■ Welfare Distribution

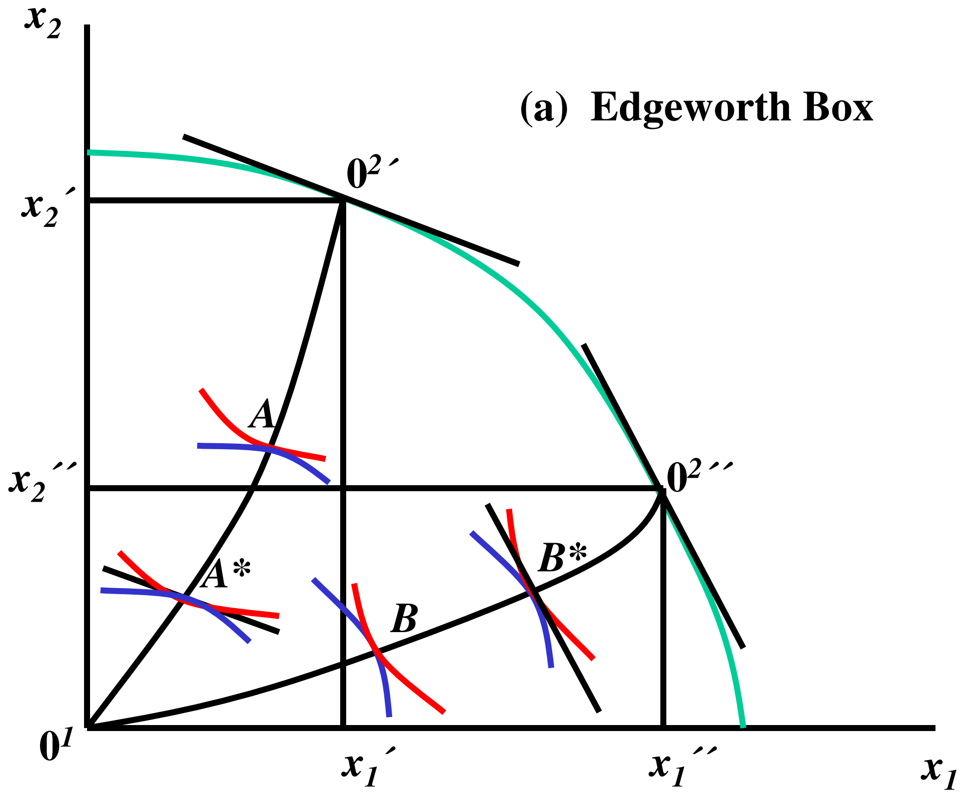
Given Second Welfare Theorem, need to explicitly consider what is meant by *welfare distribution* - natural definition in 2-household economy is a pair of utility values (U^1, U^2)

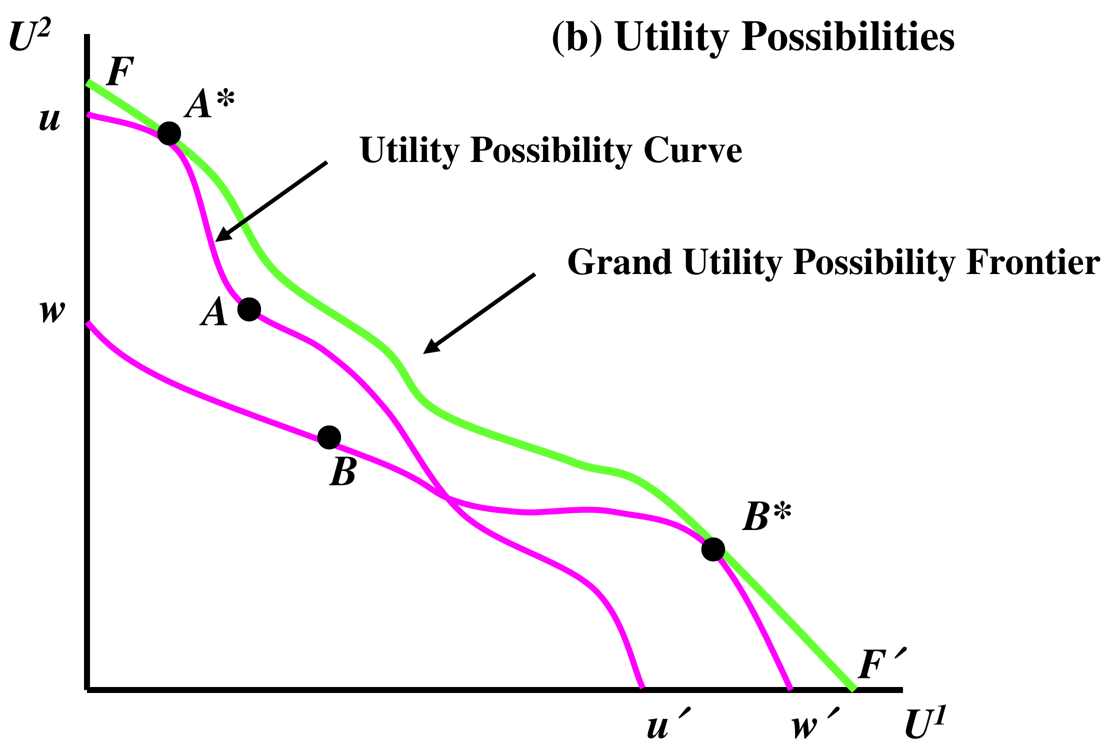
Focusing on Figure 1(a) for *given* outputs x_1' and x_2', contract curve shows Pareto optimal distributions of goods - at each tangency point, there is a pair of utility values

