■ Private Provision

- contrast so-called "first-best" outcome of Lindahl equilibrium with case of private provision through voluntary contributions of households

- need to make an assumption about how each household expects their contribution to affect contributions of others - standard Nash assumption made

- in planning contribution to public good provision, each household takes contribution of other households as given

- similar set of basic assumptions used as with the Lindahl equilibrium

- utility function is:

$$U^h = U^h(x^h, G), \quad h = 1, \ldots, H \tag{21}$$

x^h is private good, $G = \Sigma g^h$ and g^h is contribution of household h

- contribution of households other than h, $\hat{G}_h$, is defined:

$$\hat{G}_h = G - g^h \tag{22}$$

- using the budget constraint in (16), utility can be written as:

$$\begin{aligned} U^h(x^h, G) &= U^h(\omega^h - p_G g^h, g^h + \hat{G}_h) \\ &= V^h(g^h, \hat{G}_h, p_G) \end{aligned} \tag{23}$$

- household h maximizes (23) given $\hat{G}_h$ and subject to:

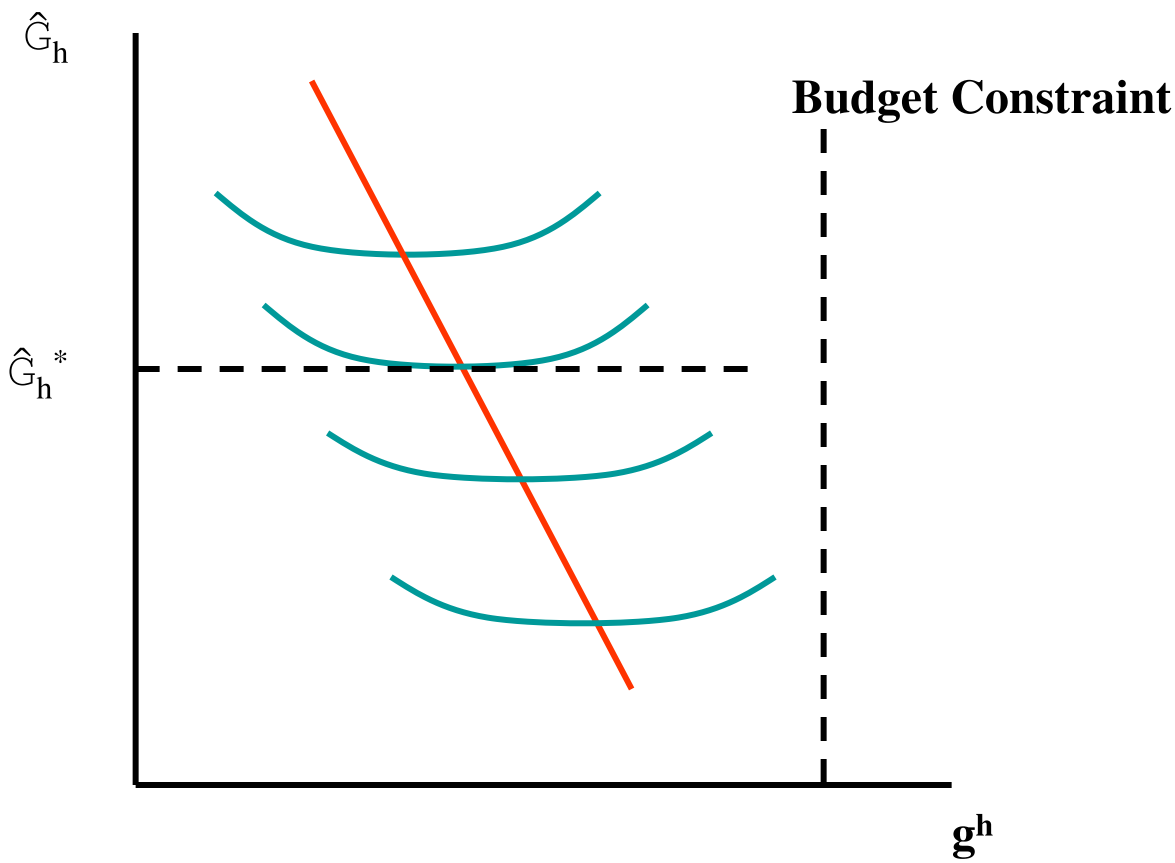
$$g^h \in \left[0, \frac{\omega^h}{p_G}\right]$$

