■ Imperfect Competition

- Imperfect Competition and General Equilibrium

Economy based on that derived in Myles (1989), there are n+1 goods, i=0,1,...,n, where good 0 is labor, labor market being competitive, and household endowments consist of labor

i=1,...,K goods are produced by competitive industries under constant returns to scale, i.e., with marginal cost pricing:

$$p_i = c^i p_0, \; i = 1,...,K \tag{1}$$

where p_0 is the wage rate, and c^i the unit labor requirement

Remaining n-K industries are imperfectly competitive, in industry i, there are m_i firms, j=1,...,m_i, and for good i, demand is:

$$X_i = X_i(p_0,...,p_n,\pi) \tag{2}$$

where π is the aggregate level of profits:

$$\pi = \sum_{i=K+1}^{n} \sum_{j=1}^{m_i} \pi_i^j \qquad (3)$$

Assuming (2) is strictly monotonic in p_i, inverse demand curve is:

$$p_i = \varphi_i(p_0,\ldots,p_{i-1},X_i,p_{i+1},\ldots,p_n,\pi) \qquad (4)$$

Firms are assumed to choose output to maximize profits:

$$\pi_i^j = \varphi_i(p_0,\ldots,p_{i-1},X_i,p_{i+1},\ldots,p_n,\pi)x_i^j - C_i^j(x_i^j) \qquad (5)$$

Firms play Cournot-Nash, so outputs of other firms in the industry are taken as given, along with the other arguments in (4), prices on all other markets and profit levels of all firms

Totally differentiating (5):

$$d\pi_i^j\left[1 - x_i^j \frac{\partial \varphi_i}{\partial \pi}\right] = dx_i^j\left[p_i + x_i^j \frac{\partial \varphi_i}{\partial X_i} - \frac{\partial C_i^j}{\partial x_i^j}\right] \qquad (6)$$

** see the appendix*

Assuming the income effect in demand is weak, the first-order condition has to satisfy:

$$p_i + x_i^j \frac{\partial \varphi_i}{\partial X_i} - \frac{\partial C_i^j}{\partial x_i^j} = 0 \qquad (7)$$

Chosen level of output must also satisfy:

$$\pi_i^j - \varphi_i(p_o,\ldots,p_{i-1},X_i,p_{i+1},\ldots,\pi)x_i^j - C_i^j(x_i^j) = 0 \qquad (8)$$

Solving (7) and (8) for the m_i firms in industry i:

$$\pi_i^j = \gamma_i^j\left(p_0,\ldots,p_{i-1},p_{i+1},\ldots,p_n, \sum_{\substack{i'=K+1 \\ i'\neq i}}^{n} \sum_{j'=1}^{m_{i'}} \pi_{i'}^{j'} \right) \qquad (9)$$

$$x_i^j = \sigma_i^j\left(p_0,\ldots,p_{i-1},p_{i+1},\ldots,p_n, \sum_{\substack{i'=K+1 \\ i'\neq i}}^{n} \sum_{j'=1}^{m_{i'}} \pi_{i'}^{j'} \right) \qquad (10)$$

** see the appendix*

(10) can be used to replace aggregate output in (4), and (9) to replace profits in (4):

$$p_i = \Phi_i\left(p_0,\ldots,p_{i-1},p_{i+1},\ldots,p_n, \sum_{\substack{i'=K+1 \\ i'\neq i}}^{n} \sum_{j'=1}^{m_{i'}} \pi_{i'}^{j'} \right) \qquad (11)$$

(1), (9), and (11) define a general equilibrium in the presence of imperfect competition - not necessary to say more about consumption side as it is the same as in a competitive economy

- Imperfect Competition and Welfare
Imperfect competition is a standard example of market failure

Under perfect competition, firms price at marginal cost, examining (7), if $\partial\varphi_i / \partial X_i$ is non-zero, imperfectly competitive firms result in non-achievement of Pareto optimality, i.e., in equilibrium with a single firm in industry i:

$$\frac{p_i + x_i \dfrac{\partial \varphi_i}{\partial X_i}}{p_k + x_k \dfrac{\partial \varphi_k}{\partial X_i}} = \frac{\dfrac{\partial F}{\partial X_i}}{\dfrac{\partial F}{\partial X_k}} \qquad (12)$$

** see the appendix*

- Imperfect Competition and Commodity Taxation

In presence of imperfect competition, motivation for commodity taxation is to reduce welfare loss, as well as raise revenue and re-distribute wealth