Figure 1(b), uu', traces out these utility values - essential characteristic is its negative slope, expressing fact that as goods are redistributed from one consumer to another, one is better off the other worse off

uu' is the *utility possibility curve* - only at point A^* do the equalized marginal rates of substitution equal the marginal rate of transformation

Figure 1: Welfare Distribution





If constraint of given outputs is relaxed, other welfare distributions can be derived

If outputs are x_1'' and x_2'', the contract curve is associated with the utility possibility curve *ww′*, and at *B the equalized marginal rates of substitution also equal the marginal rate of transformation**

FF′* is the envelope of all utility possibility curves, known as the *grand utility possibility frontier

All points on *FF′* are a full Pareto optimum, hence, *uu′* touches *FF′* at *A - while at other welfare distributions on *uu′*, the full set of necessary conditions are not met, so *uu′* lies below *FF′*, and likewise for *ww′* which only touches *FF′* at *B**, and elsewhere lies below**

In principle, what use is the grand utility possibility frontier to a policymaker?

- Second Welfare theorem implies a point on the frontier can be reached via the competitive market, given an appropriate initial distribution of wealth

- In Figure 2, policy maker may regard *C* as distributionally superior to *A, but there is always a feasible resource allocation that makes both individuals better off at *B** than at *C***

- As already noted, lump sum-transfers are infeasible, so set of welfare distributions available is not *FF′*, but *ff′*, drawn on the assumption that given a prevailing initial wealth distribution, competitive equilibrium results in *A**