- indifference curves of V^h can be drawn in $(g^h, \hat{G}_h)$ space; increasing $\hat{G}_h$ leads to a higher V for given g^h, the latter limited by the budget constraint

- given the Nash assumption, optimal choice of g^h for a given $\hat{G}_h$ occurs at a tangency of an indifference curve to line $\hat{G}_h^*$ as shown in Figure 3

- locus of these points traces out the Nash reaction function, which slopes down

Figure 3: Indirect Preferences over Private Provision



- for all g^h on the reaction function:

$$g^h = \operatorname{argmax} U^h(\omega^h - p_G g^h, g^h + \hat{G}_h)$$

$$-U_x^h p_G + U_G^h = 0 \tag{24}$$

- variations in g^h and $\hat{G}_h$ satisfying the first-order condition (24) are:

$$\frac{dg^h}{d\hat{G}_h} = \frac{U_{xG}^h p_G - U_{GG}^h}{U_{xx}^h p_G^2 - 2U_{xG}^h p_G + U_{GG}^h} \tag{25}$$

(25) is always negative when $U^h_{xG} > 0$.

- equilibrium of private provision economy is where all reaction functions are simultaneously satisfied

- for a two-household economy, private provision equilibrium is shown in Figure 4

- a property of Nash equilibria is that they are typically not Pareto-efficient; in Figure 5, Pareto-efficient allocations are points of tangency between households' indifference curves - private provision generates under-supply