Under competition, commodity taxes are passed forward by firms to households, as firms price at marginal cost

In the competitive model, it is the case that:

$$q_i = p_i + t_i, \ i = 1,...,K$$

From which:

$$\frac{\partial q_i}{\partial t_i} = 1, \ and \ \frac{\partial q_i}{\partial t_k} = 0, \ i \neq k$$

Under imperfect competition, firms may or may not fully pass through taxes to households, i.e., *tax incidence* matters

Consider a single industry of m_i firms with the same cost function and similar beliefs, inverse demand facing industry being:

$$q_i = \varphi_i\left(\sum_{j=1}^{m_i} x_i^j, q_k\right) \qquad (13)$$

Each firm aims to maximize:

$$\pi_i^j = x_i^j \varphi_i\left(x_i^j + \sum_{\substack{j'=1 \\ j' \neq j}}^{m_i} x_i^{j'}, q_k \right) - t_i x_i^j - C(x_i^j) \qquad (14)$$

Each firm has a conjecture about how other firms will react to their output choice - this allows us to parameterize problem to a range of outcomes from competition to monopoly

Conjecture is defined as:

$$\mu = \frac{\partial\left(x_i^j + \sum_{\substack{j'=1 \\ j' \neq j}}^{m_i} x_i^{j'} \right)}{\partial x_i^j} \qquad (15)$$

** see the appendix*

$\mu = m_i$ for monopoly, 0 for Bertrand behavior, and 1 for Cournot

Given (15), first-order condition is:

$$\frac{\partial \pi_i^j}{\partial x_i^j} \equiv q_i - t_i + x_i^j \mu \frac{\partial \varphi_i}{\partial X_i} - \frac{\partial C}{\partial x_i^j} = 0 \tag{16}$$

Symmetry assumption implies:

$$q_i = \varphi_i\left(\sum_{j=1}^{m_i} x_i^j, q_k\right) = \varphi_i(m_i x_i, q_k) \tag{17}$$

Totally differentiating (16) by varying all outputs and tax rate:

$$dx_i\left[m_i \frac{\partial \varphi_i}{\partial X_i} + \mu \frac{\partial \varphi_i}{\partial X_i} + x_i m_i \mu \frac{\partial^2 \varphi_i}{\partial X_i^2} - \frac{\partial^2 C}{\partial x_i^2}\right] = dt_i \tag{18}$$

From the inverse demand function:

$$dq_i = m_i \frac{\partial \varphi_i}{\partial X_i} dx_i \tag{19}$$

Using (19) in (18):

$$\frac{dq_i}{dt_i} = \frac{m_i \frac{\partial \varphi_i}{\partial X_i}}{[m_i + \mu]\frac{\partial \varphi_i}{\partial X_i} + x_i m_i \mu \frac{\partial^2 \varphi_i}{\partial X_i^2}} \qquad (20)$$

marginal cost having been assumed constant

If $\mu=0$, (20) equals 1, i.e., tax is fully shifted to households, if μ□0, there can be *under-shifting* or *over-shifting*

Condition for over-shifting is:

$$\frac{\partial \varphi_i}{\partial X_i} > -x_i m_i \frac{\partial^2 \varphi_i}{\partial X_i^2} \qquad (21)$$

(21) is a restriction on the convexity of the inverse demand function, which can be re-arranged as Seade's (1985) *E*:

$$E = \frac{-X_i \frac{\partial^2 \varphi_i}{\partial X_i^2}}{\frac{\partial \varphi_i}{\partial X_i}} \qquad (22)$$

E measures elasticity of slope of the inverse demand function, so for (21):

- if E>1, there is over-shifting
- if E=1, there is full pass-through of tax
- if E<1, there is under-shifting

In terms of effects on profits, write firm *j*'s profits as a function of the tax rate (** see the appendix*):

$$\pi_i^j(t_i) = x_i(t_i)[q_i(t_i) - t_i] - C(x_i(t_i)) \qquad (23)$$

Differentiating profits w.r.t t_i:

$$\frac{d\pi_i^j}{dt_i} = x_i\left[\frac{\partial q_i}{\partial t_i} - 1\right] + \frac{\partial x_i}{\partial t_i}\left[q_i - t_i - \frac{\partial C}{\partial x_i}\right] \qquad (24)$$