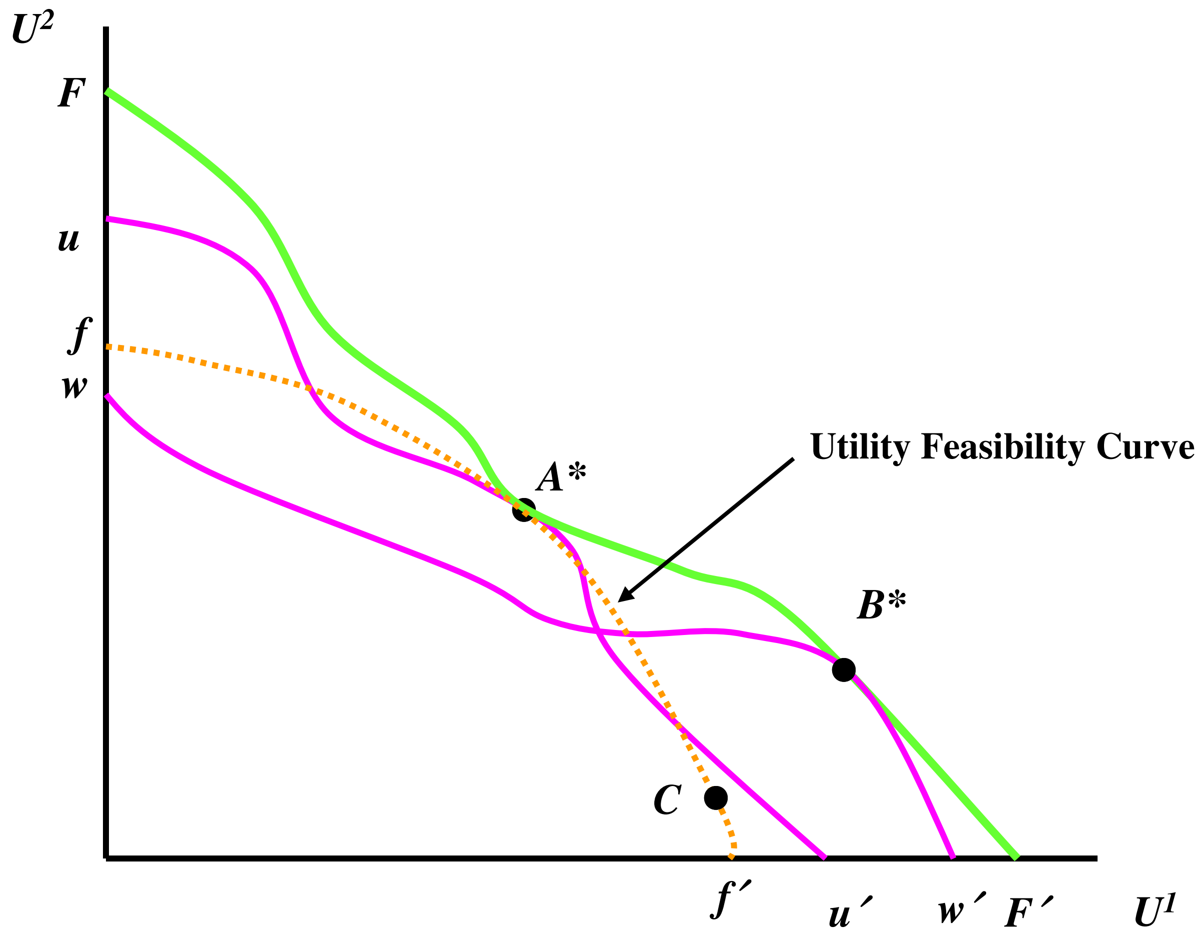
- Non lump-sum transfers will have to be used to change the welfare distribution, these will violate optimality conditions, so *ff′* is relevant *utility feasibility curve*

- Issue is to find *best* such curve that lies as close to *FF′* as possible - need to find redistribution policy that minimizes loss in allocative efficiency, while achieving desired welfare distribution

- **Choice over Welfare Distributions**

As seen, problem with Pareto principle is it cannot be applied to policies where some individuals are made better off some worse off

Figure 2: Utility Feasibility



Kaldor (1939) and Hicks (1939) suggested social welfare could be said to have increased if gainers could compensate the losers to leave them no worse off, and still retain some benefit from change

Compensation is purely hypothetical, if *actually* paid, essentially satisfies the Pareto criterion

