad (10)$$

- defining the Slutsky equation:

$$\frac{\partial x_i}{\partial q_k} = S_{ik} - x_k \frac{\partial x_i}{\partial I} \qquad (11)$$

where $S_{ik} = \delta\chi_i / \delta q_k$, the compensated demand for good i, and the change in uncompensated demand with respect to income I is evaluated at I=0

- using (11), and re-arranging (10):

$$\sum_{i=1}^{n} t_i [S_{ik} - x_k \frac{\partial x_i}{\partial I}] = -[\frac{\lambda - \alpha}{\lambda}] x_k$$

$$or \sum_{i=1}^{n} t_i S_{ik} = -[\frac{\lambda - \alpha}{\lambda}] x_k + \sum_{i=1}^{n} t_i x_k \frac{\partial x_i}{\partial I} \qquad (12)$$

- factoring out x_k:

$$\sum_{i=1}^{n} t_i S_{ik} = -[1 - \frac{\alpha}{\lambda} - \sum_{i=1}^{n} t_i \frac{\partial x_i}{\partial I}] x_k \qquad (13)$$

- symmetry of the Slutsky substitution matrix implies $S_{ik} = S_{ki}$, so re-writing:

$$\sum_{i=1}^{n} t_i S_{ki} = -\theta x_k, \tag{14}$$

$$where\ \theta = [1 - \frac{\alpha}{\lambda} - \sum_{i=1}^{n} t_i \frac{\partial x_i}{\partial I}]$$

- (14) is the Ramsey rule for a system of optimal commodity taxes, (14) having to hold for all goods, $k = 1,\dots,n$

- multiplying both sides of (14) by t_k, and summing over k:

$$\sum_{k=1}^{n} \sum_{i=1}^{n} t_i t_k S_{ki} = -\theta R \tag{15}$$

- as the Slutsky matrix is negative semi-definite, left-hand side of (15) is negative, so θ has the same sign as R

- given definition of S_{ki}, $\sum_{i=1}^{n} t_i S_{ki}$

is an approximation of total change in compensated demand for good k due introduction of tax system from position of initially no taxes

The Ramsey rule can then be written:

$$d_k = \frac{\sum_{i=1}^{n} t_i S_{ki}}{x_k} = -\theta, \qquad k = 1,\ldots,n \tag{16}$$

- optimal tax system should be such that compensated demand for each good is reduced in same proportion relative to pre-tax position

d_k is Mirlees' (1976) index of *discouragement*, which, by Ramsey rule is the same for all goods

(iii) Implications

- Ramsey rule implies that as proportional reduction in compensated demand should be same for all goods, then goods that are price inelastic will be more highly taxed

- this would imply high taxes for goods such as food and housing, implying low-income households would pay a larger proportion of their incomes in taxes

- result follows from the single-household assumption, i.e., maximization is only concerned with efficiency, not equity

- as lump-sum taxes are ruled out, commodity taxes result in substitution effects that distort optimal choices, however, Ramsey rule ensures losses are minimized

■ **Inverse Elasticities Rule**

(i) This rule was the one that dominated the textbooks for many years until the "rediscovery" of Ramsey's rule

- essentially, this rule assumes no cross-price effects between taxed goods, so taking (8), and using Roy's identity:

$$\alpha x_k = \lambda [x_k + \sum_{i=1}^{n} t_i \frac{\partial x_i}{\partial q_k}] \qquad (17)$$

- no cross-price effects implies $\delta x_i/\delta q_k = 0$, $i \neq k$, so (17) becomes:

$$\alpha x_k = \lambda[x_k + t_k \frac{\partial x_k}{\partial q_k}] \qquad (18)$$

- re-arranging (18), and dividing by $q_k = p_k + t_k$:

$$\frac{t_k}{p_k + t_k} = [\frac{\alpha - \lambda}{\lambda}][\frac{x_k}{q_k}\frac{\partial q_k}{\partial x_k}] \qquad (19)$$

- this can be re-written as:

$$\frac{t_k}{p_k + t_k} = [\frac{\alpha - \lambda}{\lambda}]\frac{1}{\varepsilon_k^d} \qquad (20)$$

(ii) This rule implies proportional rates of tax should be inversely related to the price elasticity of demand of good ε^d_k - this is an extreme interpretation of the Ramsey rule, necessities being highly taxed

- **Many Households**

Extending single-household economy to many, non-identical households, introduces equity

(i) Follow a variant of the Diamond-Mirlees (1971) economy which has the same production technology as before

- economy consists of *H* households, each having an indirect utility function:

$$U^h = V^h(q_1,\ldots,q_n,w,I^h) \qquad (21)$$

U^h vary among households, otherwise reduces to single-household problem

- revenue constraint becomes:

$$R = \sum_{i=1}^{n} \sum_{h=1}^{H} t_i x_i^h \qquad (22)$$

- social welfare function is a Bergson *swf*, defined over the indirect utilities:

$$W = W(V^1(.),\ldots,V^H(.)) \qquad (23)$$

- the problem is now to find set of commodity taxes that maximizes the *swf* subject to (22)

