

APPLIED WELFARE ECONOMICS AND POLICY ANALYSIS

■ Possibility of a *swf*

How is a *swf* formulated? Samuelson regards this as besides the point - it is the domain of political scientists

In reality, economists are never handed a *swf* - often involved in articulating social preferences

■ Arrow's Impossibility Theorem

Arrow concerned with whether a complete, transitive, and reflexive social preference ordering exists

Analysis relates to society which is individualistic, where individual preferences are aggregated into a set of social choices

e.g., alternative social states are voted on, there is a rule that determines social choice from specific voting pattern, where individual preferences determine the voting pattern

Arrow describes the *process* by which individual preferences are transformed into social ordering as a social welfare function, which can be confused with Bergson's *swf*, which is *outcome* of process

Following Sen (1970), Arrow's rule is denoted as *SWF* which associates with each set of individual preferences a social ordering

Requirement of a *SWF* is that it produce a social ordering from those of individuals - *paradox of voting* generates example of a *SWF* that does not satisfy this

Assume three individuals, 1,2,3, and three social states, *a,b,c*, over which they have strict preferences

e.g., *individual* preference ordering might be $(a,b,c)_i$, based on pair-wise orderings $(a,b)_i, (a,c)_i, (b,c)_i$, and a *social* preference ordering might be $[a,b,c]$ - implies $[a,b], [b,c]$ and $[a,c]$

Suppose a *SWF* consists of *majority voting* in this 3-person, 3-state society, individual preference orderings being:

$(a,b,c)_1, (b,c,a)_2$, and $(c,a,b)_3$

Assuming no strategic voting, from these preference orderings:

(i) Taking pair (a,b) : $(a,b)_1$ and $(b,a)_2$ and $(a,b)_3 \Rightarrow [a,b]$ - 2 outvoted by 1 and 3

(ii) Taking pair (b,c) : $(b,c)_1$ and $(b,c)_2$ and $(c,b)_3 \Rightarrow [b,c]$ - 3 outvoted by 1 and 2

(iii) Taking pair (a,c) : $(a,c)_1$ and $(c,a)_2$ and $(c,a)_3 \Rightarrow [c,a]$ - 1 outvoted by 2 and 3

i.e., axiom of transitivity is violated

Majority voting *SWF* does not produce a social ordering, and so a Bergson *swf*

If preferences had been $(a,b,c)_i, \forall_i$, there would have been unanimity on $[a,b,c]$

Arrow and others (Sen) have suggested two key restrictions on a *SWF*:

(i) Welfarism:

***U*: condition of unrestricted domain: the *SWF* should work whatever the pattern of individual preferences, i.e., any vector of individual utility functions is admissible in determining social ranking**

PI: Pareto indifference: if all households are indifferent between two social states a and b , the *SWF* must rank the two states equivalently

I: independence of irrelevant alternatives: suppose $[a,b]$, then individual orderings change in a way that leaves preferences unchanged between a and b , $[a,b]$ should continue to hold - i.e., a change only of c 's position in individual preferences should not change social ordering of a and b

(ii) Invariance Requirements:

Limits on measurability and comparability of household utility functions:

OS: ordinal scale: indifference curves can be numbered in an arbitrary manner, as long as higher indifference curves are given higher numbers

Allows re-scaling of utility function of h by any monotonic transformation, i.e., a transform of U^h , $V^h = \psi^h(U^h)$ for any $\psi^h(U^h)$ with $\delta\psi^h/\delta U^h > 0$ will convey same information as U^h , so *SWF* will be the same

NC: non-comparability: none of the information measured for individual household utility can be used in making cross-household comparisons

PW: weak Pareto principle: social state a must be preferred to state b if every household strictly prefers a to b

