■ **Income Taxation**

Taxation of income is a major source of revenue in most developed economies, and is highly contentious

(a) It is a direct means of altering the distribution of income to meet equity objectives

(b) It is also viewed as a major disincentive to effort and entrepreneurship, especially when marginal rates of tax increase with income

■ **Prior to Mirlees' (1971) work, no formal analysis of the structure and determinants of income tax that captured the efficiency-equity trade-off**

(i) Basic Structure

- 2 commodities, a consumption good and labor, where l is supply of labor by a household, $0 \leq l \leq 1$, and x is consumption of a good, $x \geq 0$

- each household characterized by a skill level s, where s reflects relative productivity of households

- s is continuously distributed through the population with support $S = [0, \propto]$, the cumulative distribution is $\Gamma(s)$, and the corresponding density function is $\gamma(s)$

- sl is the household's supply of *effective* labor, and if marginal product of labor is constant, for a worker of skill level s, marginal product is s *(technically, it should be $MP_l\ s$)*

- total productivity for l hours of work is then sl, and the supply of effective labor for a household with ability s is $z(s) \equiv sl(s)$

- normalizing the price of the consumption good at 1, $z(s)$ is also pre-tax income measured in terms of consumption units (with skill being the MP of l)

- a household can consume:

$$x(s) \leq c(z(s)) = z(s) - T(z(s)) \qquad (1)$$

where $c(z)$ is the consumption function, and $T(z)$ is the tax function

- all households have the same, strictly concave utility function:

$$U = U(x,l) \tag{2}$$

$U(.)$ Is continuously differentiable, strictly increasing in consumption, and strictly decreasing in labor, satisfying:

$$U_x > 0,\ U_l < 0,\ U_{xx} < 0$$

and: $$U_l(x,l) \rightarrow -\infty \ as\ l \rightarrow 1$$

- latter condition implies household will endeavor to avoid corner solutions with l=1

- household of ability s, chooses $x(s)$ and $l(s)$ to maximize utility subject to a budget constraint:

$$max\ U(x,l)\ subject\ to\ x(s) \leq c(sl(s)) \tag{3}$$

- define $u(s) = U(x(s), l(s))$ as maximized level of utility at optimal choices

(ii) Characterization of Optimal Tax Function

- using individual demand and supply functions, and integrating over the population, effective labor supply is:

$$Z = \int_0^\infty z(s)\gamma(s)ds \tag{4}$$

and aggregate demand is:

$$X = \int_0^\infty x(s)\gamma(s)ds \tag{5}$$

* *see the appendix*

- an optimal tax function is then chosen to maximize social welfare, where social welfare is described by a Bergson-Samuelson *swf* function:

$$W = \int_0^\infty W(u(s))\gamma(s)ds \qquad W'' \leq 0 \tag{6}$$

- the chosen allocation has to be productively feasible:

$$X \leq F(Z) \tag{7}$$

* *see the appendix*

where F is the economy's production function

- (7) can be adjusted to incorporate the amount of labor consumed by the government, z^G, so that $X = F'(Z - z^G)$

- alternatively, the government revenue constraint can be used, so denoting level of revenue as $R \equiv z^G$, the revenue constraint can be written as:

$$R \geq \int_0^\infty [z(s) - x(s)]\gamma(s)ds \qquad (8)$$

(iii) Linear Taxation

- complexity of the general model of taxation has led to focus on the restricted case of linear taxation

- with a linear income tax, there is a constant marginal rate of tax, and an identical lump-sum tax (subsidy) for all households, this restriction has two advantages:

(a) ensures household budget sets are convex, so that optimal choices are unique with strictly convex preferences

(b) tax system is described by only two parameters, the marginal tax rate and the lump-sum tax (subsidy)

- a linear tax scheme also fits in well with those countries where *negative* income taxes have been seriously discussed, and also countries that have considered reducing the number of tax rate bands

- under a linear tax scheme, a household of ability *s* supplying *l* units of labor, pays taxes of:

$$T(sl) = -\tau + tsl \qquad (9)$$

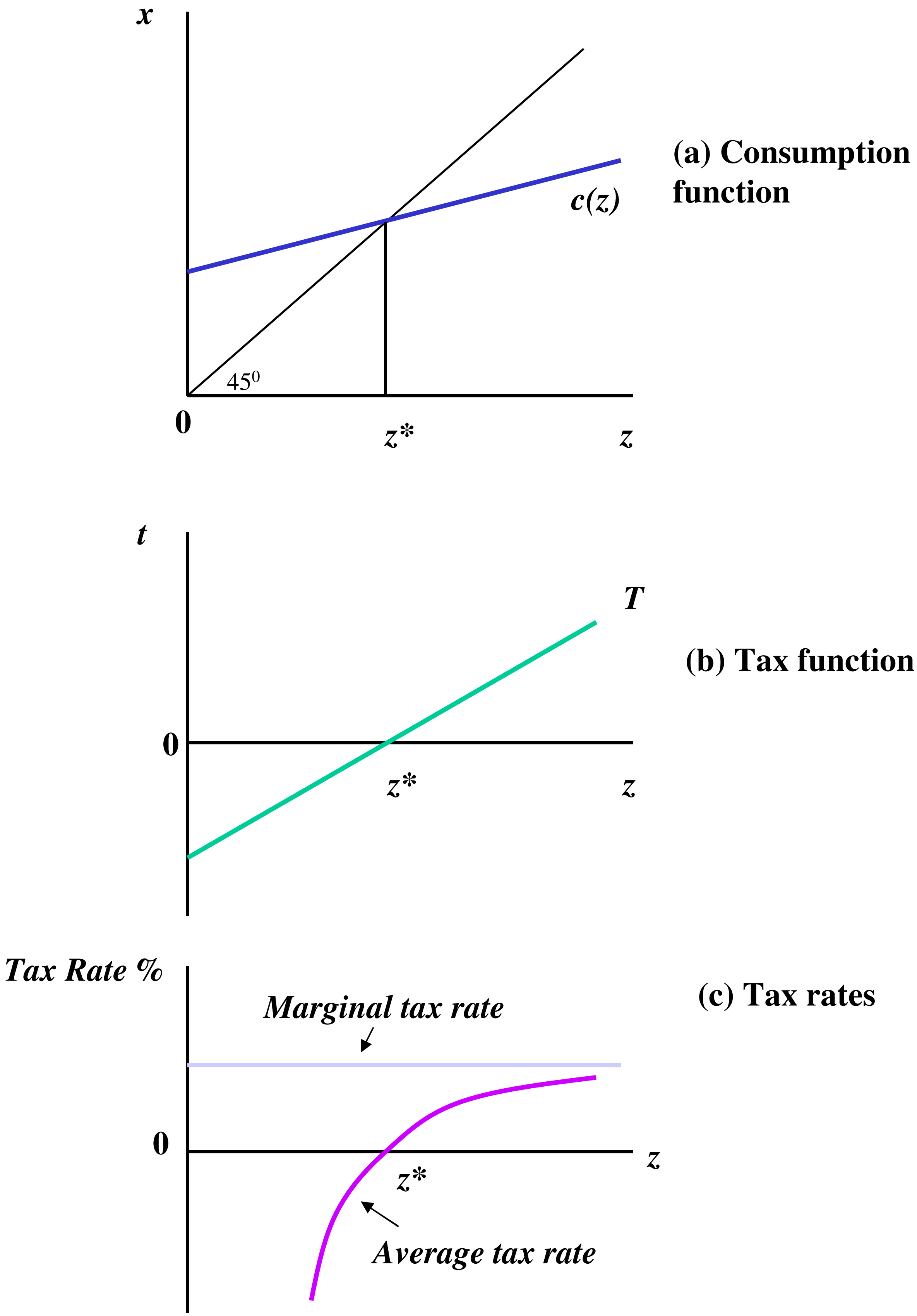
where *t* is the marginal tax rate, and $-\tau$ is a lump-sum tax (i.e., τ is negative) or a lump-sum subsidy (i.e., τ is positive)

- letting $[1-t]=\zeta$, the household consumption function becomes:

$$x = \tau + \zeta sl \qquad (10)$$

- see Figure 1

Figure 1: Linear Income Tax



- household maximizes utility function (2) subject to (10), the first-order condition being:

$$-\frac{U_l}{U_x} = \zeta s \tag{11}$$

(11) and (10) imply labor supply and demand functions of the form:

$$l = l(\zeta,\tau,s), \quad x = \tau + \zeta s\, l(\zeta,\tau,s) \tag{12}$$

- substituting (12) into the utility function (2) gives the indirect utility function:

$$U = U(\tau + \zeta s\, l(\zeta,\tau,s), l(\zeta,\tau,s)) = V(\zeta,\tau,s) \tag{13}$$

with: $$\frac{\partial V}{\partial \tau} = U_x, \quad \frac{\partial V}{\partial \zeta} = U_x s l$$