- first-order condition for an optimal tax on good *k* is:

$$\sum_{h=1}^{H} \frac{\partial W}{\partial V^h}\frac{\partial V^h}{\partial t_k} + \lambda\left[\sum_{h=1}^{H} x_k^h + \sum_{i=1}^{n}\sum_{h=1}^{H} t_i \frac{\partial x_i^h}{\partial q_k}\right] = 0 \quad (24)$$

- using Roy's identity, (24) can be re-written as:

$$\sum_{h=1}^{H} \beta^h x_k^h = \lambda\left[\sum_{h=1}^{H} x_k^h + \sum_{i=1}^{n}\sum_{h=1}^{H} t_i \frac{\partial x_i^h}{\partial q_k}\right] \qquad (25)$$

where $\beta^h = \delta W/\delta V^h . \alpha^h$, made up of the effect of a change in *h*'s utility on social welfare and *h*'s marginal utility of income α^h - Diamond and Mirlees denote this the *social marginal utiltiy of income* for *h*

- using Slutsky equation again, and re-arranging (25):

$$\frac{\sum_{i=1}^{n}\sum_{h=1}^{H} t_i S_{ki}^h}{\sum_{h=1}^{H} x_k^h} = \frac{1}{\lambda}\frac{\sum_{h=1}^{H} \beta^h x_k^h}{\sum_{h=1}^{H} x_k^h} - 1 + \frac{\sum_{h=1}^{H}\left[\sum_{i=1}^{n} t_i \frac{\partial x_i^h}{\partial I^h}\right] x_k^h}{\sum_{h=1}^{H} x_k^h} \qquad (26)$$

**** see the appendix***

- left-hand side of (25) is approximately the proportional change in aggregated compensated demand for good *k* (generalizes left-hand side of (16))

- right-hand terms show reduction in demand for *k* should be smaller when:

(a) demand for *k* is concentrated among households with high β^h - this reflects a concern for equity, i.e.,reduces rates of tax levied on necessities that low income households consume

(b) demand is concentrated among those whose tax payments change a lot as income changes - reflects an efficiency concern, i.e., if *not*, taxes would have to be higher to meet revenue constraint, increasing distortions

(26) can be re-written in a manner closer to the Ramsey rule:

$$\sum_{i=1}^{n}\sum_{h=1}^{H} t_i S_{ki}^{h} = -[H\bar{x}_k - \frac{\sum_{h=1}^{H}\beta^h x_k^h}{\lambda} - \sum_{i=1}^{n} t_i[\sum_{h=1}^{H}\frac{\partial x_i^h}{\partial I^h} x_k^h]] \quad (27)$$

where: $$\bar{x}_k = \frac{\sum_{h=1}^{H} x_k^h}{H}$$

is the mean level of consumption of *k* across households

- define: $$b^h = \frac{\beta^h}{\lambda} + \sum_{i=1}^{n} t_i \frac{\partial x_i^h}{\partial I^h}$$

- b^h is net social marginal utility of income - net as it measures both gain in social welfare β^h from an increase in *h*'s income and increase in tax payments by *h* from an increase in income, i.e., involves both efficiency and equity effects

- (27) can be re-arranged to give:

$$\frac{\sum_{i=1}^{n}\sum_{h=1}^{H} t_i S_{ki}^{h}}{\sum_{h=1}^{H} x_k^{h}} = -[1 - \sum_{h=1}^{H} \frac{b^h}{H}\frac{x_k^h}{\bar{x}_k}] \qquad (28)$$

* *see the appendix*

- reduction in aggregate compensated demand for k^{th} commodity due to commodity taxes should be inversely related to correlation between b^h and $x^h{}_k$, i.e., given b^h reflects equity concerns, goods with high b^hs should be taxed less

■ Reduction to Ramsey Rule

- Tax rule in (28) will collapse to the Ramsey rule when all households are given same social valuation, or because tax system cannot discriminate between households

- Suppose $b^h = b$ all h=1,...,H, right-hand side of (28) becomes -[1-b], and tax rule describes an efficient tax system only, i.e. proportional reduction in demand will be the same for all goods

- occurs if all households are valued equally and

- $\frac{x_k^h}{\bar{x}_k}$ is same for all k

implies no good is consumed disproportionately by rich and poor households, i.e., identical Engel curves - no way of subsidizing households with high b^hs without subsidizing those with low b^hs

- commodity taxation problem is one of *signal extraction*, i.e., state is attempting to infer household preferences and endowment from purchases in order to tax each household according to their circumstances

- if no signal can be extracted, system focuses on efficiency only

■ **Production Efficiency**

- from conditions for Pareto efficiency, know production efficiency occurs when economy is on its production frontier, and there is equality of firms' marginal rates of technical substitution

- Diamond and Mirlees (1971) proved the *production efficiency lemma*: if economy is competitive, equilibrium with optimal commodity taxation is on the frontier of the production set