Using (16), (18) and (20), (24) becomes:

$$\frac{d\pi_i^j}{dt_i} = \frac{-\left[2\mu x_i \frac{\partial \varphi_i}{\partial X_i} + x_i^2 m_i \mu \frac{\partial^2 \varphi_i}{\partial X_i^2}\right]}{[m_i + \mu]\frac{\partial \varphi_i}{\partial X_i} + x_i m_i \mu \frac{\partial^2 \varphi_i}{\partial X_i^2}} \qquad (25)$$

** see the appendix*

(25) shows profits of firms can actually increase as tax rate increases, a result first demonstrated by Seade (1986), sufficient condition being $E>2$

There are also indirect effects of taxation in an imperfectly competitive industry i due to changes in industry k

Differentiating (16) w.r.t q_k:

$$\frac{dq_i}{dq_k} = \frac{\mu\left[\frac{\partial \varphi_i}{\partial q_k}\left[\frac{\partial \varphi_i}{\partial X_i} + x_i m_i \frac{\partial^2 \varphi_i}{\partial X_i^2}\right] - x_i m_i \frac{\partial \varphi_i}{\partial X_i}\frac{\partial^2 \varphi_i}{\partial X_i \partial q_k}\right]}{[m_i + \mu]\frac{\partial \varphi_i}{\partial X_i} + x_i m_i \mu \frac{\partial^2 \varphi_i}{\partial X_i^2}} \qquad (26)$$

**** see the appendix***

Indirect effect is always non-zero, the change in q_i being of either sign

- Optimal Taxes and Imperfect Competition

Now introduce commodity taxes into general equilibrium model, assuming a single household

General equilibrium with taxation is simultaneous solution to:

$$q_i = c^i p_0 + t_i, \; i = 1,\ldots,K \tag{27}$$

$$q_i = f^i\left(q_0,\ldots,q_{i-1},t_i,q_{i+1},\ldots,q_n, \sum_{\substack{i'=K+1 \\ i'\neq i}}^{n} \sum_{j'=1}^{m_{j'}} \pi_{i'}^{j'} \right), \tag{28}$$

$$i = K+1,\ldots,n$$

$$\pi_i^j = \gamma_i^j\left(q_0,\ldots,q_{i-1},t_i,q_{i+1},\ldots,q_n, \sum_{\substack{i'=K+1 \\ i'\neq i}}^{n} \sum_{j'=1}^{m_{j'}} \pi_{i'}^{j'} \right) \tag{29}$$

$$i = K+1,\ldots,n, j = 1,\ldots,m_i$$

(28) and (29) are solved simultaneously for all imperfectly competitive firms' prices and profits

$$q_{K+1} = \Phi^{K+1}(q_0,\ldots,q_K,t_{K+1},\ldots,t_n)$$

$$\vdots \tag{30}$$

$$q_n = \Phi^n(q_0,\ldots,q_K,t_{K+1},\ldots,t_n)$$

$$\pi^{1}_{K+1} = \Omega^{1,K+1}(q_0,...,q_K,t_{K+1},...,t_n)$$

$$\vdots \qquad (31)$$

$$\pi^{m_n}_{n} = \Omega^{m_n,n}(q_0,...,q_K,t_{K+1},...,t_n)$$

** see the appendix*

The choice of optimal commodity taxes is the solution to:

$$max_{\{t_1,...,t_n\}} \mathcal{L} = V(q_0,...,q_n,\pi) + \lambda\left[\sum_{i=1}^{n} t_i X_i - R\right] \qquad (32)$$

subject to:

$$q_i = p_i + t_i,\ i = 1,...,K \qquad (33)$$

$$q_i = \Phi^i(q_0,...,q_K,t_{K+1},...,t_n),\ i = K+1,...,n \qquad (34)$$

$$\pi^j_i = \Omega^{j,i}(q_0,...,q_K,t_{K+1},...,t_n)\ \ i = K+1,...,n, j = 1,...,m_i \qquad (35)$$

Differentiating w.r.t tax rate of a typical good *k* from the competitive sector gives first-order condition:

$$\frac{\partial V}{\partial q_k} + \sum_{s=K+1}^{n} \frac{\partial V}{\partial q_s}\Phi_k^s + \frac{\partial V}{\partial \pi}\sum_{s=K+1}^{n}\sum_{j=1}^{m_s}\Omega_k^{j,s} +$$

$$\lambda[X_k + \sum_{i=1}^{n} t_i \frac{\partial X_i}{\partial q_k} + \sum_{i=1}^{n}\sum_{s=K+1}^{n} t_i \frac{\partial X_i}{\partial q_s}\Phi_k^s + \qquad (36)$$

$$\sum_{i=1}^{n}\sum_{s=K+1}^{n}\sum_{j=1}^{m_s} t_i \frac{\partial X_i}{\partial \pi}\Omega_k^{j,s}] = 0$$