Figure 4: Private Provision Equilibrium

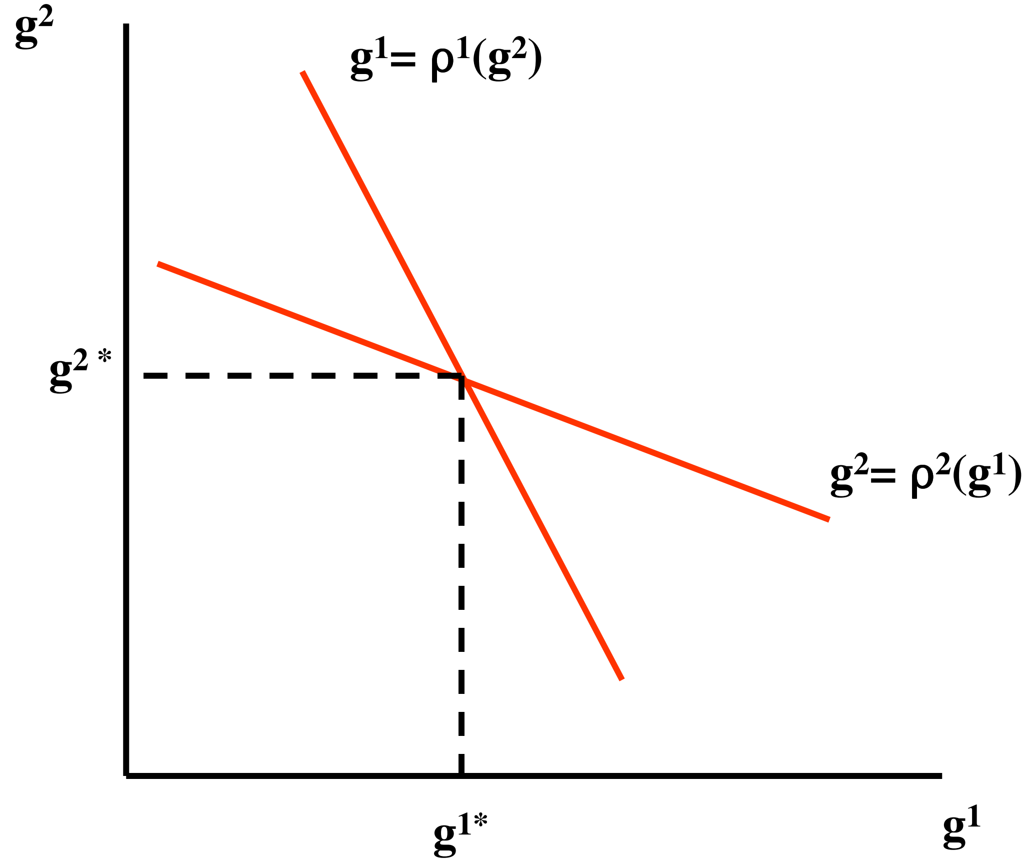
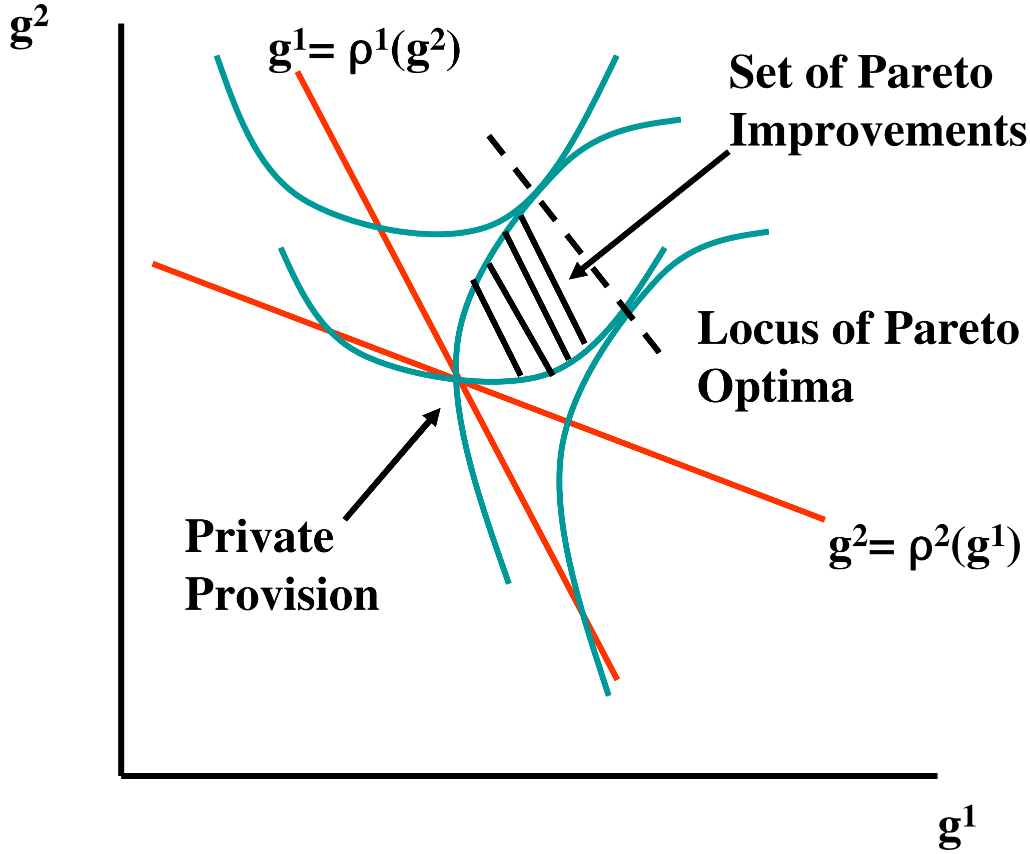


Figure 5: Non-Optimality of Private Provision



■ Variations in Provision

(i) Quantity

- result that private provision equilibrium is dominated by other allocations does *not* necessarily mean private provision leads to under-supply relative to social optimum