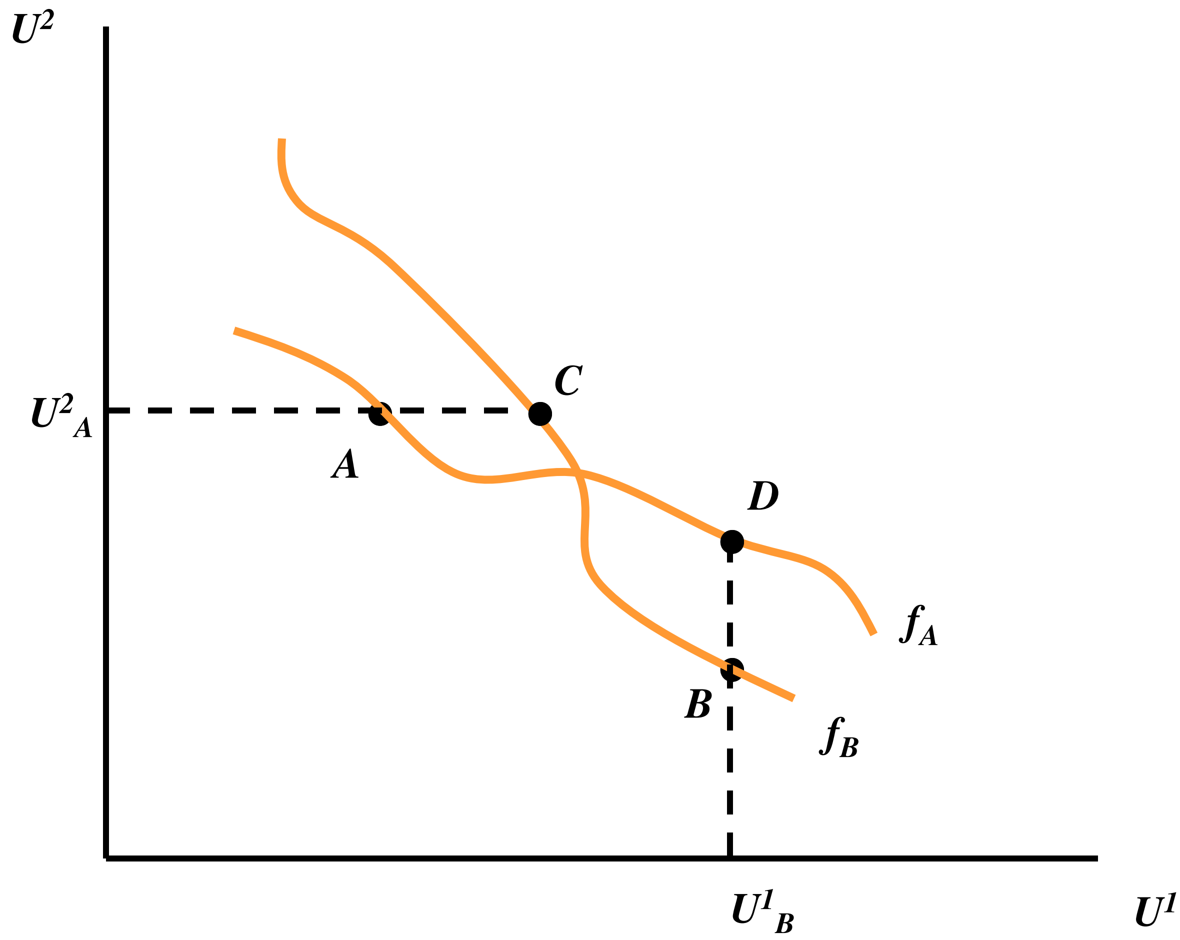
In Figure (3), suppose initial wealth distribution is at *A*, policy results in *B*, where household 1 is better off, household 2 worse

f_B is utility feasibility curve through *B*, if there is a point on f_B such as *C*, where 2 is as well off as at *A*, and 1 is better off, move to *B* should be made

Scitovsky (1941) pointed out a problem with compensation principle - possible a move from *A* to *B* is good, and a move from *B* to *A* is also good

In Figure 3, move to *B* is made on the Kaldor-Hicks criterion, and it is proposed to move back to *A* - a utility feasibility curve f_A goes through *A*, showing that at point *D*, 1 is as well of as at *B*, and 2 is better off (*the reversal paradox*)

Figure 3: Reversal Paradox



Economy could cycle indefinitely between *A* and *B*, as long as compensation is never made

Attempt by the compensation principle *not* to make value judgements is bound to fail - a policy may make the rich much richer, and the poor poorer, and be accepted on this criterion

Explicitly or implicitly, value judgements are made about welfare distributions both by individuals and governments

Accounting for these relates to the construction of a *social welfare function*

- **Social Welfare Functions**

The key concept of the *swf*, was introduced into the literature by Bergson (1938), the idea being to allow value judgements to be introduced in a systematic and objective manner

In constructing a *swf*, take as given a set of value judgements - could be from a political party, a government, an ideology, or an individual

Value judgements define a preference ordering over all relevant states of the economy

Consequences of value judgements can be evaluated by constructing preference ordering over states of system to which they give rise, and apply it to set of policy alternatives under consideration

