APPLIED WELFARE ECONOMICS AND POLICY ANALYSIS

- **General Equilibrium and Welfare**

- **Model of competitive general equilibrium developed over a long time period; earliest work by Walras (1874), seminal modern contribution by Arrow and Debreu (1954)**

- **The Arrow-Debreu economy is studied because of its two fundamental welfare properties which represent the formalization of Smith's (1776) *invisible hand***

- **The theorems provide two alternative views of economic policy:**

  - the theorems are evidence that policy *should* always be moving the economy as close as possible to the competitive ideal

  - the theorems are a description of what *could* be achieved if the economy were competitive, and why it cannot be achieved in practice

■ Characterization of Equilibrium

(i)Two types of agent in the economy:

(a) Households who own initial endowments of goods and shares in firms - engage in trade to maximize utility

(b) Firms who use inputs to produce goods subject to technology - aim to maximize profits which are then distributed as dividends

(ii) All trades occur at a given date, and only at equilibrium prices

(iii) All agents treat prices as parametric

(iv) $h=1,...,H$, households, where each has a consumption set X^h

Each household h has a strictly quasi-concave utility function:

$$U^h = U^h(x^h_1,....,x^h_n) \qquad (1)$$

x^h_i is consumption of good i by household h, if $x^h_i <$ 0, good i is supplied by household, e.g., labor

Household h has an initial endowment of the n goods:

$$\omega^h = (\omega^h{}_1,\dots,\omega^h{}_n) \qquad (2)$$

Endowment, which includes labor, is liquidated to allow purchase of goods

Shareholdings of h in m firms are:

$$\theta^h{}_1,\dots,\theta^h{}_j,\dots,\theta^h{}_m \qquad (3)$$

$\theta^h{}_j \geq 0$, so if firm j makes a profit of π^j, household h earns a dividend of:

$$\theta^h{}_j\ \pi^j \qquad (4)$$

Given profits are distributed, firm j is owned by households, and individual shares sum to 1:

$$\sum_{h=1}^{H} \theta_j^h = 1, \qquad j = 1,\dots,m \qquad (5)$$

Household h picks a consumption vector $(x^h{}_1,\dots,x^h{}_n)$ to maximize $U^h(.)$, subject to a budget constraint:

$$\sum_{i=1}^{n} p_i x_i^h \leq \sum_{i=1}^{n} p_i \omega_i^h + \sum_{j=1}^{m} \theta_j^h \pi^j \qquad (6)$$

Given strict quasi-concavity of the utility function, maximization of household problem results in demand by *h* for *i*:

$$x^h{}_i = x^h{}_i(p, \omega^h, \theta^h, \pi) \qquad (7)$$

Aggregate demand for good *i* is given as:

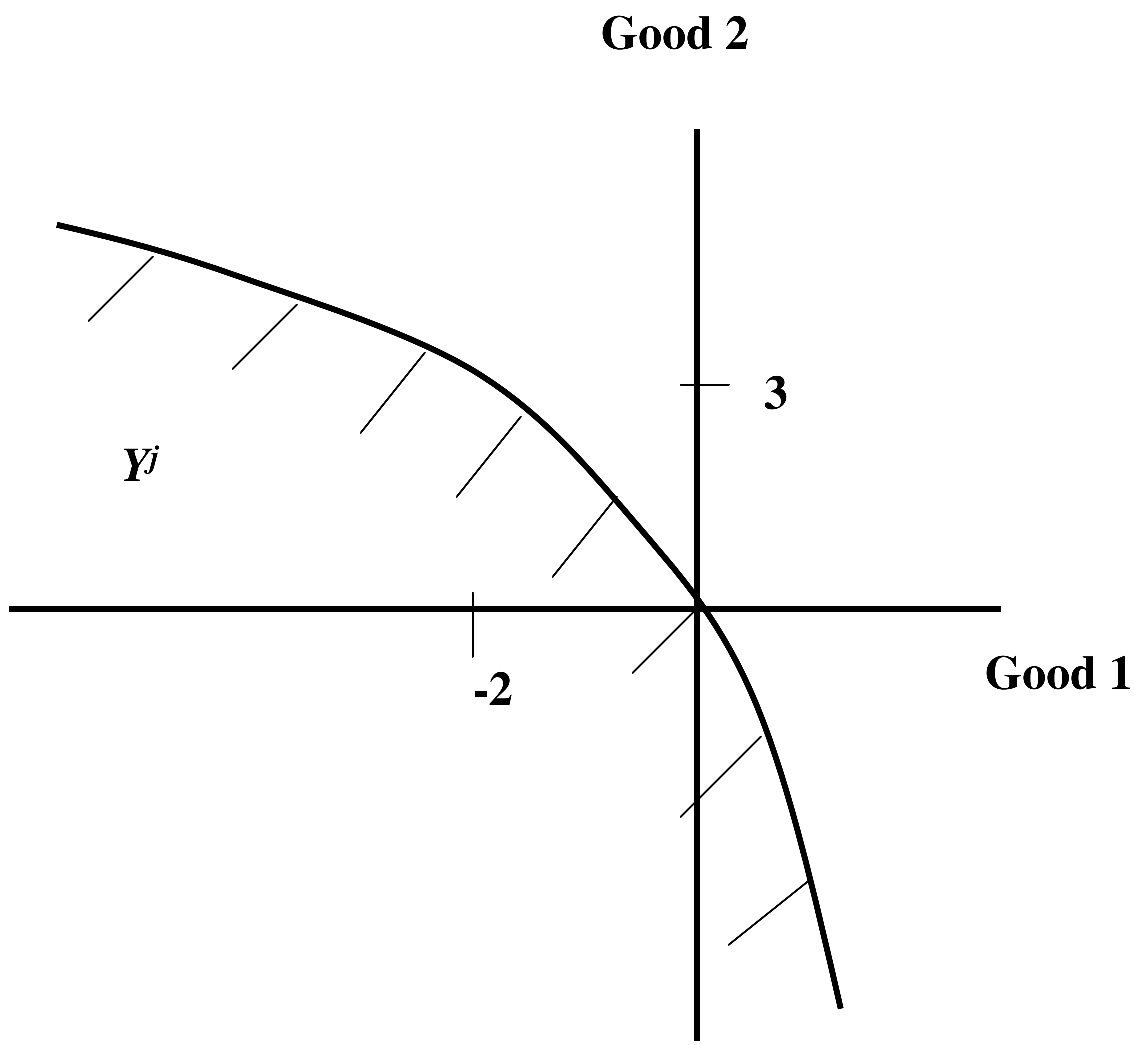
$$X_i = \sum_{h=1}^{H} x_i^h = X_i(p, \omega, \theta, \pi) \qquad (8)$$

(v) Each firm *j* has a strictly convex production set Y^j, where inputs are treated as negative and output as positive

Suppose the production plan for firm *j* is the vector $y^j = (-2, 3)$, and the price vector is $p = (2,2)$, firm profit is:

$$\pi^j = py^j = (2,2).(-2,3) = 2 \qquad (10)$$

Each firm chooses a production plan $y^j = (y^j{}_1,\ldots,y^j{}_n)$ in order to maximize profits, subject to $y^j \in Y^j$



A Production Set

Maximization determines firm *j*'s supply of good *i*:

$$y^j_i = y^j_i(p) \tag{11}$$

Summing across all firms gives aggregate supply:

$$Y_i = \sum_{j=1}^{m} y_i^j(p) = Y_i(p) \tag{12}$$

As good *i* is output for some firms and an input for others, $Y_i(p)$ is net supply; $Y(p) = (Y_1(p),\ldots,Y_n(p))$ has positive and negative elements

(vi) Using (11), level of profit for each firm is:

$$\pi^j = py^j = py^j(p) = \pi^j(p) \tag{13}$$

Given (13), and eliminating constants ω and θ, aggregate demand can be re-written:

$$X_i = X_i(p, \pi(p)) = X_i(p) \tag{14}$$

Excess demand can be defined as the difference between aggregate demand and supply:

$$Z_i(p) = X_i(p) - Y_i(p) - \sum_{h=1}^{H} \omega_i^h \tag{15}$$

(vii) **Equilibrium is where excess demand is zero or negative for all goods, with a zero price if excess demand is negative (see Figures (a)-(d)):**

$$Z_i(p) \leq 0,\ i = 1,\ldots,n, \text{ and if } Z_i(p) < 0,\ p_i = 0 \qquad (16)$$

Equilibrium is found by adjustment of the price vector until (16) is satisfied - ***existence*** **question concerns whether there is a solution to (16)**

