

APPLIED WELFARE ECONOMICS AND POLICY ANALYSIS

■ Measurement of Income

- in general equilibrium model, household income is value of endowment plus dividend income

- income then is value of consumption that can be undertaken, while leaving household with same stock of goods, i.e., zero, at end of the economy as at the start

- Fisher (1930) proposed income was set of pleasurable experiences over period of measurement

- as this would be difficult to measure, Fisher proposed as an approximation - level of expenditure, less disutility of labor

- Hicks (1939) provided standard definition of income,

“...income is the maximum value which a man can consume during a week and still expect to be as well-off at the end of the week as he was at the beginning...”

- Hicks' definition embodies consumption aspect emphasized by Fisher, and reduces to the stock-of-goods measure in a static environment

- difficulty in applying definition lies in the word *expect*, which gives the concept a forward-looking nature, e.g., a literal application of definition would not count windfall gains as income as they are not expected

- Simons (1938) adopts a workable definition, measuring income *ex post*,

“...Personal income may be defined as the algebraic sum of (1) the market value of rights exercised in consumption and (2) the change in the value of the store of property rights between the beginning and end of the period in question...”

- these definitions highlight *ex ante* versus *ex post* ways of measuring income; typically tax assessment adopts a backward-looking view, measuring income as all payments received over stated period

■ Equivalence Scales

- income level of a household often treated as a proxy for its level of welfare, as income is means to achieve welfare

- as household composition varies, need to use equivalence scales as a means of adjusting income into comparable quantities - i.e., need to adjust observed household income to take account of *demographic* differences

(i) Minimum Needs

- this is an approach based on identifying minimum needs household requires to survive at some chosen welfare level - typically just above poverty

- calculation of such scales traced back to Rowntree (1901)

- scales calculated by determining cost of a bundle of goods and services representing minimum household needs - Rowntree's bundle included food, rent and property taxes, and household sundries

- many studies since Rowntree employing methodology (Table 1), in each case a household with zero children is assigned an index value of 100

- equivalence scales assume there are returns to scale in household size

Table 1

Minimum Needs Scales			
	Rowntree (1901)	Beveridge (1942)	US Poverty Scale (1942)
Single person	60	59	78
Couple	100	100	100
+ 1 child	124	122	123
+ 2 children	161	144	152
+ 4 children	222	188	208

Note: For the Beveridge scale, children are taken to be in the 5-9 age group

- shortcoming of scales is that by focusing on cost of minimum level of consumption, inappropriate for applying to incomes above minimum level

- do not take account of optimization by households, so do not measure true economic cost of demographic differences

(ii) Engel and Rothbarth

- equivalence scales for Engel and Rothbarth based on observation:**
- if welfare level of a household can be judged by consumption of a specific commodity, equivalent incomes lead different households to consume same quantity of commodity**
- Engel's (1895) approach rests on view that welfare can be measured by proportion of income spent on food - i.e., appeals to *Engel's law***
- equivalence scales constructed for demographically different households by calculating income levels where expenditure share on food is equal**
- Figure 1 shows, using Engel method, that incomes M^1 and M^2 are equivalent, the scale being ratio M^2/M^1 , where d^1 and d^2 are demographic characteristics**
- two problems with the Engel approach:**

(a) scale alone does not provide a basis for welfare comparisons

(b) Engel method over-estimates cost of additional children in a household

- Rothbarth (1943) procedure selects “adult goods” such as tobacco and alcohol as measure of welfare, where changes in demographic characteristics only affect demand via income effects

- Figure 2 illustrates procedure for x^* the chosen consumption level, and three different households, M^1, M^2, M^3 being equivalent incomes

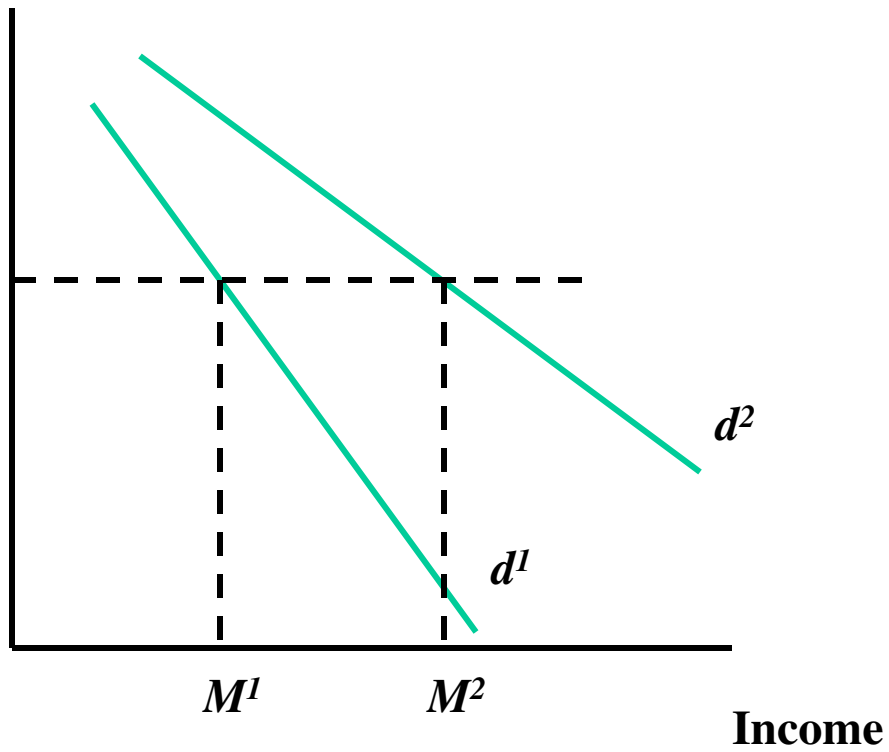
- problems with this approach:

(a) rests upon hypotheses that consumption of adult goods accurately measures welfare, and that they are affected only via income effects

(b) ratios of M^1 to M^2 and M^3 depend on level of demand chosen for comparison - except in special case where Engel curves are straight lines through origin

