APPLIED WELFARE ECONOMICS AND POLICY ANALYSIS

- **Welfare Properties of General Equilibrium**

- **What can be said about *optimality* properties of resource allocation implied by general equilibrium?**

- **Any criterion used to compare resource allocations must be based on value judgements - typically use the *Pareto* criterion (see Figure)**

- **Why is this criterion adopted?**

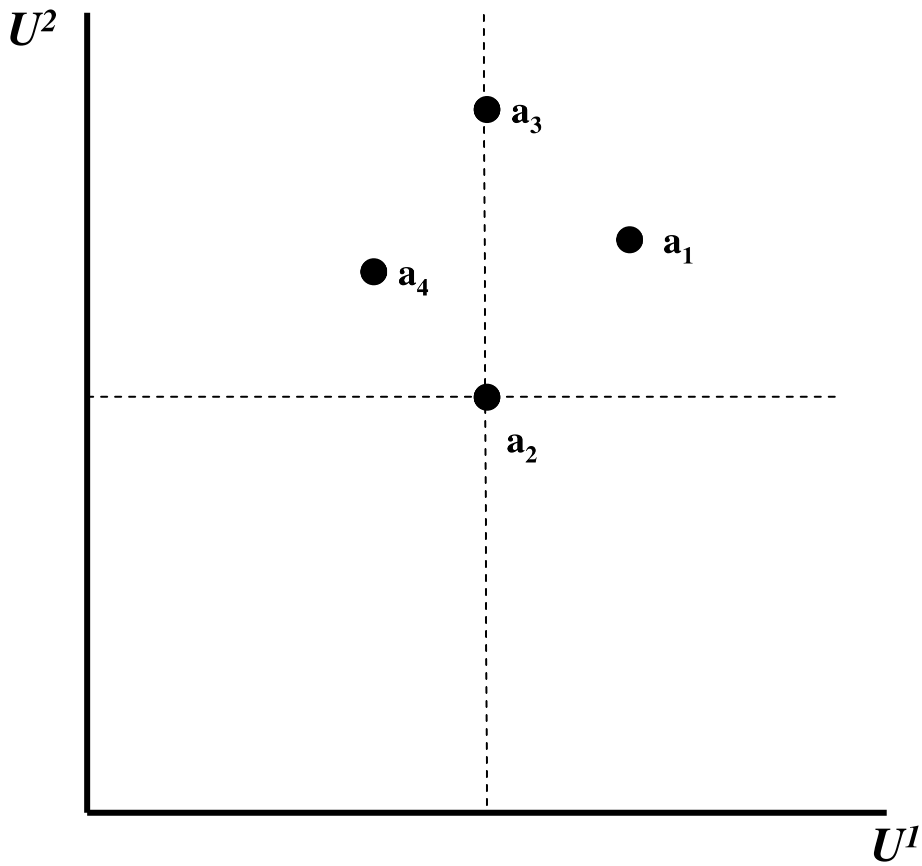
(a) Suppose levels of individual well-being $h=1,...,H$ are given as U^h, and suppose some set of explicit value judgements gives rise to a social ordering over U^h, given by the *swf*:

$$W=W(U^1,...,U^H) \tag{1}$$

Suppose that an increase in the well-being of one individual is a good thing and a decrease is bad, the well-being of no-one else changing, implying:

$$\frac{\delta W}{\delta U^h} > 0 \qquad \text{for all } h \tag{2}$$

The Pareto Criterion



- *weak* Pareto criterion states a_1 is superior to a_2 if and only if both individuals 1 and 2 are better off at a_1

- *strict* Pareto criterion states a_3 is preferred to a_2 if everyone is at least as well off, and at least one person is better off

- provides only a *partial* ordering, e.g., a_4 cannot be compared to a_2 by this criterion

This can be used to show that a resource allocation that maximizes *W* must be a Pareto optimum