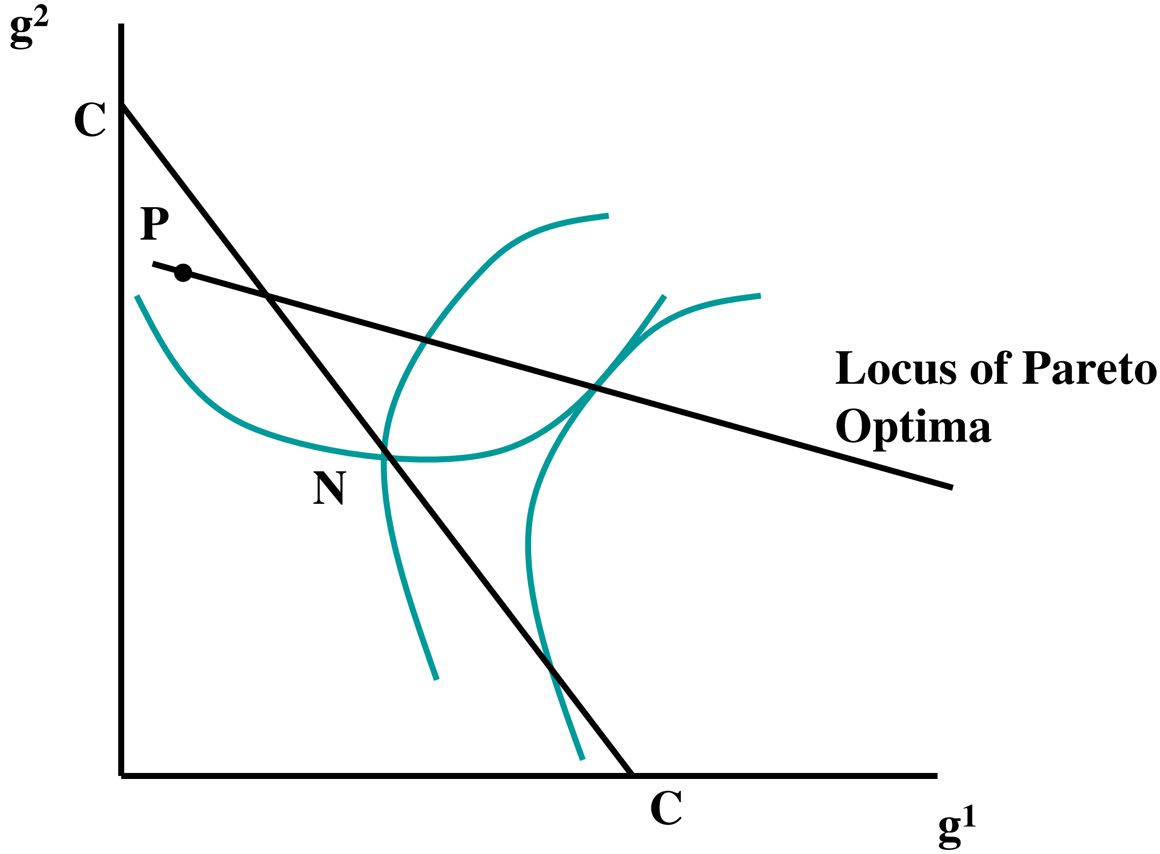
- this follows from notion that a social optimum is not necessarily on that part of locus that Pareto dominates the private provision equilibrium

- in Figure 6, private provision equilibrium is at N, and set of Pareto efficient allocations given by locus of tangencies between indifference curves

- CC is an aggregate level of public good supply equal to that at N, so if locus of Pareto optima cuts CC, and social optimum is at P, there will be a reduction in provision of public good

- Diamond and Mirlees (1973) show such anomalies can only be ruled out if restrictions are placed on the second derivatives of households' utility functions

Figure 6: Private Provision and Over-Supply



(ii) Number of Households

- might be expected that an increase in number of households would lead to greater divergence between private provision and optimal provision