Suppose there are three value judgements relating to the 2-household, 2 good economy, where relevant states of the economy are goods consumed (x^h_1, x^h_2, z^h), $h = 1,2$:

(i) Individual preferences to count - *social* preference ordering over bundles consumed by one individual coincide exactly with that of individual

(ii) Well-defined social preference ordering exists over pairs of bundles for two households, i.e., for $\{(x^1_1{}', x^1_2{}', z^1{}')(x^2_1{}', x^2_2{}', z^2{}')\}$ and $\{(x^1_1{}^\circ, x^1_2{}^\circ, z^1{}^\circ)(x^2_1{}^\circ, x^2_2{}^\circ, z^2{}^\circ)\}$ one is either preferred or indifferent to the other, choices being transitive and reflexive

(iii) Social state *A*, where one household has a preferred bundle and other no worse a bundle than in state *B*, will always be socially preferred (*strong* Pareto principle)

(i)-(iii) extend Paretian value judgements by adding (ii) which introduces explicit inter-personal comparisons of well-being - sufficient to allow choice among *any* allocations

A continuity axiom is also required, and assuming this, the Bergson *swf* for the 2-household economy is:

$$W = W[U^1(x_1^1, x_2^1, z^1), U^2(x_1^2, x_2^2, z^2)] = W(U^1, U^2) \qquad (1)$$

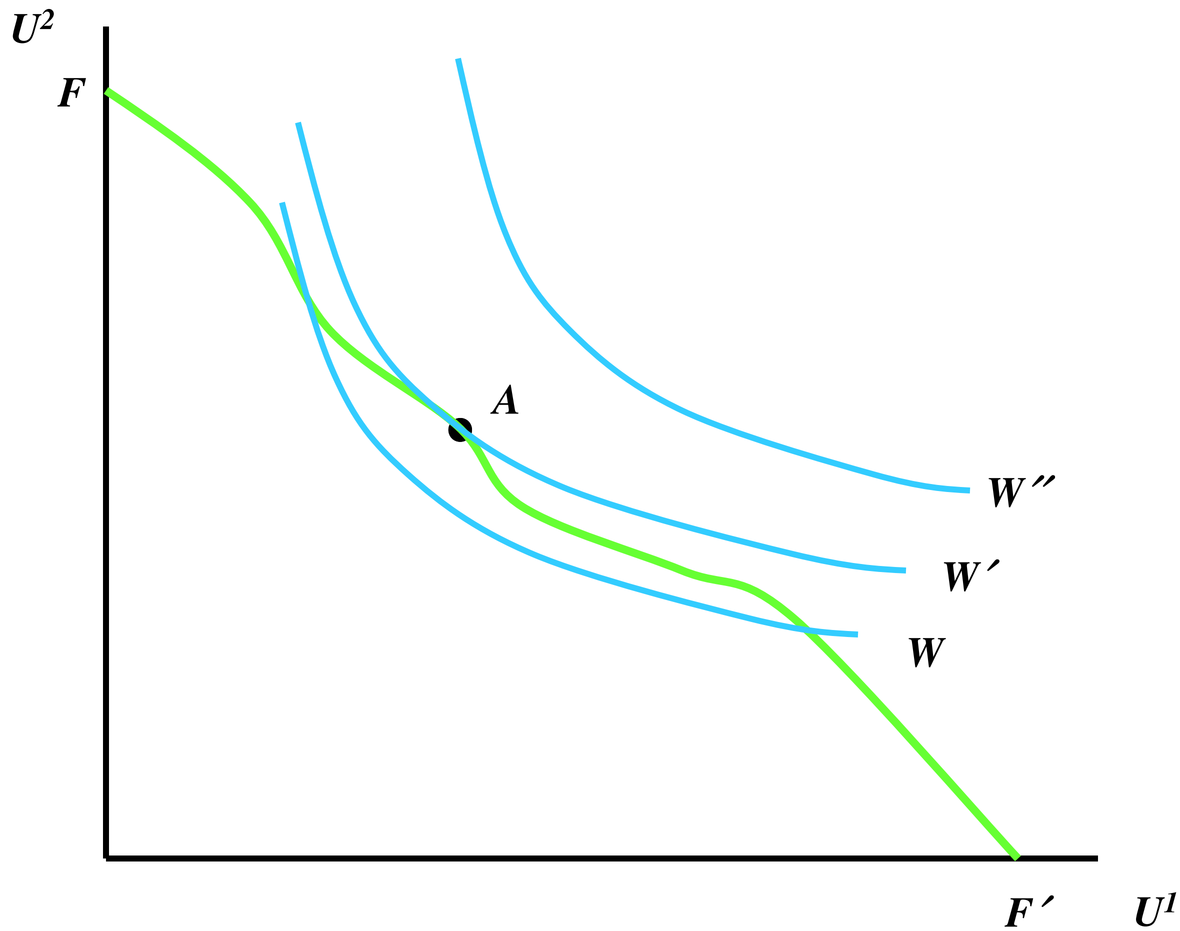
If (1) is differentiable, the following conditions are assumed to hold, $dW/dU^h > 0$, and $d^2W/dU^{h2} < 0$, the latter often being strengthened to strict concavity