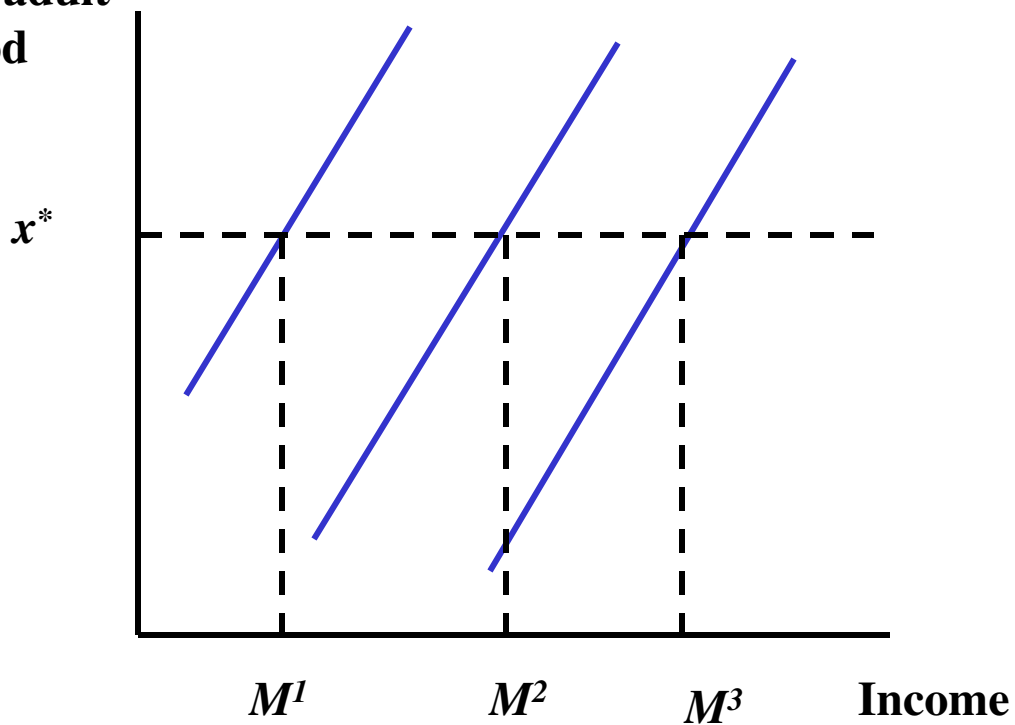
Figure 1: Engel Scale

**Expenditure
share on
food**



**Demand
for adult
good**

Figure 2: Rothbarth Scale



(iii) Barten

- shortcomings of methods due to lack of choice-theoretic foundation, Barten (1964) approached derivation via a direct utility function, method extended using duality by Gorman (1978) and Muellbauer (1974)

- certain goods may not be consumed by children, while children consume relatively large amounts of food

- let household of type d be equivalent to $a^i d$ adults for consumption of good i , $i=1, \dots, n$, household utility is then:

$$U^h = U^h \left(\frac{x_1^h}{a^1(d)}, \dots, \frac{x_n^h}{a^n(d)} \right) = U^h(\bar{x}_1^h, \dots, \bar{x}_n^h) \quad (1)$$

$$\text{where } \bar{x}_i^h = \frac{x_i^h}{a^i(d)}$$

- household chooses quantities of goods to maximize (1), subject to a budget constraint:

$$M^h = \sum_{i=1}^n \bar{p}_i \bar{x}_i^h \quad (2)$$

where $\bar{p}_i = a^i(d)p_i$

“...when you have a wife and a baby, a penny bun costs threepence...” (Gorman)

- solving the maximization problem gives the demand functions:

$$\bar{x}_i^h = \bar{x}_i^h(\bar{p}, M^h) \quad (3)$$

$$x_i^h = a^i(d) \bar{x}_i^h(p_1 a^1(d), \dots, p_n a^n(d), M^h)$$

* see the appendix

- change in demographic composition has two effects on demand:

(i) directly via the equivalence term $a^i(d)$

(ii) indirectly via equivalence term affecting demographically adjusted prices

- in elasticity form, effect of a change in demographic characteristic k is:

$$\frac{d_k}{x_i^h} \frac{\partial x_i^h}{\partial d^k} = \frac{\partial a^i}{\partial d_k} \frac{d_k}{a^i} + \sum_{j=1}^n \left[\frac{p_j}{x_i^h} a^i \frac{\partial \bar{x}_i^h}{\partial p_j} \right] \left[\frac{d_k}{a^j} \frac{\partial a^j}{\partial d_k} \right] \quad (4)$$

* see the appendix

(4) provides a basis for estimating effects of household composition on demand elasticities

- to construct the equivalence scale, expenditure function dual to (1) is defined:

$$E^h(\bar{p}, U^h) = \min \{ \bar{p} \bar{x}^h \text{ subject to } U^h(\bar{x}^h) \geq U^h \} \quad (5)$$

$$E^h(\bar{p}, U^h) = E^h(a^1(d)p_1, \dots, a^n(d)p_n, U^h) \quad (6)$$

- hence, expenditure function captures demographic information; for some given level of welfare, equivalence scale for two households of different compositions d' and d'' :

$$\frac{E^h(a^1(d')p_1, \dots, a^n(d')p_n, \bar{U}^h)}{E^h(a^1(d'')p_1, \dots, a^n(d'')p_n, \bar{U}^h)} \quad (7)$$

- if (1) is correct, (7) gives an exact equivalence scale, as it measures true cost of demographics
- 2-good case is illustrated in Figure 3; outward shift in indifference curve caused by increase in number of family members requiring increase in household consumption to keep utility constant
- extent of shift in budget line determines extra income required to compensate for change in demographics
- specification of utility is quite precise, rejected in econometric tests (Muellbauer, 1977), also assumes demographic variables are exogenous which may be inappropriate (see Myles)

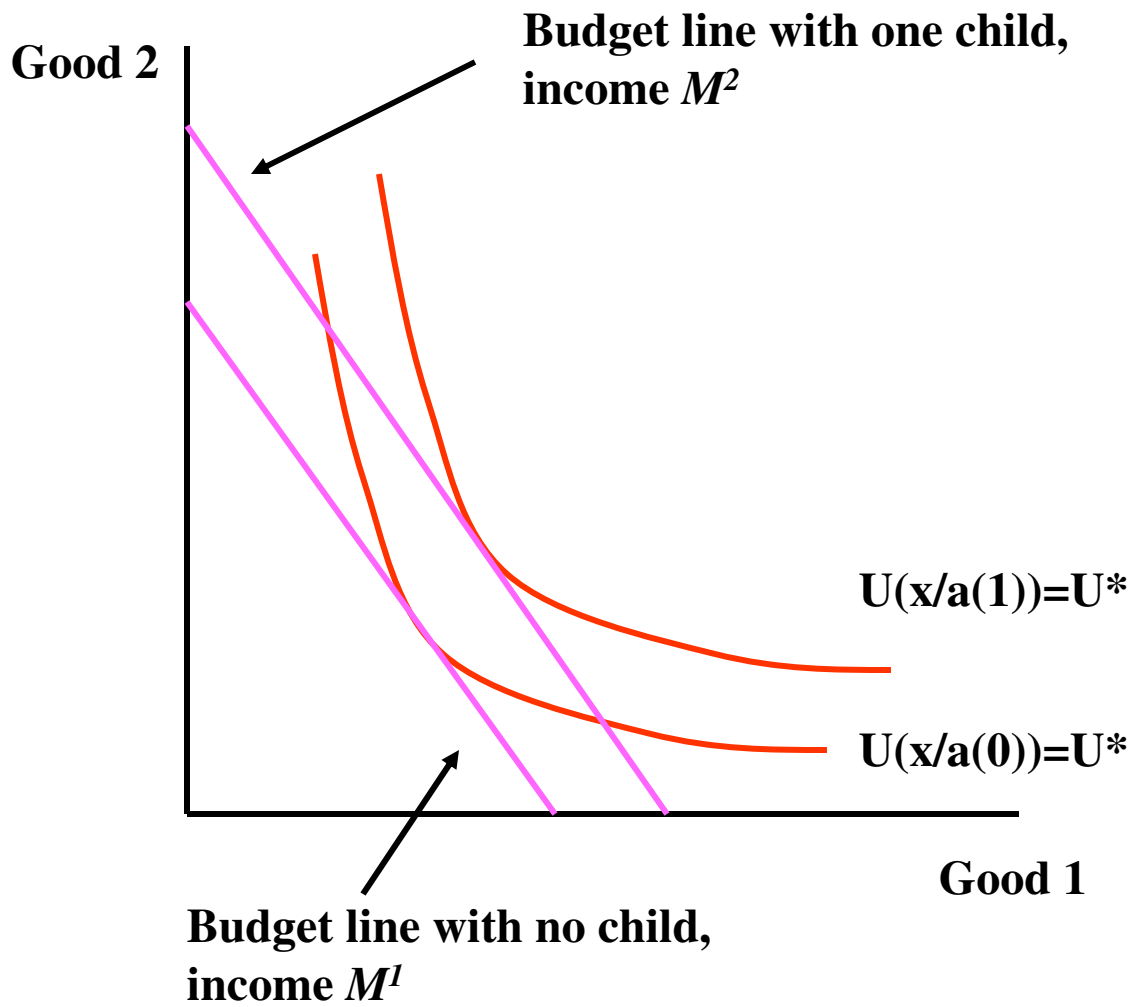
(iv) Practice

- Buhman *et al.* (1989) list 34 equivalence scales, and they summarize their content with the approximation:

$$\overline{M}^h = \frac{M^h}{[d^h]^\epsilon} \quad (8)$$

where d^h measures number of family members

Figure 3: The Barten Scale



- the elasticity ϵ varies between 0 and 1
- four types of equivalence identified:
 - (i) *expert statistical scales*, designed to count number above and below a given standard of living, $\epsilon = 0.75$
 - (ii) *expert program scales*, designed to assist with welfare programs, $\epsilon = 0.55$
 - (iii) *consumption scales*, based on observed expenditures, $\epsilon = 0.36$
 - (iv) *subjective scales*, based on questionnaire values of income, $\epsilon = 0.25$
- expert statistical scales give little weight to economies of scale in household consumption, while subjective scales give a large weight

■ Measurement of Inequality

- given income levels, adjusted by an appropriate equivalence scale, issue is one of measuring inequality of incomes

- measures of inequality divide into three groups:

(i) *statistical* measures

(ii) *welfare* measures, based on an explicit formulation of social welfare

(iii) *axiomatic* measures, based on specified properties such an index should possess

- Basic Definitions

Measures of income inequality defined either as discrete or continuous distributions of income

With a discrete distribution, there are H households, $h=1,\dots,H$, with labeling chosen so that incomes M^h form an increasing sequence, i.e.,

$$M^1 \leq M^2 \leq M^3 \leq \dots \leq M^H$$

the mean income being:

$$\mu = \frac{1}{H} \sum_{h=1}^H M^h$$

For a continuous distribution, basic data is a density function for income, $\gamma(M)$, assuming support is,

$[0, \bar{M}]$ so that,
$$\int_0^{\bar{M}} \gamma(M) dM = 1$$

Mean level of income is:

$$\mu = \int_0^{\bar{M}} M \gamma(M) dM$$

** see the appendix*

Objective of inequality measurement is to assign a single number to an income distribution, describing inequality and allowing ranking of different income distributions

Given an income vector $M = \{.\}$, an index that assigns same inequality to vectors M and λM for any $\lambda > 0$, is a *relative index*

i.e., if $I(M) = I(\lambda M)$, relative index is homogeneous of degree zero - scaling up of incomes leaves inequality unchanged

- Statistical Measures

(i) The Range

Consider distributions over H persons, $h = 1, \dots, H$, and let M^h be income of person h . Average income is μ so that:

$$\sum_{h=1}^H M^h = H\mu$$

The relative share of income going to h is x^h :

$$M^h = H\mu x^h$$

Simplest measure is to compare the extreme values of the distribution, the range being the gap between these levels as a ratio of the mean income:

$$E = (\text{Max}^h M^h - \text{Min}^h M^h) / \mu$$

If income is divided equally, $E=0$, and at other extreme, if one person gets all the income, $E=H$, and in general, E lies between 0 and H

In Figure 4, distribution AA' has wider range than BB' but most people under AA' enjoy μ , while under BB' , the population is divided into distinct classes of rich and poor

(ii) Relative Mean Deviation

Rather than look at extreme values, compare income levels to the mean, sum absolute differences, then look at proportion of total income:

$$R = \frac{\sum_{h=1}^H |\mu - M^h|}{H\mu}$$

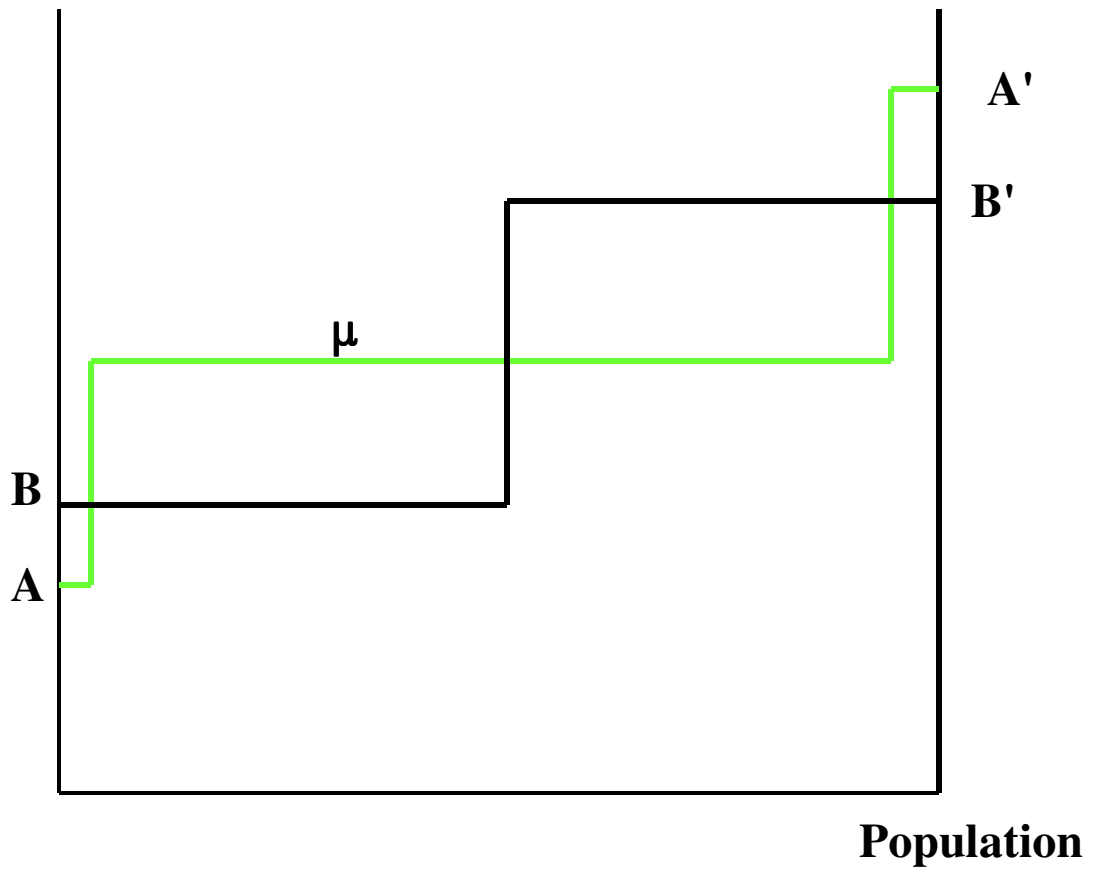
With perfect equality, $R=0$, and with all income going to one person, $R=2(H-1)/H$

Unlike E , R takes account of the whole distribution, e.g. in Figure 4, value of R is higher for BB' than AA'

Problem with R is it is not sensitive to transfers from a poorer to a richer person, if they both lie on same side of μ , as it adds to one gap and reduces other gap by same amount