Take an allocation that is *not* a Pareto optimum, we know it is possible to change this and increase some U^h, with no U^h decreasing, hence, *W* must increase by (2)

Proves that an allocation that is *not* Pareto optimal will not maximize *W*, hence, a welfare maximum must lie in a set that *is* Pareto optimal

(b) Under certain conditions, equilibrium of a market economy is Pareto optimal, i.e., the First Welfare theorem

- **Characteristics of a Pareto Optimum**

Focus on a 2 consumer, 2 firm, 2 good, 2 input economy, and assume a *central planner* that wants to establish the conditions that will be satisfied by a Pareto optimal allocation

The model of the economy is:

(a) Utility functions:

$$U^h = U^h(x^h{}_1, x^h{}_2, z^h) \qquad i=1,2 \tag{3}$$

$x^h{}_i$ are quantities of goods consumed, giving positive utility, and z^h is input h supplied by household h, generating negative utility; initial endowments of each input are $\check{z}^h$, $h=1,2$

(b) Production functions:

$$x_i = f_i(z_i^1, z_i^2) \qquad i = 1,2 \tag{4}$$

where x_i is output of each good, and each firm produces only one good

(c) Market Clearing Conditions:

$$\sum_{h=1}^{2} x_i^h - x_i = 0 \qquad i = 1,2 \tag{5}$$

$$z^h - \sum_{i=1}^{2} z_i^h = 0 \qquad h = 1,2 \tag{6}$$

- Need to find necessary conditions for resource allocation satisfying constraints, and which maximizes one household's utility function subject to a given level of utility for the other household

- Define the problem:

$$\max\ U^1(x_1^1, x_2^1, z^1) \tag{7}$$

$$s.t.\ \breve{U}^2 = U^2(x_1^2, x_2^2, z^2) \tag{8}$$

Lagrange multipliers

$$\mu_i, \lambda_i, \upsilon^h \text{ and } \rho$$

can be associated with
(4),(5), (6), and (8) respectively, the necessary conditions for Pareto optimality are:

$$\left.\begin{aligned} U_i^1 - \lambda_i &= 0 \\ U_3^1 - \upsilon^1 &= 0 \\ \rho U_i^2 - \lambda_i &= 0 \\ \rho U_3^2 - \upsilon^2 &= 0 \end{aligned}\right\} \quad i = 1,2$$

$$-\mu_i + \lambda_i = 0 \quad i = 1,2 \qquad \text{total output condition}$$

$$\mu_i f_i^h - \upsilon^h = 0 \quad h = 1,2 \qquad \text{input allocation condition}$$

Conditions for Pareto optimality given by taking appropriate ratios:

$$\frac{U_1^1}{U_2^1} = \frac{\lambda_1}{\lambda_2} = \frac{U_1^2}{U_2^2} \qquad (9)$$

$$\frac{U_3^h}{U_i^h} = \frac{v^h}{\lambda_i} = f_i^h \qquad h = 1,2 \quad i = 1,2 \qquad (10)$$

$$\frac{f_1^1}{f_1^2} = \frac{v^1}{v^2} = \frac{f_2^1}{f_2^2} \qquad (11)$$

$$\frac{U_1^1}{U_2^1} = \frac{U_1^2}{U_2^2} = \frac{\lambda_1}{\lambda_2} = \frac{\mu_1}{\mu_2} = \frac{v^h/f_1^h}{v^h/f_2^h} = \frac{f_2^h}{f_1^h} \qquad (12)$$

- (9) states households' marginal rates of substitution between goods have to be equal

- (10) states household's marginal rate of substitution between good *i* and input z^h has to equal input's marginal product

- (11) states firms' marginal rates of technical substitution between inputs have to be equal

- (12) states that equalized marginal rates of substitution have to be equal to the marginal rate of transformation

λ_1/λ_2 *and* v_1/v_2 **are shadow price ratios,**

(9) - (12) are presented graphically in Figure 1

(a) At α, marginal rates of substitution are equal, and equal to the marginal rate of transformation; total consumption also equals total production