- assume all households are identical in terms of preferences and endowments, so that with *H* households, the symmetric equilibrium is:

$$g = \frac{\hat{G}}{H - 1} \qquad (26)$$

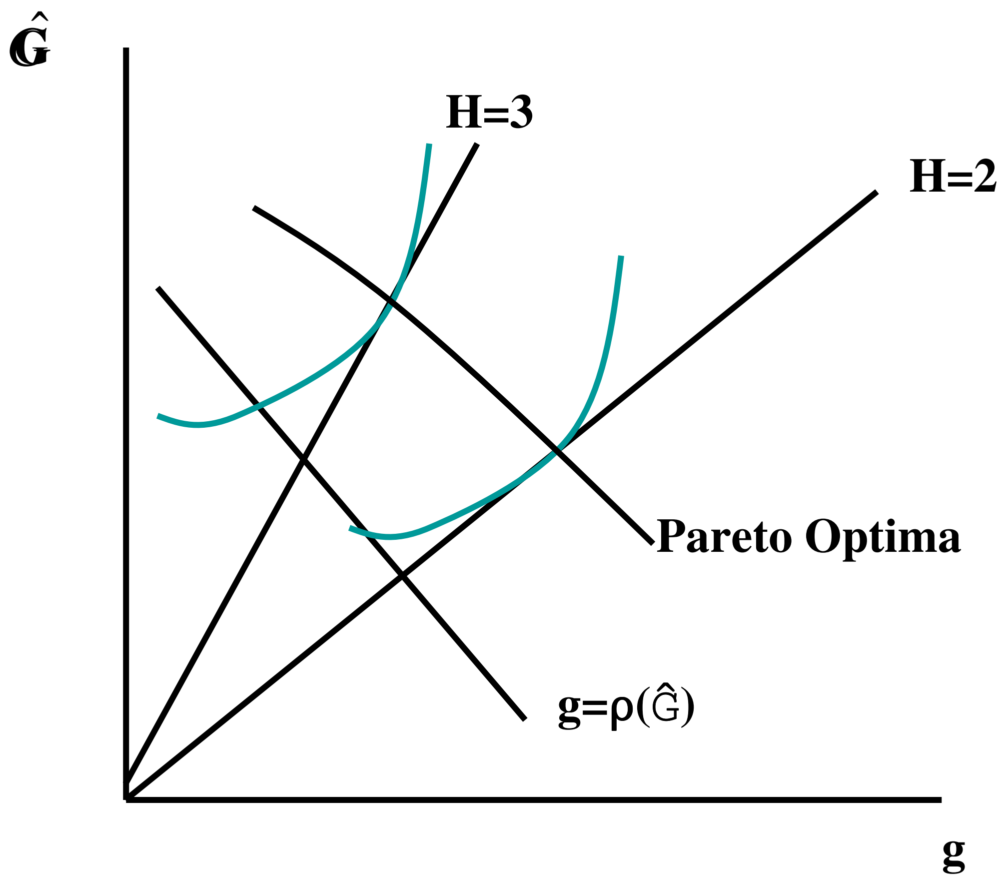
where *g* is common contribution of a household; in *g,* Ĝ space, an allocation satisfying (26) must lie on a ray from the origin with slope of (*H*-1)

- in Figure 7, for a level of *H,* private provision where ray cuts reaction function, and welfare optima are where ray is tangent to an indifference curve

- quantity of public good at private provision varies in *H* as function of the gradient of the reaction function, i.e., if gradient is < 1 (>1), provision *Hg*, is an increasing (decreasing) function of *H*

**** See the appendix***

Figure 7: Equilibria and Optima for H



(iii) Alternative Formulations

- notion that an increase in contribution of one household leads to a reduction in that of others has been criticized

- behavioral assumptions other than Nash have been investigated, such that in (24), household's consider the choices of others as being dependent on their decision, i.e., a conjectural variation:

-first-order condition becomes:

$$- U_x^h p_G + U_G^h \left[1 + \sum_{j=1, j \neq h}^{H} \frac{\partial g^j}{\partial g^h} \right] = 0 \qquad (27)$$

* *see the appendix*

- Cornes and Sadler (1984) show that if conjectures are positive, equilibrium has greater private provision of the public good

- if conjectures are consistent, i.e., they agree with actual responses of households, Sugden (1985) has shown the only consistent conjectures are negative, so that zero provision of the public good is possible

- conjectural variations are ultimately arbitrary

■ Mechanism Design

-analysis of the Lindahl equilibrium assumed households were honest in revealing their reactions to cost shares for a public good