■ **Walras' Law**

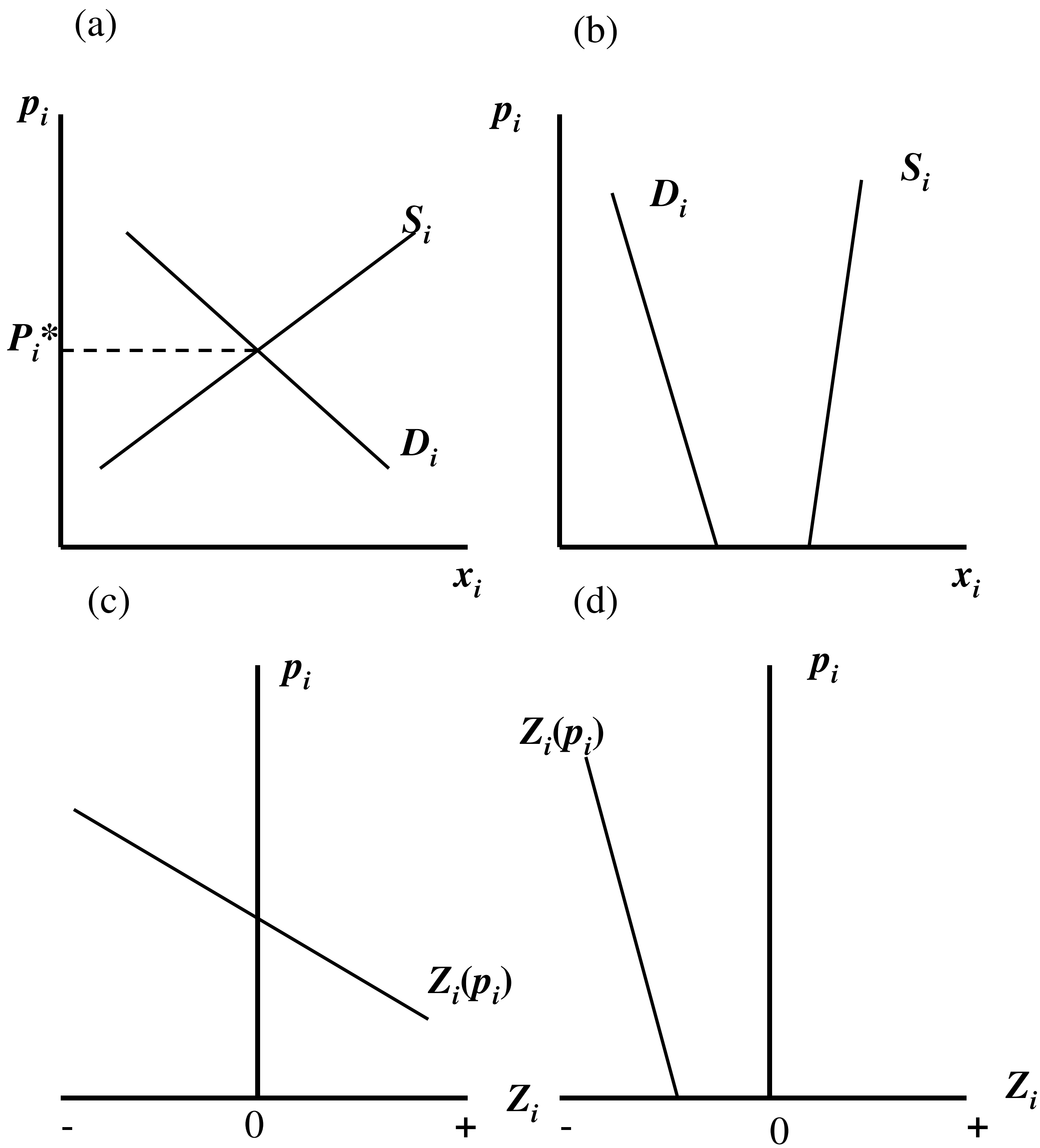
There are ***n*** **equations in (16) to be solved simultaneously - by Walras' law, the** ***n*** **equations are not independent, so only** ***n-1*** **need to be solved**