Figure 4

Income



In Figure 5, distribution $ABCDEF$ is transformed into $ABGHJEF$, by transferring to some of the poorest from some of the richer class

Value of R remains unchanged since $BGIC$, the diminution of gap, is exactly matched by $DIHJ$, the increase in the gap

R takes no account of income transfers unless they cross the mean μ , hence it is rather arbitrary, and it fails to capture commonly accepted ideas on inequality, i.e., $ABCDEF$ would be regarded as more unequal than $ABGHJEF$

(iii) Variance and Coefficient of Variation

Squaring the gaps would accentuate differences from the mean, so a transfer like that in Figure 5 would reduce measure of inequality

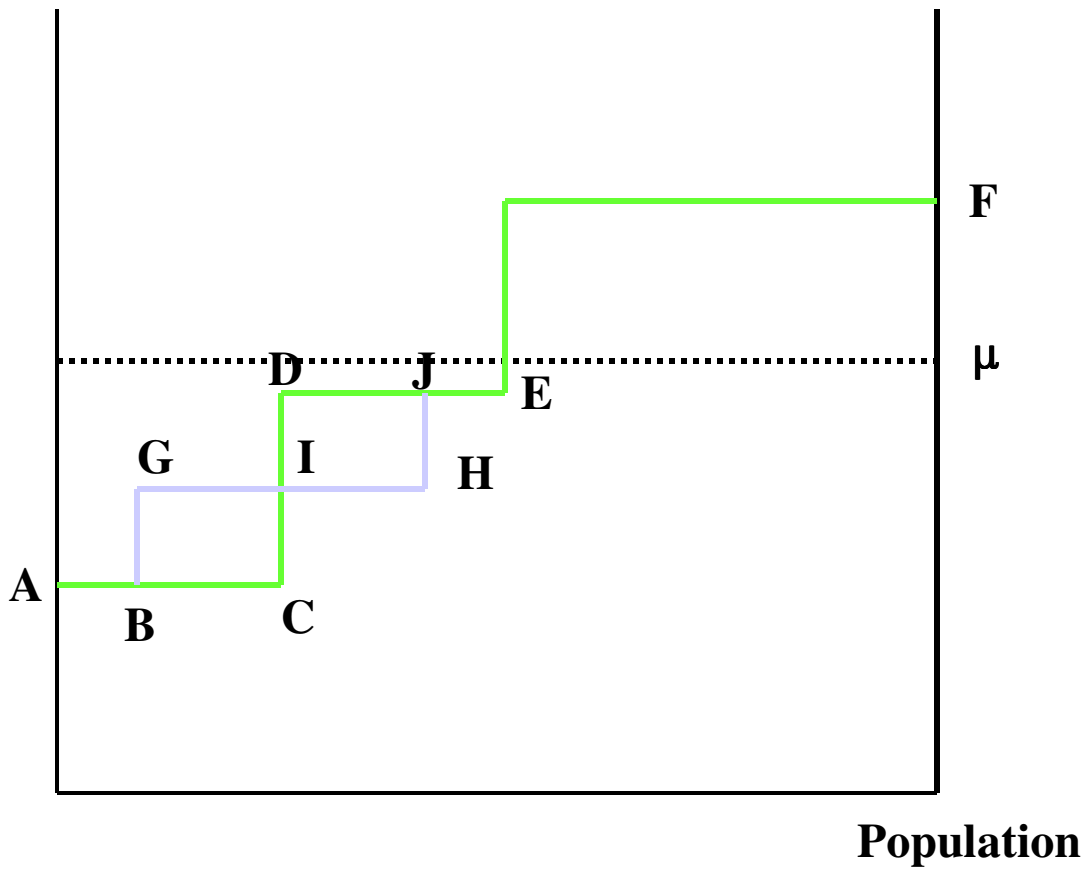
Variance has this property:

$$V = \frac{\sum_{h=1}^H (\mu - M^h)^2}{H}$$

In Figure 5, $ABCDEF$ has a higher variance than $ABGHJEF$, as BG has a stronger impact than JH

Figure 5

Income



Any transfer from a poorer to a richer person, *ceteris paribus*, always increases variance, which seems an attractive property for an inequality measure

Dalton (1920) argued a measure of inequality should have this condition, following Pigou (1912)

Pigou-Dalton (1920) Principle of Transfers

An inequality index must decrease if there is a transfer of income from a richer to a poorer household, which preserves their ranking in the distribution and leaves total income unchanged

Variance depends on mean income μ , so one distribution may show greater *relative* variation, and still have a lower variance if mean is lower than that of another distribution

Coefficient of variation does not have this deficiency:

$$C = V^{0.5}/\mu$$

While C captures property of being sensitive to income transfers for all income levels, and is independent of μ , why the squaring procedure rather than some other operation?

(iv) Standard Deviation of Logarithms

To attach greater weight to income transfers at lower end, need a transformation that staggers income levels

Logarithm does this, and also eliminates arbitrariness of units:

$$L = \left[\sum_{h=1}^H (\log \mu - \log M^h)^2 / H \right]^{0.5}$$

As income levels suffer increasingly severer contraction as they get higher, L is not concave at all at high income levels - if social welfare is to be a concave function of all individual incomes, L has problems

Shares limitation with V and C of taking differences only from μ

(v) The Lorenz Curve

While this is not an inequality index, it has been a key in other inequality indices

Curve is constructed by arranging population in order of increasing income, and graphing proportion of income going to each proportion of population

If incomes are identical, Lorenz curve is the diagonal connecting points (0,0) and (1,1) (see Figure 6)