- a household can play strategically and misrepresent their preferences, assuming other households do not do likewise

- in Figure 9, *EL* is the Lindahl equilibrium if households play honestly

- suppose, household 2 knows household 1's preferences, and claims its preferences to be $L^{2\prime}(\tau^2)$, rather than $L^2(\tau^2)$, equilibrium can be pushed to E′, where household 2 maximizes utility subject to household 1's reaction function

-household 1's reaction function can be thought of as inverse supply curve for public good facing household 2 – measures per unit or average cost to household 2

- *m*(*G*) is marginal cost to household 2 of public good, so household 2 chooses level of *G* where marginal value of public good from true reaction function is
just equal to marginal cost, and reports $L^{2\prime}(\tau^2)$, giving G´ and *τ´*

- household 2' tax bill is reduced by *abG*G´*, which offsets their reduction in consumption of public good *acGG´*

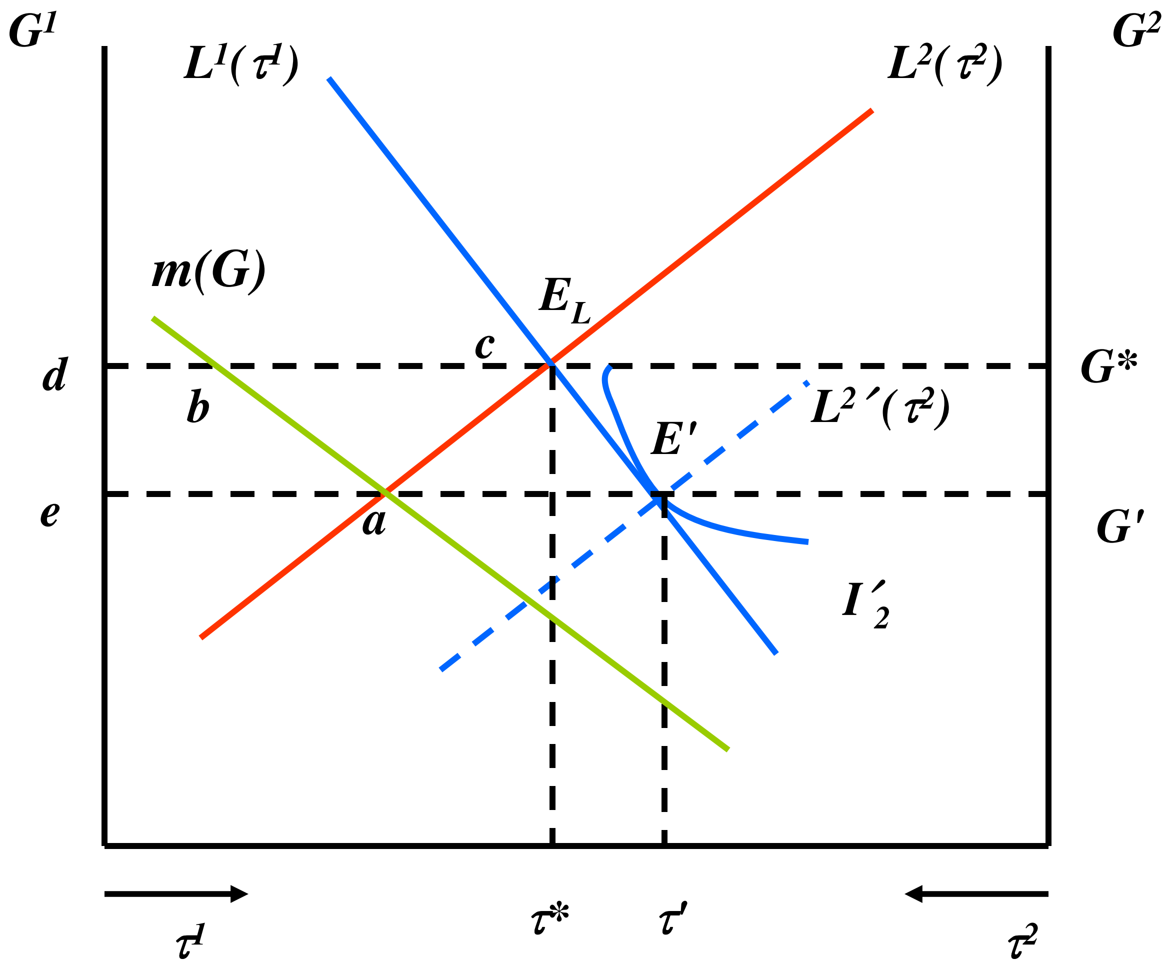
- cost saving on public good is *edG*G´*, losses to household 1 is *cdeE´* and *acG*G´* for household 2, so efficiency loss is *acE´*

