■ Finance of Public Goods

- if lump-sum taxes are infeasible, level of public good provision must take account of the method of finance

- gain in welfare due to provision of public good must be offset by any distortions due to method of finance

- focus on the case of a single-consumer economy where only commodity taxes are available

- each consumer maximizes utility function $U(x,G)$, subject to a budget constraint $qx = 0$, where q is vector of post-tax prices and x is a vector of net demands

- government revenue must equal expenditure on the public good:

$$H\sum_{i=1}^{n} t_i x_i = G \tag{28}$$

- revenue constraint can be used interchangeably with the production constraint because of market clearing

- **production constraint is used, where $F(X,G) = F(H_x,G) = 0$, and assumed that:**

$$F_1 = \frac{\partial F}{\partial X_1} = 1 \tag{29}$$

good 1 is chosen as the numeraire, $q_1 = p_1 = 1$, and pre-tax prices are chosen so that $F_k = p_k$

- **the Lagrangean is:**

$$\mathcal{L} = HV(q,G) - \lambda F(X(q,G),G) \tag{30}$$

- **first-order condition for choice of G is:**

$$\frac{\partial \mathcal{L}}{\partial G} \equiv H\frac{\partial V}{\partial G} - \lambda\left[\sum_{i=1}^{n} F_i \frac{\partial X_i}{\partial G} + F_G\right] = 0 \tag{31}$$

- **using definition of pre-tax prices, (31) becomes:**

$$H\frac{\frac{\partial V}{\partial G}}{\alpha q_k} = \frac{p_k}{q_k}\frac{\lambda}{\alpha}\frac{F_G}{F_k} + \frac{\lambda}{\alpha q_k}\sum_{i=1}^{n} p_i \frac{\partial X_i}{\partial G} \tag{32}$$

**see the appendix*

- **where α is marginal utility of income for each household, and from each households' first-order condition, $\partial U/\partial x_k = \alpha q_k$, Hence,**

$$H \frac{\frac{\partial V}{\partial G}}{\alpha q_k} = H \frac{\frac{\partial U}{\partial G}}{\frac{\partial U}{\partial x_k}} \qquad (33)$$

The LHS of (32) is the sum of the marginal rates of substitution between public good and private good *k* (equals the ratio of utility derivatives)

- evaluating (32) for *k*=1, so that q_1=1, it can be re-arranged:

$$\frac{F_G}{F_1} = \frac{\alpha}{\lambda} H \frac{\frac{\partial U}{\partial G}}{\frac{\partial U}{\partial x_1}} - \sum_{i=1}^{n} [q_i - t_i] \frac{\partial X_i}{\partial G} \qquad (34)$$

- consumer budget constraints imply:

$$\sum_{i=1}^{n} q_i \frac{\partial X_i}{\partial G} = 0$$

- re-writing (34):

$$MRT_{G,1} = \frac{\alpha}{\lambda}\sum_{h=1}^{H} MRS_{G,1}^{h} + \frac{\partial \sum_{i=1}^{n} t_i X_i}{\partial G} \qquad (35)$$

- (35) is the Samuelson rule in the presence of distorting commodity taxes, which is different to (7) in two ways:

(i) the sum of the marginal rates of substitution is multiplied by α/λ which may not be =1

(ii) an additional term on right-hand side that measures effect of public good provision on tax revenue due to substitutability/complementarity in demand between private goods and public good

- latter effect implies that if provision of public good increases tax revenue, i.e., if it is a complement to private goods, this reduces cost of providing the public good

- this tends to increase provision above Samuelson level; converse is true if provision of public good reduces tax revenue

- to isolate the first effect, assume the second term on right-hand side is zero, i.e., public good is revenue neutral, so *α/λ* determines departure from first-best

- from the Lagrangean, tax rate for good *k* is:

$$H \frac{\partial V}{\partial q_k} = \lambda \sum_{i=1}^{n} F_i \frac{\partial X_i}{\partial q_k} = \lambda \frac{\partial \sum_{i=1}^{n} p_i X_i}{\partial t_k} \qquad \textbf{(36)}$$

- using Roy's identity, and fact that:

$$\frac{\partial \sum_{i=1}^{n} p_i X_i}{\partial t_k} + \frac{\partial \sum_{i=1}^{n} t_i X_i}{\partial t_k} = \frac{\partial \sum_{i=1}^{n} q_i X_i}{\partial t_k} = 0 \qquad \textbf{(37)}$$

- (36) is re-written:

$$\frac{\alpha}{\lambda} = \frac{\dfrac{\partial \sum_{i=1}^{n} t_i X_i}{\partial t_k}}{X_k} \qquad \textbf{(38)}$$

* *see the handout*

- using the Slutsky equation:

$$\frac{\alpha}{\lambda} = 1 - \sum_{i=1}^{n} t_i \frac{\partial X_i}{\partial I} + \sum_{i=1}^{n} t_i \frac{S_{ik}}{X_k} \qquad \textbf{(39a)}$$

** see the handout*

the divergence of *α/λ* from 1 is separated into two parts, the first a revenue effect, the second a distortionary effect, i.e.:

$$\sum_{i=1}^{n} t_i \frac{\partial X_i}{\partial I} \quad \textit{and} \quad \sum_{i=1}^{n} t_i \frac{S_{ik}}{X_k}$$

- the revenue effect cannot be unambiguously signed, unless all goods are normal, and the effect is positive, so that $\alpha < \lambda$

- as the Slutsky matrix is negative semi-definite, it follows that the distortionary effect is negative, so α tends to be reduced below λ

- implies that the true benefit of the public good is less than the sum of the marginal rates of substitution

■ Level of Provision in Presence of Taxes

- can the analysis outlined answer question: will more or less of public good be provided in presence of distortionary taxation?

- consider an economy where finance is possible through a lump-sum tax *T* or a tax *t* levied upon a single factor of production *L*, the Lagrangean being:

$$\mathfrak{L} = HV(t,T,G) + \lambda[HT + tHL - G] \qquad \textbf{(39b)}$$

* *see the handout*

with first-order conditions:

$$V_t + \lambda\left[L + t\frac{\partial L}{\partial t}\right] = o \qquad \textbf{(40)}$$

$$HV_G - \lambda = 0 \qquad \textbf{(41)}$$

$$HT + HtL - G = 0 \qquad \textbf{(42)}$$

where *T* can vary from 0 to *G*

-in Figure 8, (41) can be drawn as the "demand" curve for public goods, $\lambda = HV_G$

$$\lambda = \alpha/(1 + \frac{t}{L}\frac{\delta L}{\delta t}) \tag{43}$$

where t is a function of G from (42)

- in the case of lump-sum taxes, $\lambda = \alpha$

-in Figure 8, for $G_{CT} < G_{LS}$, requires that $\lambda_{CT} > \lambda_{LS}$ ($= \alpha_{LS}$); Stiglitz and Dasgupta have shown that $\lambda_{CT} > \alpha_{CT}$, if the supply curve of labor is upward sloping, but this is not sufficient to show $G_{CT} < G_{LS}$, has to be shown $\alpha_{CT} > \alpha_{LS}$

-Atkinson and Stern have shown provision is lower with commodity taxation with utility function:

$$U(X, L, G) = a \log X + [1 - a] \log[1 - L] + f(G) \tag{44}$$

* *See the appendix*

Figure 8: Lump-sum vs. Commodity Taxes