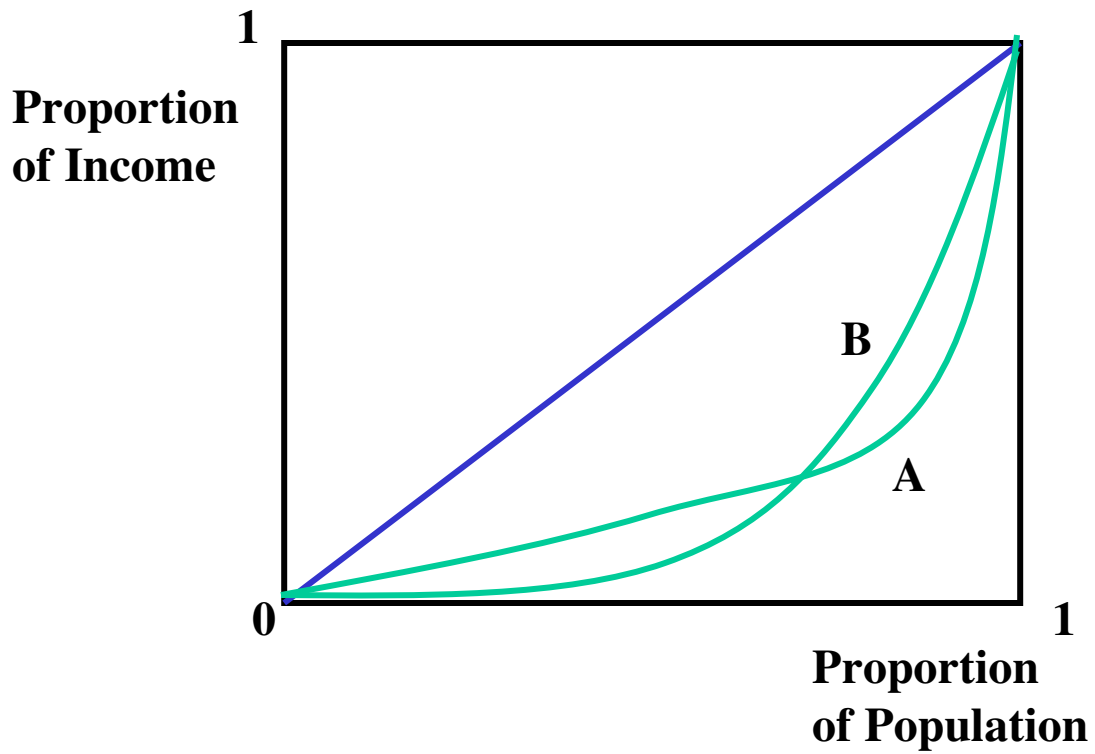
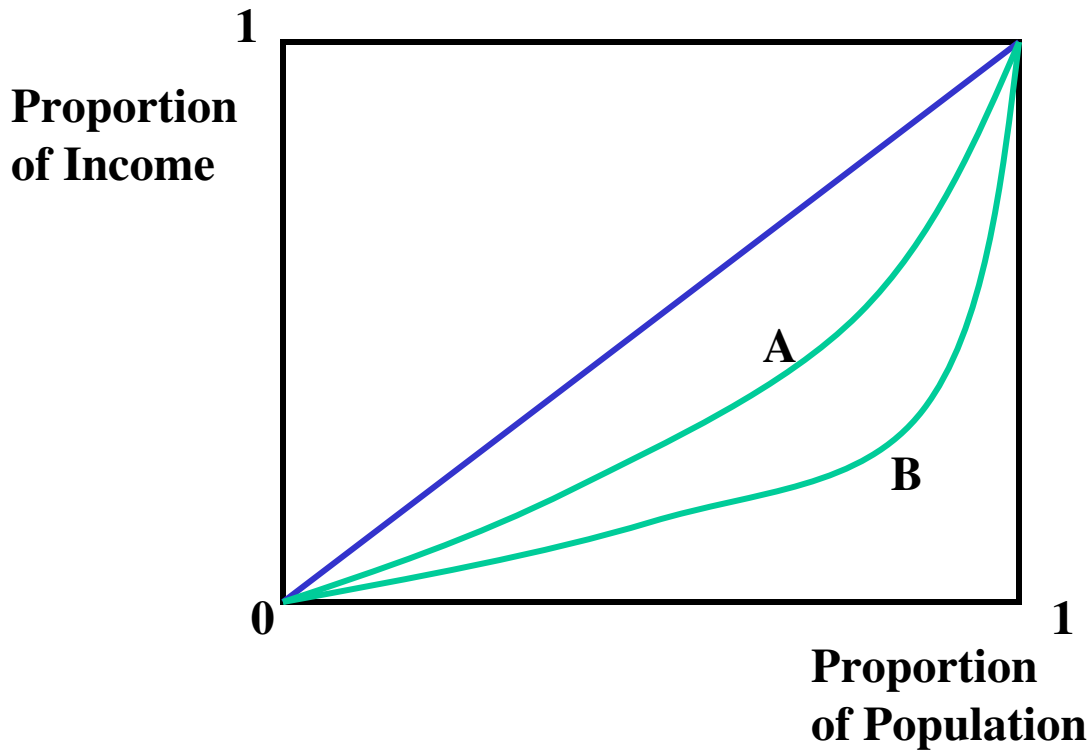
If there is any inequality, ordering of population ensures Lorenz curve lies below diagonal, i.e., if poorest half of population has less than half the proportion of income

For continuous distribution, coordinates (x,y) of the curve are:

$$\left(\int_0^m \gamma(M) dM, \frac{1}{\mu} \int_0^m M \gamma(M) dM \right)$$

** see the appendix*

Figure 6: Lorenz Curves



Lorenz curve can be used to rank some income distributions (Figure 6), e.g. if curve for distribution B lies entirely outside that for distribution A, B is less equal

B could have been derived from A by transfers from poor to rich, A *Lorenz dominates* B

If distributions A and B cross, an unambiguous ranking of distributions is not possible, so *Lorenz domination* provides only a partial ordering of income distributions

The Lorenz curve moves further from the diagonal if there is a transfer of income from poor to rich - so it satisfies the Pigou-Dalton Principle

(vi) Gini Coefficient

Gini (1912) coefficient is equal to area between Lorenz curve and line of equality as a proportion of area of triangle beneath line of equality

Considers all possible pairs of incomes, and out of each pair selects the minimum income level, summing and normalizing

$$\begin{aligned}
G &= (0.5H^2\mu) \sum_{h=1}^H \sum_{i=1}^H |M^h - M^i| \\
&= 1 - (1/H^2\mu) \sum_{h=1}^H \sum_{i=1}^H \text{Min}(M^h, M^i) \\
&= 1 + (1/H) - (2/H^2\mu) \sum_{h=1}^H \sum_{i=1}^H \text{Min}(M^h, M^i)
\end{aligned}$$

** see the appendix*

The Gini coefficient varies between 0 and 1

In taking differences over all pairs of incomes, avoids total concentration on differences relative to μ

Gini coefficient can be used to rank distributions when Lorenz curves cross, as areas are always well-defined

Gini coefficient is a relative index of inequality, so it is independent of scale, and it satisfies Pigou-Dalton

- Comparison of Measures

In comparing measures, E , and R , are non-starters, real choice being between C , L and G

(i) As far as Pigou-Dalton condition is concerned, C and G pass, i.e., a transfer from a richer to a poorer person always reduces the value of both, but L can violate this condition

(ii) In terms of relative sensitivity, C is equally sensitive at all levels, whereas L is more sensitive for transfers at lower levels of income

L however becomes so insensitive to transfers among the rich that it may violate Pigou-Dalton, so does Gini coefficient bridge the gap?

Essentially no - G does not depend on income levels but on number of people in between them - it implies a welfare function that is the weighted sum of different peoples' income levels with weights given by rank-order in ranking of incomes

- Welfare-Theoretic Indices

Rather than examine implicit welfare assumptions of statistical measures of inequality, welfare judgements can be made explicit in deriving an inequality index

Dalton (1920) argued any measure of economic inequality must be concerned with welfare to be of relevance

Dalton's measure followed directly from utilitarian framework, based on comparison between actual levels of aggregate utility and total utility that would obtain if income were equally divided

Based on a strictly concave utility function, and the same function for all, maximization of aggregate welfare required equal division

Dalton took ratio of actual social welfare to maximal social welfare as measure of inequality:

$$D = \frac{\sum_{h=1}^H U(M^i)}{H U(\mu)}$$

Atkinson (1970) points out measure is not invariant with respect to positive linear transformations of the utility function

Possible to redefine measure in such a way that this is not a problem

Consider method of *equally distributed equivalent incomes* (Kolm, 1969; Atkinson, 1970)

Assume social welfare is a utilitarian function:

$$W = \sum_{h=1}^H U(M^h) \quad (9)$$

where $U(M)$ is increasing and strictly concave, $U'(M) > 0$, and $U''(M) < 0$

$U(M)$ is either the true cardinal utility function for households, or state chooses it so that social welfare function captures welfare judgements

Before specifying $U(M)$, possible to derive measure of inequality specified by Kolm, and Atkinson; define M_{EDE} as solution to:

$$\sum_{h=1}^H U(M^h) = H U(M_{EDE}) \quad (10)$$

M_{EDE} is the equally distributed equivalent income - the level of income if given to all households would generate same level of social welfare as initial income distribution