- implies intermediate goods should not be taxed, which contrasts with the predictions of Lipsey and Lancaster (1956) which suggest distortions induced by commodity taxes should be offset by a similar distortion in input prices

- focus on a single household, 2 good economy, where one good is a consumption good, the other an input such as labor (see Figure 1)

- vertical axis depicts output of consumption good, horizontal axis input use

- given constant returns to scale, shaded area is economy's production set, where frontier is moved left by amount *a* to reflect the tax revenue required in units of the input

- household supplies input, and consumes output, so its budget constraint is upward-sloping, and, in absence of lump-sum taxes, it goes through origin

- budget constraint corresponds to optimal post-tax set of prices *q*

- as supplying labor gives disutility, indifference curves are downward-sloping

- output/input combinations household is willing to trade to at these prices is given by the price consumption locus or *offer curve*

- optimal tax equilibrium is highest point on the offer curve that is in production set, at point *e*, on indifference curve I_0

- I_1 is both preferred and feasible, but can only be achieved by the use of a lump-sum tax, which is ruled out by assumption

- why is the optimum on frontier?

> **(a) if at *f*, household utility can be increased by reducing use of labor, keeping output constant, which is feasible**
>
> **(b) if at *g*, output can be increased without using more labor, which is feasible**

- suppose labor is used to produce an intermediate good, and this is then an input into producing consumption good

- there is a direct link between units of labor and intermediate good, allowing preferences and budget constraint to be defined directly on labor and consumption good

- production efficiency implies intermediates should not be taxed, as it would violate Pareto efficiency in production, firms' marginal rates of technical substitution not being equal

■ **Some Numerical Results**

- tax rules as defined do not have precise implications, and, hence, no practical policy applications

- limited amount of research done on developing methodology for applying the rules

- Ray (1986), Murty and Ray (1987) and Srinivasan (1989) calculated optimal commodity taxes for many-household economy, using Indian data - 80% tax revenue raised by indirect taxation

- to calculate tax rates, first have to specify a *swf*, procedure has been to use an additive social welfare function, and then define a social utility of income function for each household:

- denote aggregate expenditure of *h* as μ^h, which is equal to income, social utility of income being:

$$U^h = \frac{K[\mu^h]^{1-\nu}}{1-\nu} \qquad \nu \neq 1 \qquad (29)$$

$$or \qquad K\log\mu^h \qquad \nu = 1$$

where ν = constant relative inequality aversion

- with additive social welfare function, $\delta W/\delta V^h = 1$, so β^h, marginal social utility of income is:

$$\beta^h = \frac{\partial U^h}{\partial \mu^h} = K[\mu^h]^{-\nu} \qquad (30)$$

- households in sample ranked by expenditure, lowest expenditure (income) being first in ranking

- setting $\beta^1 = 1$ for lowest expenditure household:

$$\beta^h = \left[\frac{\mu^1}{\mu^h}\right]^\nu \qquad (31)$$

** see the appendix*

- (31) implies that as μ^h increases relative to μ^1, β^h declines at rate v - as higher values of v reduce β^h for h>1, v reflects concern for equity

- method fixes β^hs exogenously, and can be determined by observed expenditure levels in data set, and concern for equity can be varied parametrically

- given β^hs, optimal taxes are determined from (22) and (25); requires specification of demand system and estimation of system from data set

- estimated demand functions are then substituted into (22) and (25), allowing solution for n taxes

- Murty and Ray (1987) estimated a demand system based on the indirect utility function:

$$V^h(.) = \frac{\mu_h^{\alpha} - \sum_{i=1}^{n} \gamma_i^{\alpha_1} w_h^{\alpha_2}}{\prod_{k=1}^{n} p_k^{\alpha\beta_k} w_h^{\alpha\beta_o}} \qquad (32)$$

LES system, where γ = subsistence requirements, numerator is discretionary allocation, and denominator is weighted geometric mean of prices

- (32) permits evaluation of separability assumptions, i.e., if α_2 and β are zero, there is separability between goods and leisure in utility, and non-separability between goods if $\alpha_1 \neq 1$

- tax rates calculated on basis of estimated values of γ_i, β_i, and α_1 given in Ray (1986), β_0 was set at zero, and a value of α_2 assumed

- defining θ as the wage as a proportion of expenditure, which is imposed on the analysis, results from Murty and Ray are shown in the table for v=2 and $\alpha_2 = 0.025$

Table 1: Optimal Tax Rates

Item	$\theta = 0.05$	$\theta = 0.1$
Cereals	-0.015	-0.089
Milk and milk products	-0.042	-0.011
Edible oils	0.359	0.342
Meat, fish, eggs	0.071	0.083
Sugar and tea	0.013	0.003
Other food	0.226	0.231
Clothing	0.038	0.082
Fuel and light	0.038	0.014
Other non-food	0.083	0.126

- redistribution occurs through products that are subsidized

- Atkinson and Stiglitz (1972) calculated optimal tax rates satisfying the Ramsey rule in a single household economy; using two types of demand system, results showed food and rent bear highest tax rates, while durable goods bear the lowest