(b) At β, marginal rates of technical substitution are equal, firms are on isoquants corresponding to x_1* and x_2*, and input allocations sum to what is made available by households z^1* and z^2*

(c) At γ, marginal rate of substitution between $x^1{}_1$ and z^1 is equal to the marginal product of z^1 in producing x_1 (could be expressed in terms of good 2)(m_1 shows valuation of increment of z^1 in terms of x_1 consumer would have to be paid to supply z^1)

(d) At δ, marginal rate of substitution between $x^2{}_1$ and z^2 is equal to the marginal product of z^2 in producing x_1 (could be expressed in terms of good 2)(m_2 has similar interpretation to m_1)

- **<u>First Welfare Theorem:</u> A competitive equilibrium is a Pareto Optimum**

This theorem indicates that equilibrium price signals are sufficient to coordinate decentralized economic activities in a way that is Pareto optimal

By individual maximization behavior, each economic agent responds to prices by equating marginal rate of substitution (marginal rate of technical substitution) to these prices

As all agents face the same prices, all marginal rates are equated to each other in equilibrium. Combined with market equilibria, these equalities characterize Pareto optima in a convex environment, i.e., non-increasing returns and convex preferences

**** see the appendix for a proof of the theorem***

- **<u>Second Welfare Theorem</u>: Given a Pareto optimum, there exists a set of prices that will lead to a competitive equilibrium**

- **Second theorem has important policy implication:**

  By making economy competitive, selecting equilibrium to be decentralized, and by using lump-sum transfers to ensure each household has enough income to afford their allocation, policy maker can achieve a Pareto optimum (Figure 2)

  Normative question of *which* allocation to achieve is separated from *how* to achieve that allocation

- **Problems with Welfare theorems**

  (a) First Welfare theorem:

  - in presence of market failures, theorem no longer applicable to any economy where they are present (theory of second-best)

  - highly inequitable allocations may be optimal

  "A society or an economy can be Pareto optimal and still be perfectly disgusting" (A. Sen, 1970)

(b) There are three key problems with the Second Welfare theorem:

- with market failures, there will not be a Pareto optimum

- requires convexity assumptions to hold

- relies on the use of lump-sum transfers, but these may be costly to collect, characteristics on which they are based are unobservable, giving households an incentive to make false revelations

"It is therefore best to treat the Second Theorem as being of considerable theoretical interest but of very limited practical relevance" (G. Myles, 1995)

Appendix (Welfare Properties of General Equilibrium)

■ **Proof of the First Welfare Theorem**

***Definitions*:**

(i) Feasibility:

An array of consumption vectors $\{x^1,\ldots,x^h,\ldots,x^H\}$ is feasible if $x^h \in X^h$, all h, and there exists an array of production vectors $\{y^1,\ldots,y^j,\ldots,y^m\}$, each $y^j \in Y^j$, such that:

$$x \leq y + \omega$$

where:

$$x = \sum_{h=1}^{H} x^h \ , \ \omega = \sum_{h=1}^{H} \omega^h \ , \ y = \sum_{j=1}^{m} y^j \ .$$

i.e., an allocation of consumption bundles to consumers is feasible if it can be produced with initial endowment and production technology

(ii) Pareto Optimality:

A feasible consumption array $\{x^{h*}\}$ *is* Pareto optimal if there does not exist another array $\{x^h\}$ such that:

$$U^h(\underline{x}^h) \geq U^h(x^{h*}),\ h = 1,...,H,$$

$$U^h(\underline{x}^h) > U^h(x^{h*}),\ for\ at\ least\ one\ h.$$

$\{x^h*\}$ is Pareto optimal if there is no alternative feasible array that gives each household as much utility as $\{x^h*\}$, and gives strictly more utility to at least one household

(iii) Competitive Equilibrium (CE):