■ Non-Comparability and Dictatorship Possibilities

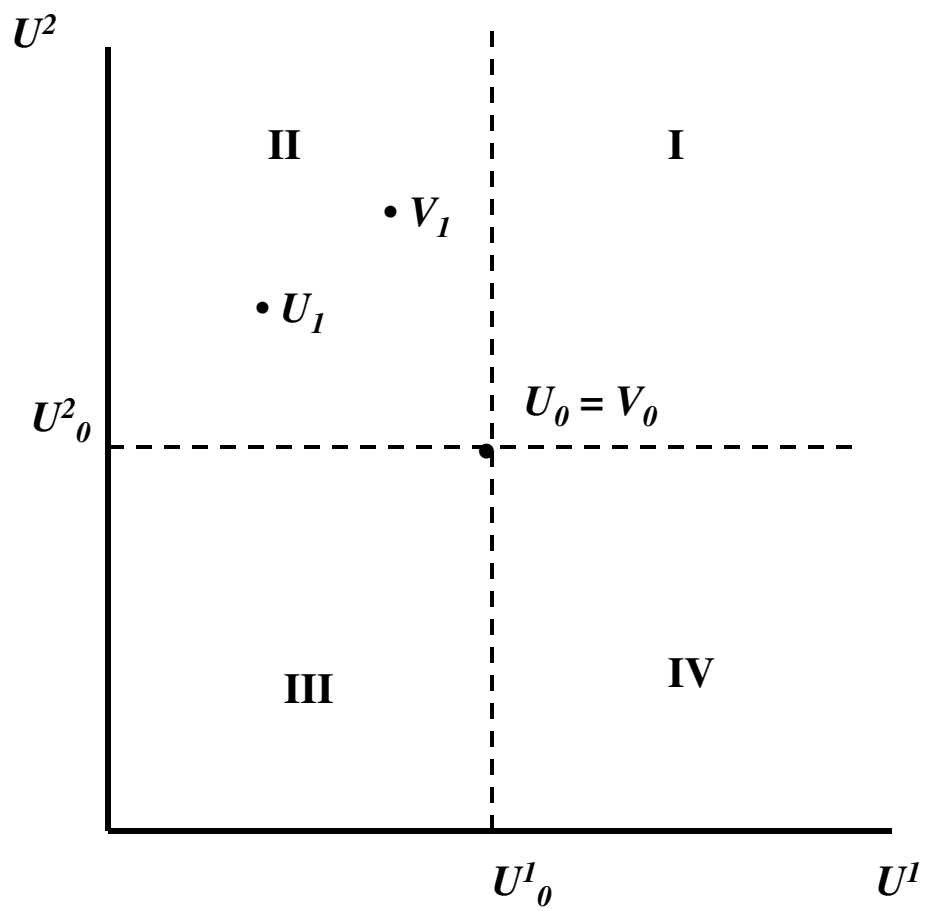
Arrow (1951) was interested in a situation where if utility functions are *OS* and *NC*, what possibilities are permitted if one restricts the *SWF* to incorporate *PW* and welfarism?

Social orderings must coincide with preferences of some individual (a dictator), regardless of preferences of others

Examine case of a 2-household economy in Figure 1. Need to rank all utility points relative to $U_0 = [U^1_0, U^2_0]$

Ignoring boundaries, points in quadrants I and III are easy to rank, all points in I being ranked higher than U_0 , while U_0 must be ranked higher than all points in III

Figure 1



With the *OS-NC* assumption, can be shown all points in quadrant II must be ranked in same way

Consider U_1 , where $U^1_1 < U^1_0$, and $U^2_1 > U^2_0$. By completeness of the *SWF*, either $U_1 > U_0$, $U_0 > U_1$, or U_1 is equivalent to U_0

Suppose $U_1 > U_0$, this ranking must be preserved given monotonic transformations of U^1 and U^2 - suppose transformation is $V^1 = \psi^1(U^1)$, $V^2 = \psi^2(U^2)$, such that $V^1_0 = U^1_0$, and $V^2_0 = U^2_0$

By choice of $\psi^h(\cdot)$, point $[\psi^1(U^1_1), \psi^2(U^2_1)]$ can be mapped anywhere in II, as long as $V^1_1 < V^1_0$, and $V^2_1 > V^2_0$, i.e., all points in II must be ranked the same with respect to U_0

Possible to rule out that all points in II are equivalent to U_0 - suppose this were actually the case, given the relevant transformation, $V^1_1 > U^1_1$, and $V^2_1 > U^2_1$, then by the weak Pareto principle, *PW*, $V_1 > U_1$, which would violate transitivity, given that all points in II were assumed equivalent to U_0

Either all points in II are ranked above U_0 , or U_0 is ranked above all points in II. By same line of reasoning, can prove all points in IV ranked above U_0 , or U_0 is ranked above all points in IV

It can also be established that if all points in II are ranked above U_0 (or vice-versa), U_0 must be ranked above all points in IV (or vice-versa)