** see the appendix*

Compared to the competitive model, effects of induced price and profit changes are included, i.e., $\Phi^s{}_k$, and $\Omega^{j,s}{}_k$

Using Roy's identity and Slutsky equation, (36) can be written:

$$\sum_{i=1}^{n} t_i S_{ki} = -\theta X_k + \left[\frac{1}{\lambda}\right]\Gamma^k,\ k = 1,\ldots,K \qquad (37)$$

where:

$$\theta = 1 - \frac{\alpha}{\lambda} - \sum_{i=1}^{n} t_i \frac{\partial X_i}{\partial \pi} \qquad (38)$$

and:

$$\Gamma^k = \alpha \sum_{s=K+1}^{n} X_s \Phi_k^s - \alpha \sum_{s=K+1}^{n} \sum_{j=1}^{m_s} \Omega_k^{j,s}$$
$$+ \lambda \sum_{i=1}^{n} \sum_{s=K+1}^{n} t_i \frac{\partial X_i}{\partial q_s} \Phi_k^s + \lambda \sum_{i=1}^{n} \sum_{s=K+1}^{n} \sum_{j=1}^{m_s} t_i \frac{\partial X_i}{\partial \pi} \Omega_k^{j,s} \qquad (39)$$

If $[1/\lambda]\Gamma^k$ were not included, (37) would be identical to the standard Ramsey rule

(39) is determined by the induced price and profit effects not present in competitive model, reduction in compensated demand for good *k* is smaller when a tax on this good:

- increases prices of imperfectly competitive goods $\Phi^s_k > 0$

- reduces profits $\Omega^{j,s}_k < 0$

- induced tax changes lower tax revenue

Derivation can be repeated for a typical good *k* from the imperfectly competitive sector:

$$\frac{\partial V}{\partial q_k}\Phi_k^k + \sum_{\substack{s=K+1 \\ s\neq k}}^{n} \frac{\partial V}{\partial q_s}\Phi_k^s + \frac{\partial V}{\partial \pi}\sum_{s=K+1}^{n}\sum_{j=1}^{m_s}\Omega_k^{j,s} +$$

$$\lambda\,[X_k + \sum_{i=1}^{n}\sum_{s=K+1}^{n} t_i \frac{\partial X_i}{\partial q_s}\Phi_k^s + \qquad (40)$$

$$\sum_{i=1}^{n}\sum_{s=K+1}^{n}\sum_{j=1}^{m_s} t_i \frac{\partial X_i}{\partial \pi}\Omega_k^{j,s}\,] = 0$$

** see the appendix*

which after using Roy's identity and the Slutsky equation becomes:

$$\sum_{i=1}^{n} t_i S_{ki} = -\theta^k X_k + \left[\frac{1}{\lambda}\right]\Gamma^k,\; k = K+1,...,n \qquad (41)$$

where:

$$\theta^k = \frac{1}{\Phi_k^k} - \frac{\alpha}{\lambda} - \sum_{i=1}^{n} t_i \frac{\partial X_i}{\partial \pi} \qquad (42)$$

and:

$$\Gamma^k = \frac{1}{\Phi_k^k}[\alpha \sum_{\substack{s=K+1 \\ s\neq k}}^{n} X_s \Phi_k^s - \alpha \sum_{s=K+1}^{n} \sum_{j=1}^{m_s} \Omega_k^{j,s}$$

$$+\lambda \sum_{\substack{i=1 \\ i\neq k}}^{n} \sum_{s=K+1}^{n} t_i \frac{\partial X_i}{\partial q_s} \Phi_k^s + \lambda \sum_{i=1}^{n} \sum_{s=K+1}^{n} \sum_{j=1}^{m_s} t_i \frac{\partial X_i}{\partial \pi} \Omega_k^{j,s}] \tag{43}$$

Again $[1/\lambda]\Gamma^k$ is a correction to the Ramsey rule

Key here is that result is dependent on good *k* under consideration due to tax-shifting term in (43) - the greater the extent of over-shifting, the smaller the reduction in compensated demand

Market conduct is as important as tastes in determining relative rates of taxation