Re-writing the individual budget constraint:

$$\sum_{i=1}^{n} p_i x_i^h \leq \sum_{j=1}^{m} \theta_j^h \pi^j + \sum_{i=1}^{n} p_i \omega_i^h \qquad (17)$$

and summing over all individuals:

$$\sum_{i=1}^{n} p_i X_i \leq \sum_{j=1}^{m} \sum_{h=1}^{H} \theta_j^h \pi^j + \sum_{h=1}^{H} \sum_{i=1}^{n} p_i \omega_i^h \qquad (18)$$



Zero excess demand **Negative excess demand**

Using (5) and (13), (18) can be re-written as:

$$\sum_{i=1}^{n} p_i X_i \leq \sum_{i=1}^{n} \sum_{j=1}^{m} p_i y_i^j + \sum_{h=1}^{H} \sum_{i=1}^{n} p_i \omega_i^h \qquad (19)$$

** see the appendix*

Using (12) and (14), (19) can be expressed as:

$$\sum_{i=1}^{n} p_i X_i(p) \leq \sum_{i=1}^{n} p_i Y_i(p) + \sum_{i=1}^{n} p_i \sum_{h=1}^{H} \omega_i^h \qquad (20)$$

Then using (15), (20) is:

$$\sum_{i=1}^{n} p_i Z_i(p) \leq 0 \qquad (21)$$

Walras' law states that the aggregate value of excess demand is non-positive; if all households are non-satiated, (21) holds with equality, all household income being spent

This implies that if *n-1* markets have zero excess demand, so must the n^{th}

■ A Sketch of Existence

$Z_i(p)$ defines a *mapping* from the set of price vectors into the set of excess demand vectors, where the mapping is *continuous* and *homogeneous of degree zero*

To prove existence, make use of Brouwer's *fixed point theorem*: a continuous mapping of a closed, bounded, convex set into itself, always has a fixed point

P is a set of prices, where an element of the set is a vector of prices $(p_1,...p_n)$ that is bounded below by $p_i \geq 0$, but not from above

To apply the fixed point theorem, need to restrict the price set such that it is bounded and closed - this requires a *normalization* rule

A new price vector is formed with the following rule:

$$p_i{}' = p_i \frac{1}{\Sigma p_i e_i} \qquad i = 1,...,n \tag{22}$$

$e = (e_1,...,e_n) = (1,...,1)$ is a vector whose i^{th} component is one unit of good i, i.e. $\Sigma p_i e_i$ is the cost in \$ at prices p of a bundle of goods consisting of one unit of each good

Normalized set *P′* is bounded, closed and convex, case of $n=2$ illustrated in Figure 1

Positive quadrant is the set *P*, i.e., all pairs of non-negative price vectors (p_1,p_2); *ab* joins price vectors *(0,1)* and *(1,0)*, and is the locus of normalized prices *P′* - any point on a ray such as *0c* will be collapsed onto a single point on *ab*

Figure 2(a) reproduces Figure 1, while Figure 2(b) is the space of excess demand vectors (Z_1,Z_2)

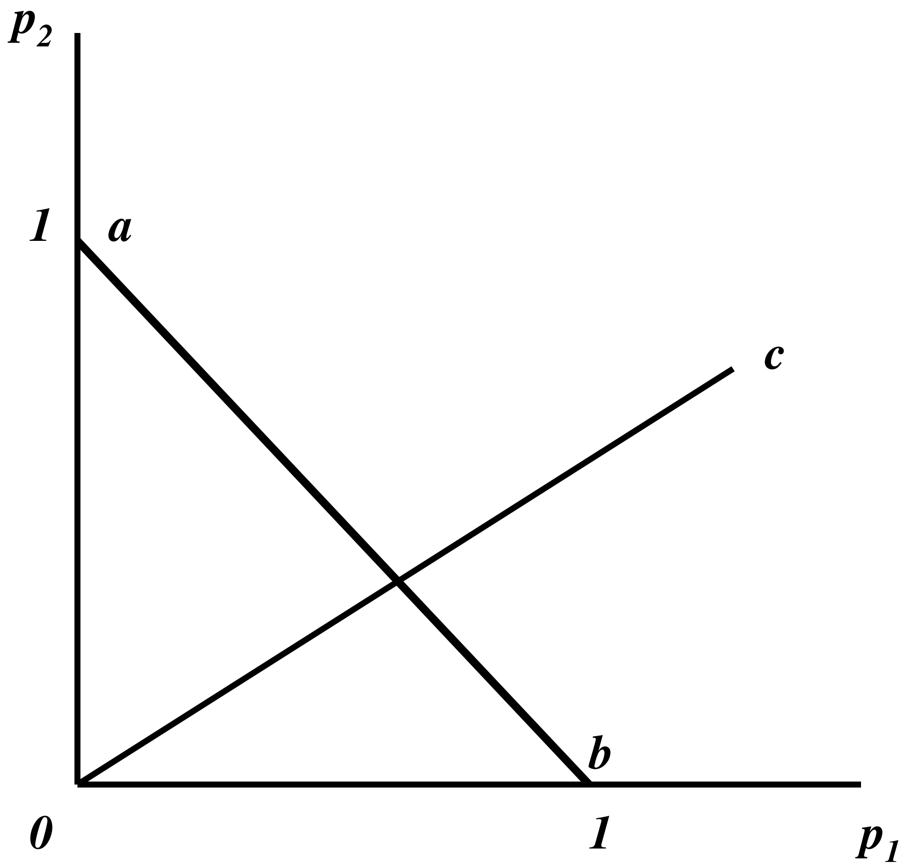
Start with point *a* in (a), which is the price vector *(0,1)*, and assume this is *not* an equilibrium, i.e., the corresponding excess demand vector cannot be at the origin of *(b)*, i.e. the excess demand vector *(0,0)*