The latter assumptions means that *W* can be treated in much the same way as an individual utility function - for a specific level of *W*, it is possible to draw a *social indifference curve* (see Figure 4)

Reflects *egalitarian* ethic that inequalities between households, *per se*, is socially undesirable

What is precise meaning of defining a social welfare function on individual utility functions which are themselves *ordinal*? The value judgements (i)-(iii) place certain restrictions on *W*:

Figure 4: Social Welfare Function



(a) Due to (i), social marginal rate of substitution (SMRS) between two goods for a given household must be the same as MRS for household itself - if not, household may be regarded as socially better/worse off when they would regard themselves as neither

(b) Also due to (i), *W* must increase when household moves to preferred bundle or remain constant when indifferent

(c) Value of *W* remains same over pairs of consumption bundles that are indifferent in the social ordering, and increases over pairs that are preferred - *W* need only be an ordinal numerical representation of social preferences

Objective is to maximize *swf* (Figure 4) at *A*:

$$SMRS = \frac{\delta W / \delta U^1}{\delta W / \delta U^2} = \frac{W^1}{W^2} = -\frac{\lambda^1}{\lambda^2}$$

λ^1 / λ^2 is ratio of marginal utilities of income for households 1 and 2, also:

$$\frac{W^1}{\lambda^1} = \frac{W^2}{\lambda^2} = \ldots = \varphi$$

where φ is social marginal utility of income ***(See Appendix)***

■ Other Social Welfare Functions

(i) *Utilitarian* or *Benthamite swf*:

$$W = \sum_{h=1}^{H} U^h$$

where social welfare is the un-weighted sum of household utilities (see Figure 5)

(ii) *Generalized Utilitarian*:

$$W = \sum_{h=1}^{H} a_h U^h$$

where the weights a_h, $h=1,...,H$, are positive constants

(iii) *Bernoulli-Nash*:

$$W = \prod_{h=1}^{H} U^h$$

where social welfare is the product of un-weighted household utilities (see Figure 6)

(iv) *Generalized Bernoulli-Nash*:

$$W = \prod_{h=1}^{H} (U^{h})^{a_h}$$

which is utilitarian in logarithms of utility

(v) *Rawlsian-maximin*: Rawls (1971) thought in a "just" economy that welfare of worst-off is as large as possible:

$$W = \min [U^{1},\ldots,U^{H}]$$

where social welfare is identified with utility of the worst-off household - i.e., maximization of minimum value of utility vector (see Figure 7)

Principle does not imply removing all inequalities in well-being - differences in primary goods are justifiable insofar as they benefit least well-off, i.e., everything for the greater utility of the poor

Suppose there is a two-class economy, where U_1 and U_2 represent groups of households