** see the appendix*

Index is:

$$A = 1 - \frac{M_{EDE}}{\mu} = \frac{\mu - M_{EDE}}{\mu} \quad (11)$$

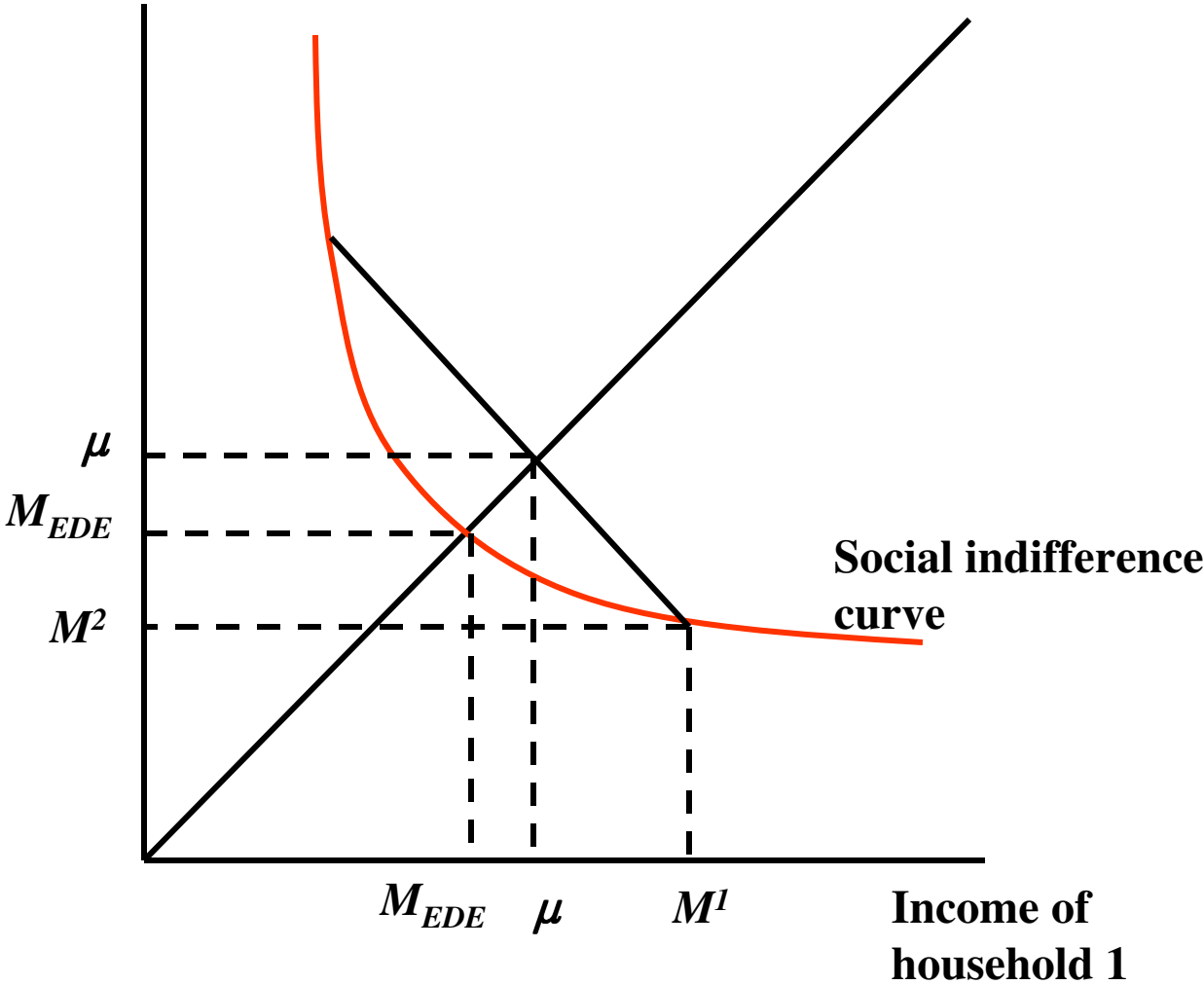
Concavity of household utility function ensures A is non-negative

Figure 7 illustrates case of a 2-household economy; initial income distribution is M^1 and M^2 , which determines contour of social welfare function

M_{EDE} is found by moving round indifference curve to line of equal incomes, and because of concavity of utility function, $M_{EDE} < \mu$ the mean level of income

Figure 7: Equally Distributed Equivalent Income

Income of household 2



If income is equally distributed, then M_{EDE} is equal to μ , and $A = 0$, so for any distribution, A lies between 1 and 0

Flexibility of index A lies in freedom of choice over the utility function, which then determines importance attached to inequality by the index

Atkinson uses utility function:

$$U(M) = \frac{M^{1-\varepsilon}}{1-\varepsilon}, \quad U'(M) = M^{-\varepsilon}, \quad \varepsilon \neq 1 \quad (12)$$

(12) is concave if $\varepsilon \geq 0$, and welfare judgements of policy maker contained in value of ε as this determines degree of concavity of utility function

Increasing ε makes utility function more concave, reducing importance given to high incomes in determination of social welfare

As Sen (1972) notes, Atkinson's idea of inequality is totally dependent on the form of social welfare function, and use of utilitarian framework

- Statistical Measures and Welfare

There has been a clarification of link between statistical measures of inequality and social welfare (Atkinson, 1970; Dasgupta *et al.*, 1973; Rothschild and Stiglitz, 1973)

Suppose Lorenz curves x and y both lie inside Lorenz curve z , but x and y both intersect

Treating L as the relation of being strictly inside, xLz , and yLz , but neither xLy nor yLx - does Lorenz ranking L catch essence of inequality?

Atkinson (1970) developed a theorem on Lorenz ranking using a normative approach - suppose social welfare is sum of individual utility functions which are strictly concave functions of income M^h :

$$W(M) = \sum_{h=1}^H U(M^i)$$

Let xLy , and total income is same for both distributions, without knowing $U(\cdot)$, it can be said that:

$$W(x) > W(y)$$

Furthermore, if $W(x) > W(y)$, then xLy ; so xLy implies $W(x) > W(y)$ irrespective of the precise utility function chosen

Sen and others show it is sufficient to consider strict quasi-concavity as opposed to strict concavity of the social welfare function in order to incorporate the egalitarian bias