From Walras' law, the following must hold:

$$0.Z_1 + 1.Z_2 = 0$$

implying $Z_2 = 0$, and $Z_1 > 0$ - if $Z_1 \leq 0$, *a* will be an equilibrium, which has been ruled out by assumption - so corresponding to price vector *a* must be an excess demand vector such as α; by a similar argument, *b* in (a) maps into point β in (b)

Figure 1: Price Normalization



As successive price vectors are chosen along *ab*, the resulting excess demand vectors lie along a continuous curve in excess demand space such as $\alpha\beta$

Due to Walras' law, excess demands cannot *both* be in the positive or negative quadrants, which means that a continuous curve going from α to β has to pass through the origin, i.e., there must be a price vector that generates the demand vector *(0,0)*, which will be the equilibrium price vector

More generally, a point such as *c* maps into γ, which maps back into *d*; i.e., *c* maps into a point where $Z_2 < 0$, and $Z_1 > 0$, which maps back into *d* where p_2 has fallen and p_1 has risen

Likewise *e* maps into ϵ where $Z_1 < 0$, and $Z_2 > 0$, which maps back into *f* where p_1 has fallen and p_2 has risen

Given mapping of *ab* into itself is continuous, there must be, by Brouwer's theorem, a point such as p^*, which maps back into itself, and is, therefore, the equilibrium price vector

■ Stability of Equilibrium

Interest is in whether system will converge to some equilibrium price vector, given it is initially in disequilibrium

Assume that there is at least one equilibrium price vector p^*, and at time t, there is a price vector $p(0) \neq p^*$; issue of stability concerns under what conditions the time path of the price vector converges on the equilibrium price vector

The general equilibrium system is globally stable if:

$$\lim_{t \to \infty} p(t) = p^*$$

given any initial price vector $p(0)$, and an equilibrium price vector p^*

An adjustment process known as the *tâtonnement* process (groping) is assumed, i.e., an "auctioneer" operates by the following set of rules:

- **a new price vector is announced if the previous vector was not an equilibrium**

- trading is only permitted at equilibrium prices

- rate at which a given price is changed is proportionate to excess demand for that good

$$\frac{dp_i}{dt} = \lambda_i Z_i, \qquad i = 1,\ldots,n. \qquad \lambda_i > 0$$

If $Z_i>0$, p_i is increased, if $Z_i<0$, p_i is decreased, and if $Z_i=0$, p_i is left unchanged

With many goods, negatively sloped excess demand is *not* sufficient to ensure stability

Need to assume that all goods are *gross substitutes*, and this is sufficient to ensure stability, i.e., in a two-good world:

$$\frac{dZ_1}{dp_2} > 0, \qquad \frac{dZ_2}{dp_1} > 0$$

Problems with the *tâtonnement* process:

- explicitly ignores *search* by buyers and sellers when there is no auctioneer calling out prices

- assumes there is no *trading out of equilibrium*

**** see the appendix***