An array $[p^*, \{x^h*\}, \{y^j*\}]$ is a competitive equilibrium if:

$$x^{h*} \in X^h, p^* x^{h*} \leq p^* \omega^h + \sum_{j=1}^{m} \theta_j^h p^* y^{j*}, h = 1,...,H. \quad (13)$$

$$y^{j*} \in Y^j,\ j = 1,...,m \quad (14)$$

and:

$$(i)\ U^h(x^{h*}) \geq U^h(x^h)\ for\ all\ x^h \in X^h$$

$$such\ that\ p^* x^{h*} \leq p^* \omega^h + \sum_{j=1}^{m} \theta_j^h p^* y^{j*},$$

(ii) $p^{*} y^{j*} \geq p^{*} y^{j}$*, all* $y^{j} \in Y^{j}$,

(iii) $x^{*} \leq y^{*} + \omega$

(13) and (14) require household demands to be both affordable and in their consumption sets, and firms' choices are in their production sets

(i) implies households maximize utility, (ii) implies that firms maximize profits, and (iii) that the equilibrium is feasible

To develop first theorem, suppose household *h* has locally non-satiated preferences, and let x^{h*} be their consumption plan.

Lemma:

***Let x^{h*} be a locally non-satiating choice for household h at prices p^{*}. Then*:**

$$(a)\ U^{h}(x^{h}) > U^{h}(x^{h*}) \mapsto p^{*} x^{h} > p^{*} x^{h*},$$

$$(b)\ U^{h}(x^{h}) = U^{h}(x^{h*}) \mapsto p^{*} x^{h} \geq p^{*} x^{h*}.$$

Proof:

If (a) were false, then x^h would have satisfied the household's budget constraint and would have been chosen instead of x^{h*}.

To prove (b), suppose $p^*x^h < p^*x^{h*}$. As x^{h*} is not a point of local satiation, then neither is x^h. There then exists $x^{h\prime}$ at a distance ε from x^h with $U^h(x^{h\prime}) > U^h(x^h) = U^h(x^{h*})$. Suppose ε is small such that $p^*x^{h\prime} < p^*x^{h*}$. This then contradicts the assumption that x^{h*} was the optimal choice.

First Welfare Theorem:

Let $[p^*,\{x^{h*}\},\{y^{j*}\}]$ be a competitive equilibrium with no household locally satiated at $\{x^{h*}\}$. Then $[\{x^{h*}\},\{y^{j*}\}]$ is a Pareto optimum

Proof:

Suppose $[\{x^{h*}\},\{y^{j*}\}]$ is not a Pareto optimum, then there exists $[\{\underline{x}^h\},\{\underline{y}^j\}]$ with $\underline{x}^h \in X^h$, $\underline{y}^j \in Y^j$, and:

$$(i)\ \underline{x} \le \underline{y} + \omega$$

$$(ii)\ U^h(\underline{x}^h) \ge U^h(x^{h*})\ \text{all } h,$$

$$(iii)\ U^h(\underline{x}^h) > U^h(x^{h*})\ \text{some } h.$$

Given (ii) and (iii), (a) and (b) imply:

$$\sum_{h=1}^{H} p^* \underline{x}^h > \sum_{h=1}^{H} p^* x^{h*}.$$

Under local non-satiation, (iii) of CE gives $p^*x^* = p^*y^* + p^*\omega$, so it follows that $p^*\underline{x} > p^*y^* + p^*\omega$.

Profit maximization, (ii) of CE, implies that $p^*y^{j*} \ge p^*y^j$ all $y^j \in Y^j$, and, specifically $p^*y^{j*} \ge p^*\underline{y}^j$. Summing over j, $p^*y^* \ge p^*\underline{y}$. Hence, $p^*\underline{x} > p^*\underline{y} + p^*\omega$.

From this inequality, it follows that $[\{\underline{x}^h\},\{\underline{y}^j\}]$ is not feasible, proving the theorem by contradiction