If two quadrants are ranked the same way w.r.t. to U_0 , then points on boundary between two quadrants ranked in same way, i.e., either I and II (and common boundary) are ranked above III and IV, or I and IV are preferred to II and III

In former case, have not ranked points along horizontal line through U_0 , or in latter case along vertical line through U_0

Two possibilities:

***Strong dictator*: possible all points on horizontal line through U_0 are socially indifferent - i.e., line is a social indifference curve**

Implies household 2 is a strong dictator, since if it is indifferent between states, states are ranked indifferent socially, and entire *swf* would be a series of horizontal lines

If household 1 were a dictator, the *swf* would be a set of vertical lines

Lexicographic Dictatorship: Assumptions do not require all points on horizontal line through U_0 be socially indifferent, i.e., possible is U_0 preferred to any point to the left but not the right

As U_0 is chosen arbitrarily, any point on horizontal line is preferred to points left of it, i.e., ranking of points increases as one moves right

Such a social ordering corresponds to a lexicographic dictatorship - i.e., if household 2 is prior dictator, but is indifferent between two social states, household 1 becomes dictator as long as they strictly prefer one social state, if not, next household becomes dictator, and so on

If social ordering is lexicographic, it is not continuous, so an SWF cannot be represented by a swf - i.e., there is no possibility of indifference between social states

Arrow's Impossibility Theorem: no SWF exists which satisfies welfarism, the weak Pareto principle, and non-dictatorship, and which can generate a transitive preference ordering over social states

Implication is that a SWF that does satisfy these conditions must be because of a dictator

Strong dictator can be ruled out if the strong Pareto principle is used, i.e., if U_0 is preferred by household 1 to any point to its left, while the dictator is indifferent between such points, and hence is no worse off moving to U_0

Nevertheless, still left with lexicographical dictatorship

- **Is there any way out of this? Much of the public choice literature has focused on relaxing Arrow's axioms (see Mueller):**

(i) Dictatorship - is it irrational to select one individual to make decisions on behalf of the rest of society?

Notion of an omniscient dictator around since Plato's *Republic*, and Hobbes argued life under anarchy terrible compared to that under a unanimously accepted dictator

Such arguments are inconsistent with basic democratic ideals

(ii) Transitivity - this axiom can be relaxed to allow dictatorial power to be spread across a wider group

Quasi-transitivity requires transitivity only of the preference relation, but not of indifference, e.g., indifference between a and b , and between b and c , can co-exist with a being preferred to c

Gibbard (1969) has shown that a quasi-transitive ordering produces an oligarchy which can impose its unanimous preference on rest of community

Sen (1970) has shown spreading dictatorial power across a wider group can result in a degree of arbitrariness

Society can pick between either a or b , but if c is in the set, a must be picked. If points a , b , and c are all Pareto efficient, those most favored under distribution b may question the ethical basis for a then being chosen

(iii) Unrestricted domain - axiom implies freedom of choice for individuals, which should then be reflected in social ordering - however, conflict easily arises

Is it possible to place constitutional constraints on types of issue that need to be subject to collective choice?

Single peakedness of preferences, and uni-dimensional choices will work with majority rule, i.e., the median voter outcome

How are other multi-dimensional issues to be resolved? Implies individuals with multi-peaked preferences have to be isolated and excluded from collective choice

Such homogeneity of preferences may come about if individuals adopt, or already have a common set of values - might be true in a Rawlsian world

(iv) Independence of irrelevant alternatives - by restricting social choice between a and b to only rankings of these alternatives, one is excluding information that can be used to cardinalize and interpersonally compare utilities

Why? Because cardinal measurement is difficult and arbitrary and, hence, open to abuse by those public officials doing the measurement.

What if cardinal utility information was provided by voters themselves?

Problem here is there may be strategic misrepresentation of preferences, i.e., an objective of Arrow was to ensure collective decision process was strategy proof

■ **Where does this leave public choice?**

- drop the transitivity axiom, abandon search for best alternative, and simply seek a collective choice process that is fair or democratic

- relaxing unrestricted domain and independence of irrelevant alternatives assumptions raises questions over what issues are to be decided, who is to decide, and how to weigh their preferences - i.e., involves interpersonal utility comparisons

■ **Conclusion**

Public economics theory proceeds upon assumption that a Bergson *swf* exists, i.e. policy is chosen to maximize a *swf* which accounts for the weights society may attach to individual households