-due to this type of result, there has been a focus design of *preference revelation* mechanisms

-these are games where each household chooses a strategy to maximize their payoff, the game often being one of incomplete information, households only having knowledge of their own payoff function

-a number of equilibrium concepts have been employed in the literature including dominant strategies, Nash, and more recently Bayesian

Figure 9: Manipulating the Lindahl Equilibrium



■ Clarke-Groves-Vickrey (CGV) Mechanism

Board asks individuals to report their marginal valuations $\hat{B}'_i(G)$ of different amounts of public good, and supply is given by equating summed marginal valuations to marginal cost of the public good:

$$\sum_i \hat{B}'_i(G) = c \qquad (1)$$

First part of CGV tax is a per unit tax price t_i (with $\sum_i t_i = c$) which is fixed and cannot be altered by individual *i*'s announcement

Second part of tax on *i* is determined by calculating G_{-i} the amount of the public good which would be supplied if *i* announced a constant marginal valuation equal to their tax price:

$$\sum_{j \neq i} \hat{B}'_j(G_{-i}) + t_i = c \qquad (2)$$

With *i*'s announcement of $\hat{B}'_i$, supply of the public good changes to *G*, satisfying:

$$\sum_j \hat{B}'_j(G) = c \qquad (3)$$

Second part of tax on *i* is change in reported benefits to all other individuals, less their payments of per unit taxes:

$$T_i(G) = \sum_{j \neq i}[\hat{B}_j(G_{-i}) - \hat{B}_j(G)] - \sum_{j \neq i} t_j [G_{-i} - G] \quad (4)$$

This calculation is possible because $\hat{B}_j(G_{-i}) - \hat{B}_j(G)$ is the area under individual *j*'s reported demand curve or marginal valuation schedule between G_{-i} and *G*

Under the CGV tax, individual *i* has utility of:

$$B_i(G) + x_i - t_i G - T_i(G) \quad (5)$$

They choose *G* to maximize (5), so *G* satisfies:

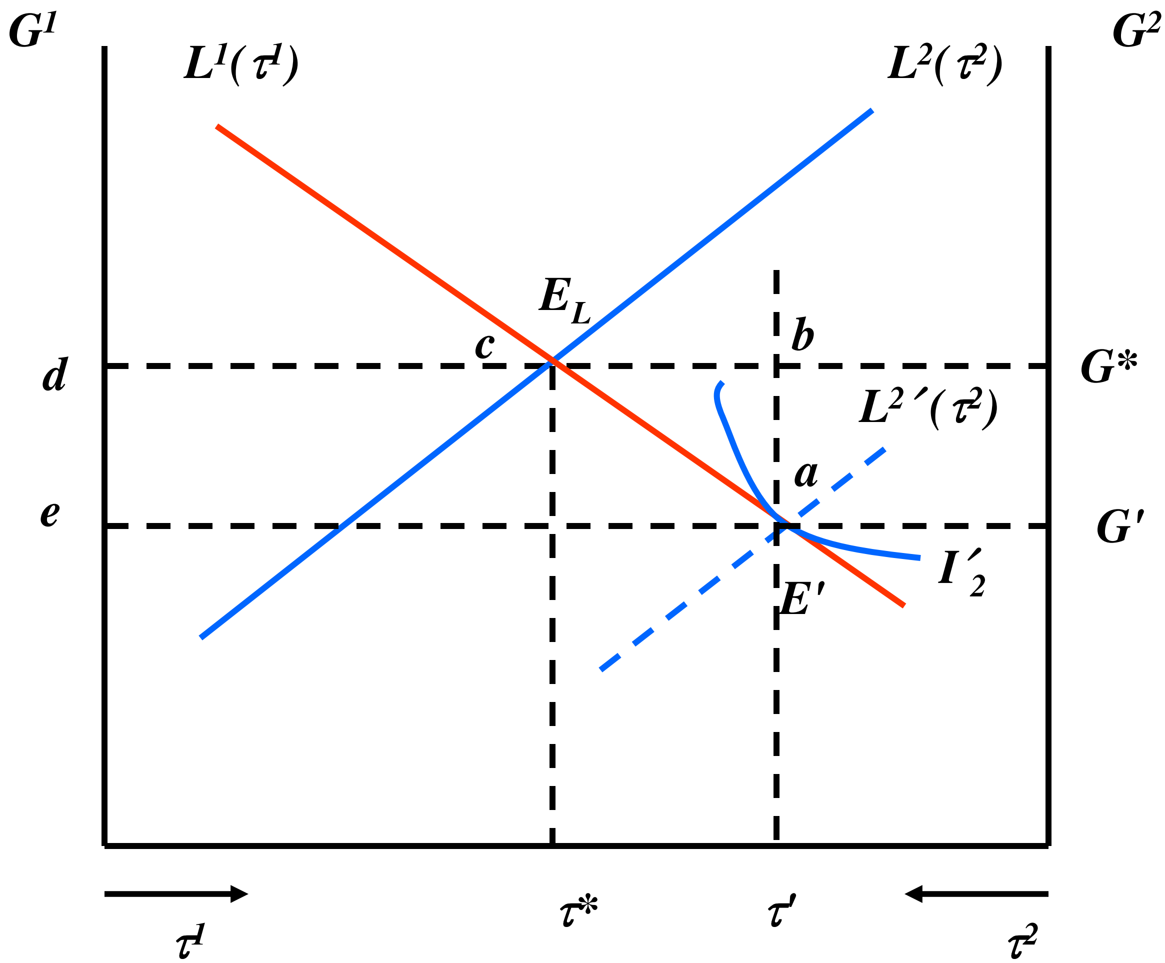
$$B'_i(G) - t_i - \frac{dT_i(G)}{dG} = B'_i(G) - t_i + \sum_{j \neq i} \hat{B}'_j(G) - \sum_{j \neq i} t_j$$

$$= B'_i(G) + \sum_{j \neq i} \hat{B}'_j(G) - c$$

Thus, *i* chooses a level of public good which is efficient given announced valuations of other individuals, i.e., CGV mechanism is a dominant strategy mechanism

As all are motivated to reveal true marginal valuations, there is an efficient supply of the public good

Figure 10: The CGV Mechanism



CGV mechanism is illustrated in figure 10

Suppose tax price is $t_i = \tau'$, $G_{-2} = G'$, where $B'_1 + \tau' = c$

If supply is increased to G^*, individual 1 will have a gross benefit equal to *acde*, and pays extra taxes of *abde*, therefore, $T_2(G^*) = abc$

In order to increase G to G^*, individual 2 has to pay additional taxes of $acG^*G' = \tau'(G^*-G') + T_2(G^*)$

acG^*G' is also the social cost of the increase in G, i.e., the difference in the resource cost $G'G^*de$, and its gross value to individual 1 of *acde*

The CGV mechanism confronts individual 2 with a marginal cost of the public good equal to the height of individual 1's demand curve, so that they choose G^*, where their true marginal valuation cuts the marginal cost curve under the mechanism

There are two key features of the process:

- **per unit taxes τ' do not change with individual's announcement, so they can be set arbitrarily, as long as they add up to c**

- **as per unit taxes cover cost of public good, there is surplus of $\sum_i T_i$, which should be wasted if there is correct revelation, although this becomes arbitrarily small as the population of individuals becomes very large**