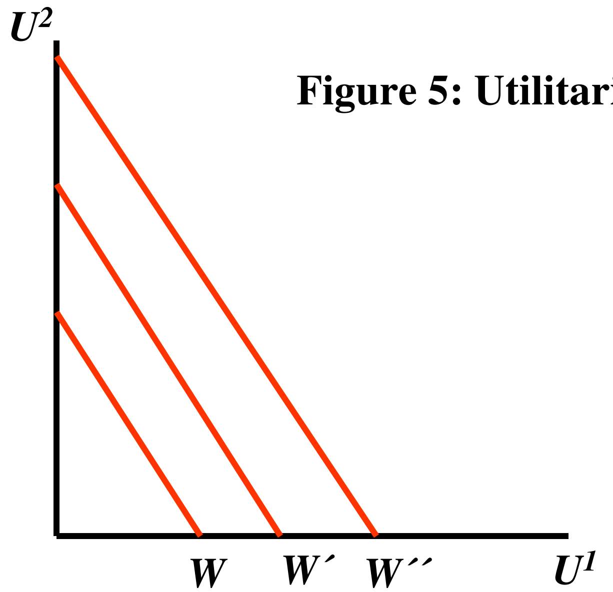


Figure 5: Utilitarian

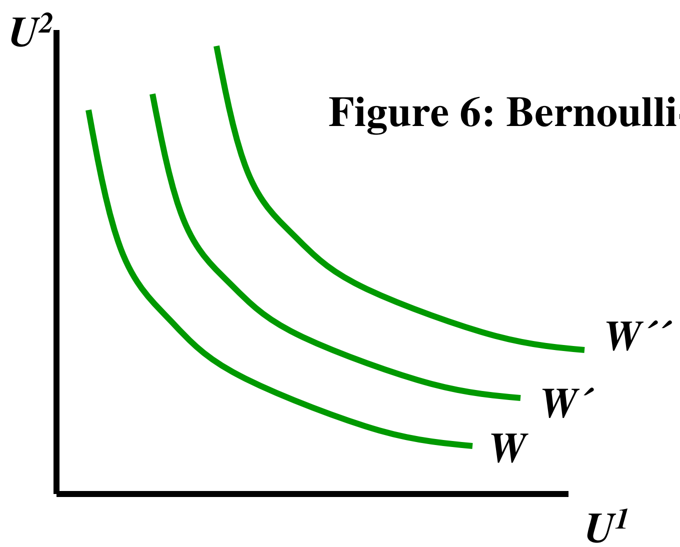


Figure 6: Bernoulli-Nash

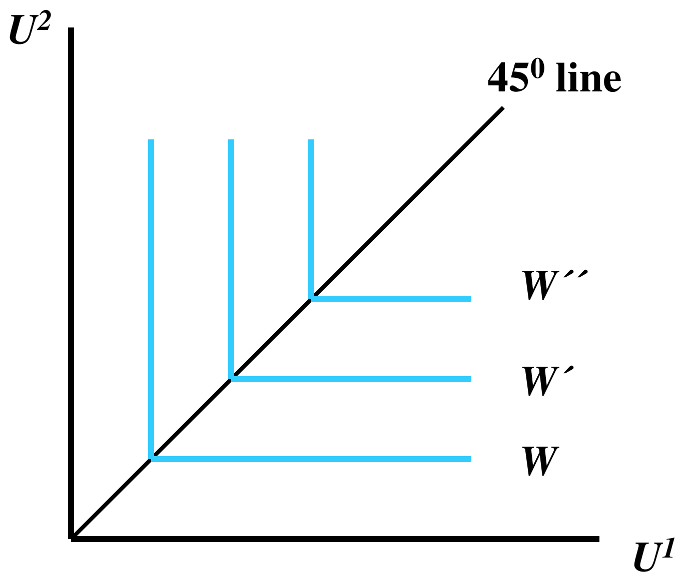
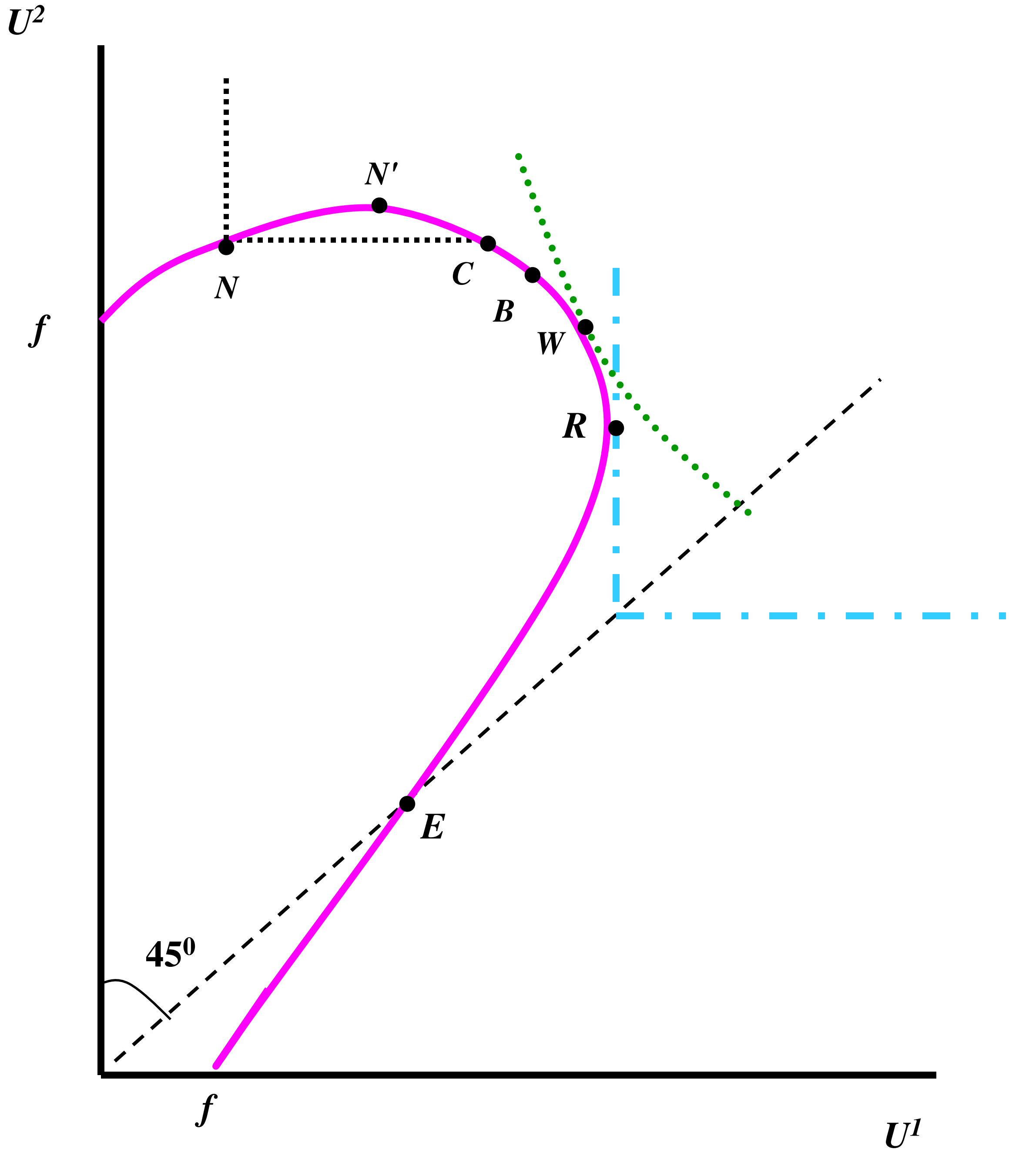


Figure 7: Rawlsian

Figure 8: Comparison of Welfare Distributions



In Figure 8, *ff* is a utility feasibility frontier, given policy instruments at government's disposal

(i) *N* – minimal, "night-watchman" state, given initial endowments

(ii) *NC* – any move along this section is Pareto-improving (contractarian solution)

(iii) *E* – egalitarian solution

(iv) *B* – Benthamite/utilitarian solution

(v) *W* – Bernoulli-Nash solution

(vi) *R* – Rawlsian solution

(iv) - (vi) are all special cases of an iso-elastic function (Atkinson, 1970):

$$W = \frac{\sum_{h=1}^{H} a_h (U^h)^{1-\rho}}{1-\rho}$$

ρ is interpreted as "inequality-aversion" parameter

- if $\rho = 0$, $a_h = 1$, all h, reduces to utilitarian case
- if $\rho \to 1$, $a_h = 1$, reduces to Bernoulli-Nash case
- if $\rho \to \alpha$, reduces to maximin/Rawlsian case

Note - Rawls himself objected to notion that one can move from one ethical concept to another by varying ρ