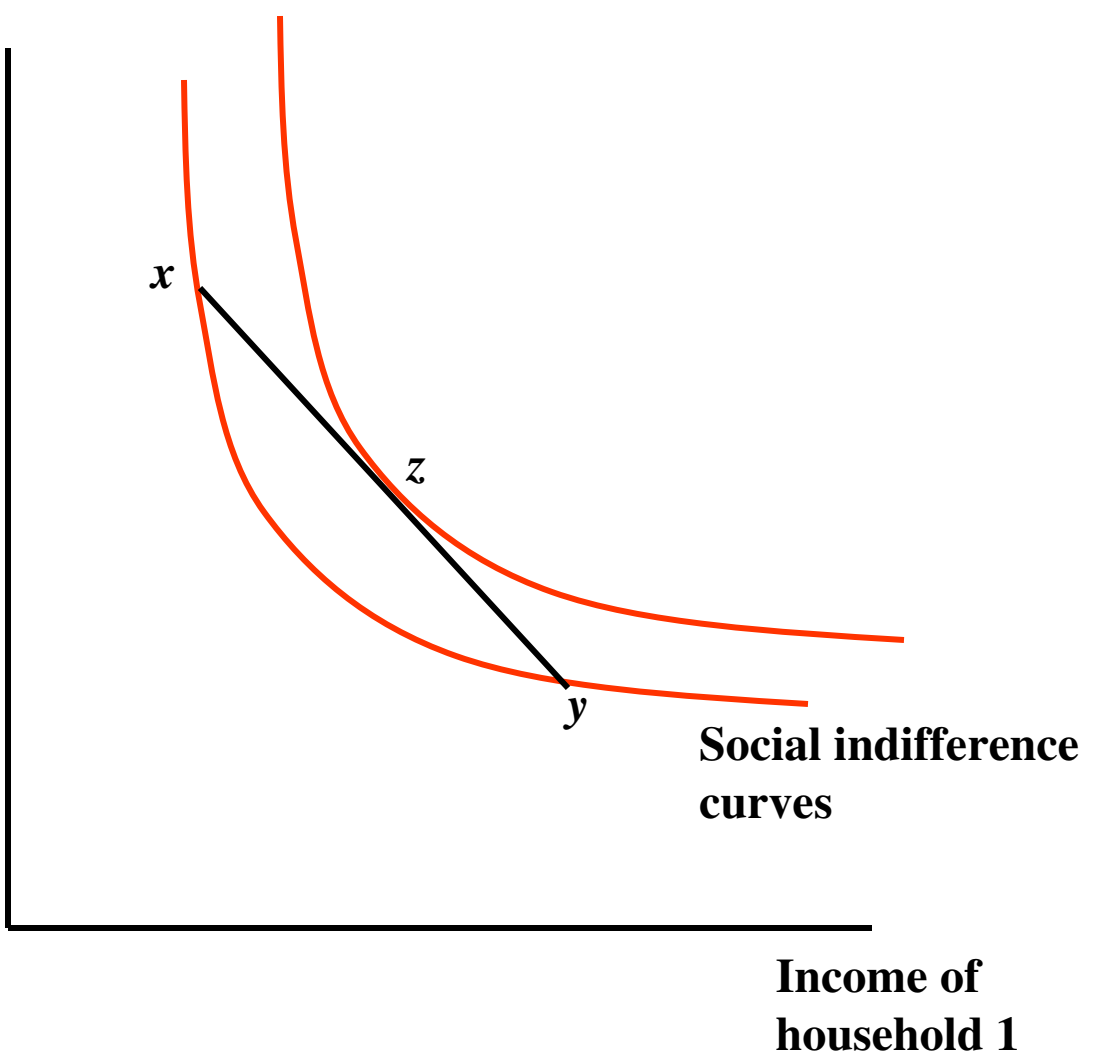
Strict quasi-concavity requires the social indifference curves must be as in Figure 8, x and y lie on the same indifference curve giving same level of social welfare, and z is weighted average of x and y

Strict quasi-concavity requires that social welfare at z be strictly larger than at x and y , i.e., the social indifference curve through z must be higher than that through x and y

Strict quasi-concavity means that as income level of one person is increased, given that of others, less importance is attached to the income of that person

Figure 8

Income of household 2



Social indifference curves

Income of household 1

Consider a social welfare function F defined over individual incomes, and let F be symmetric and strictly quasi-concave

Dasgupta *et al.* state following theorem:

Taking F to be symmetric and strictly concave, if for income distributions x and y with same total income, yLx , then $F(y) > F(x)$, and if not yLx , then for some F , $F(y) < F(x)$

To provide a complete ranking when Lorenz curves cross, more restrictions will have to be placed on the social welfare function

■ Poverty and Inequality

- Before measuring poverty, important to define it**
- Clear that it means a lack of income and low level of consumption and welfare, but what is the level of income against which to judge?**

(i) *absolute poverty*, assumes some fixed minimum level of consumption (income) constituting poverty, independent of time and place

Consumption level is a diet sufficient to maintain health and provision of housing and clothing

If incomes of all households rise, eventually there will be no poverty - as a result of this, concept of absolute poverty rejected

(ii) *relative poverty*, defined in terms of standards and norms of a given society at a given time - as standard of living of society rises, income level required to be out of poverty must increase

- the *poverty line* is defined as that level of income on or below which a household is defined as being in poverty

- poverty line defined as z , so any income at or below z represents poverty; for household h , $g^h \equiv z - M^h$, is extent to which household falls below the poverty line

- given z , and distribution of income M , number of households in poverty is $q = q(M, z)$

- *headcount ratio* measures degree of poverty by counting number of households whose incomes are not above poverty line, taken as a proportion of population:

$$\mathcal{E} = \frac{q}{H} \quad (9)$$

- easy to calculate, but pays no attention to how far households are below poverty line, i.e. how costly will it be to alleviate observed poverty?

- *aggregate poverty gap*:

$$\mathcal{F} = \sum_{h=1}^q g_h \quad (10)$$

- *income gap ratio*:

$$\mathcal{L} = \frac{1}{q(M, z)} \sum_{h=1}^q \frac{g^h}{z} \quad (11)$$

- both account for income shortfalls, but equal weight is given to all income shortfalls

- Sen defined an index based on an axiomatic approach:

$$\mathfrak{S} = \mathcal{E} \left[\mathcal{L} + [1 - \mathcal{L}] G_p \left[\frac{q}{q + 1} \right] \right]$$

where G_p is the Gini coefficient for households below poverty line

This poverty index combines a measure of the shortfall of income of the poor with one of income distribution among the poor

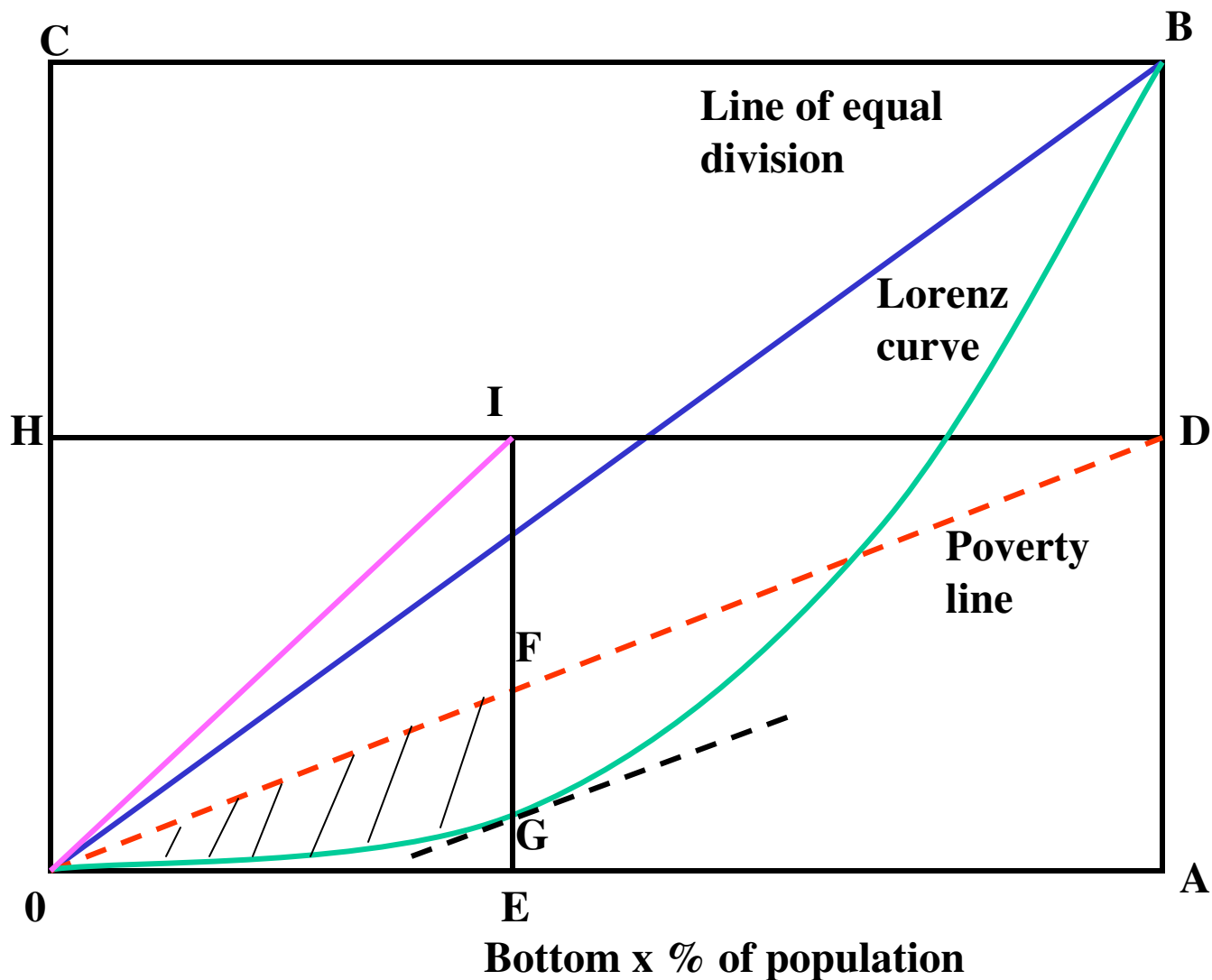
\mathfrak{S} lies in the interval $[0,1]$, where $\mathfrak{S} = 0$, if everyone has income greater than z , and if $\mathfrak{S} = 1$, everyone has zero income

When poor all have same income, $G_p = 0$, the lower the income of the poor, the closer \mathfrak{S} lies to the head-count measure \mathcal{E} , and the larger the proportion of the poor, the closer \mathfrak{S} approaches the income gap measure \mathcal{L}

Essentially \mathfrak{S} is a translation of the Gini coefficient from the measurement of inequality to measurement of poverty

Figure 9: Poverty Measure and Gini Coefficient

**% share of income of bottom
x % of population**



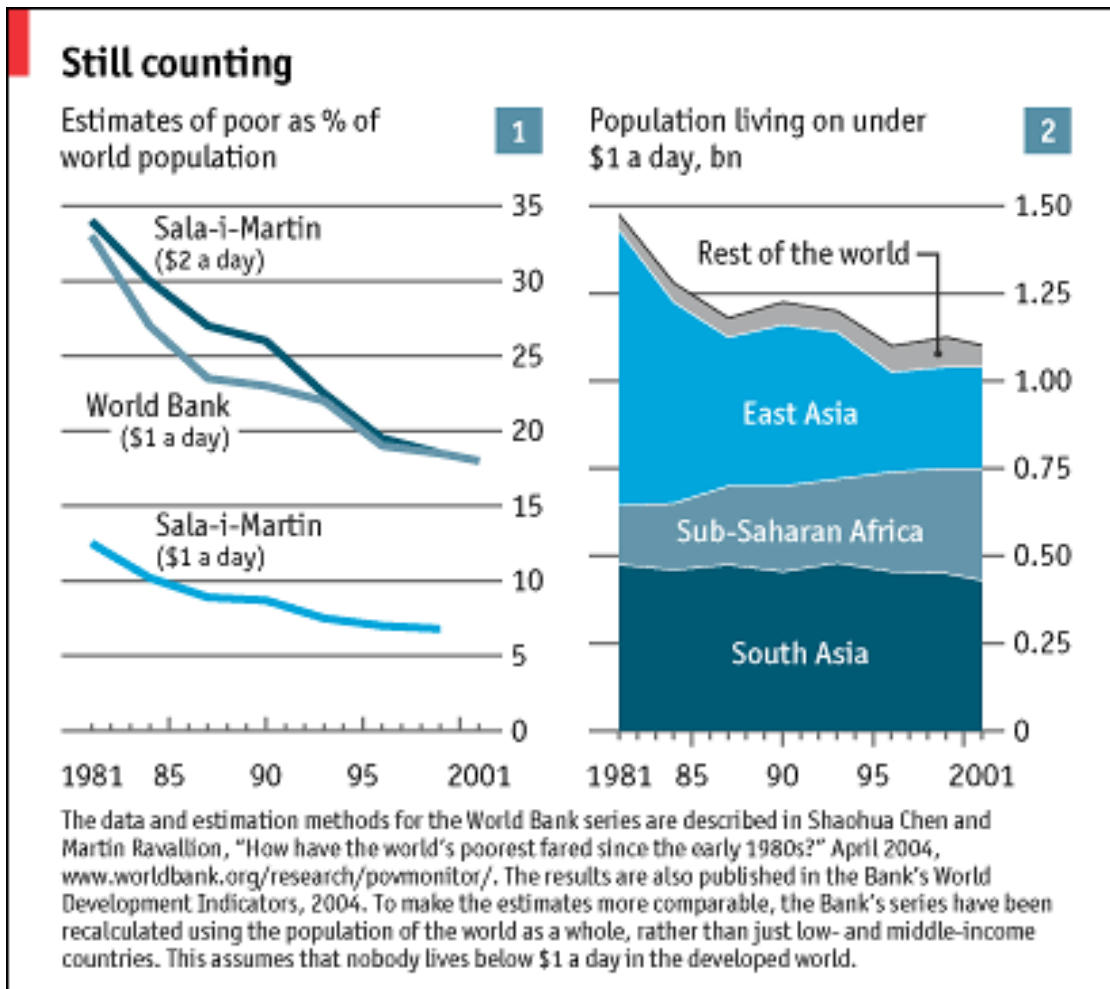
Gini coefficient is given by $0GB/0AB$, and the slope of the line $0D$ gives the poverty line in normalized units, and $0E$ is the number of the poor

$\$$ corresponds to area $0GF/0EI$. Difference to Gini coefficient lies in:

- slope of line $0D$, the poverty line, being different from slope of $0B$, normalized mean income**
- counting only the poor, $0E$ in the poverty measure, as opposed to all, $0A$**

■ Estimates of Income Inequality and Poverty

- Bourignon and Morrison (2002, *AER*) – world income inequality increased from early 19th Century until 1980 (Table 1)**
- work of Chen and Ravallion (2004, World Bank), and Sala-i-Martin (2002, NBER) most recent estimates of poverty (see Deaton, 2004 for discussion)**
- Chen and Ravallion based on household survey data, i.e., focus on basic food and consumption needs**
- Sala-i-Martin based on GDP from national accounts used to measure average income per person of households, i.e., includes more than just consumption**



-Chen and Ravallion show by \$1/day standard (1993 ppp) global poverty rate fell to 18% of population in 2001 (1.1 billion)

-Sala-i-Martin shows global poverty rate fell to 7% of population in 1998 by the \$1/day standard

-composition of world poverty has changed, falling in Asia, but increasing in Africa