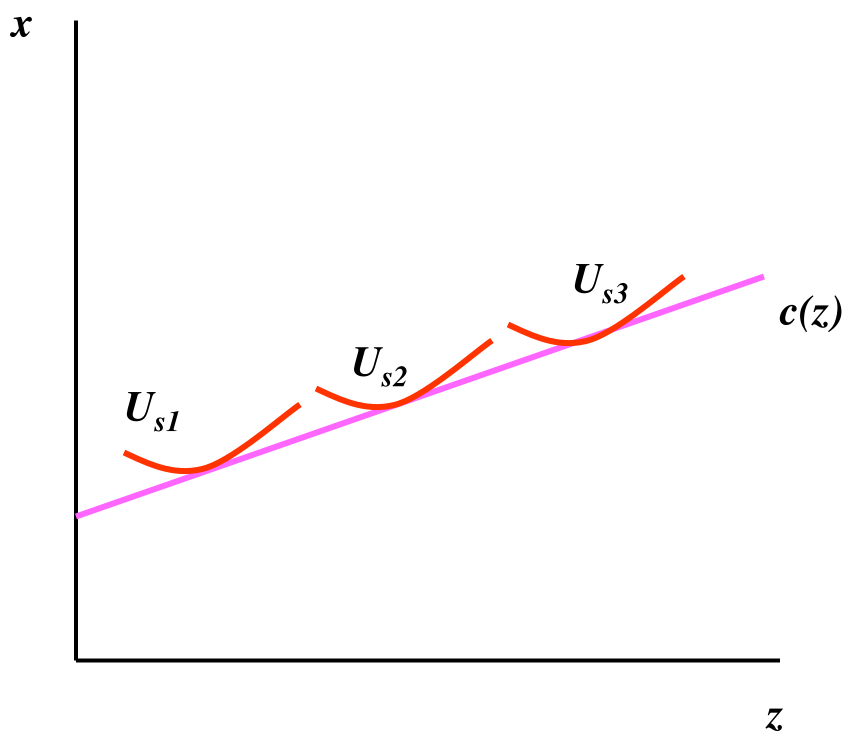
**** See the appendix***

where $\partial V/\partial \tau$ is the marginal utility of income

- optimization problem is to choose tax parameters to maximize *swf* subject to *R* the revenue constraint:

$$max_{\tau,\zeta} \int_0^\infty W(V(\zeta,\tau,s))\gamma(s)ds \tag{14}$$

Figure 2: Household Maximization



Subject to:

$$\int_0^\infty [-\tau + [1-\zeta] s\, l(\zeta,\tau,s)]\gamma(s)\,ds = R \qquad (15)$$

- given definition of marginal utility of income, social marginal utility of income for a household of ability *s* is:

$$\beta(s) = W'(V(\zeta,\tau,s))\frac{\partial V(\zeta,\tau,s)}{\partial \tau} \qquad (16)$$

- necessary conditions for optimal choice of lump-sum tax(subsidy) and marginal tax rate are:

$$\int_0^\infty \beta\gamma(s)\,ds = \lambda\left[H - \int_0^\infty [1-\zeta]\frac{\partial z}{\partial \tau}\gamma(s)\,ds\right] \qquad (17)$$

$$\int_0^\infty \beta z\gamma(s)\,ds = \lambda\int_0^\infty \left[z - [1-\zeta]\frac{\partial z}{\partial \zeta}\right]\gamma(s)\,ds \qquad (18)$$

** see the appendix*

H **is population size,** $H = \int_0^\infty \gamma(s)\,ds$

- (17) and (18) are used to derive an expression for the optimal t, dividing (18) by (17):

$$\frac{\int_0^\infty \beta z \gamma(s) ds}{\int_0^\infty \beta \gamma(s) ds} = \bar{z}(\beta) = \frac{\bar{z} - \int_0^\infty [1-\zeta] \frac{\partial \bar{z}}{\partial \zeta} \gamma(s) ds}{1 - \int_0^\infty [1-\zeta] \frac{\partial \bar{z}}{\partial \tau} \gamma(s) ds} \quad (19)$$

where: $\bar{z} = z/H, \quad \frac{\partial \bar{z}}{\partial (.)} = \frac{\partial \bar{z}}{\partial (.)} \cdot \frac{1}{H}$

** see the appendix*

- left-hand side of (19) is interpreted as the welfare-weighted average labor supply

- totally differentiating the government revenue constraint, holding $r=R/H$ constant:

$$\left.\frac{d\tau}{d\zeta}\right|_{r=const} = \frac{-\bar{z} + \int_0^\infty [1-\zeta] \frac{\partial \bar{z}}{\partial \zeta} \gamma(s) ds}{1 - \int_0^\infty [1-\zeta] \frac{\partial \bar{z}}{\partial \tau} \gamma(s) ds} \quad (20)$$

** see the appendix*

- from (19) and (20):

$$\bar{z}(\beta) = -\left.\frac{d\tau}{d\zeta}\right|_{r=const} \tag{21}$$

- welfare-weighted average labor supply equals constant revenue effect of tax rate changes on guaranteed income τ , so it follows that:

$$\left.\frac{d\bar{z}}{d\zeta}\right|_{r=const} = \frac{d\bar{z}}{d\zeta} + \left.\frac{d\bar{z}}{d\tau}\frac{d\tau}{d\zeta}\right|_{r=const} = \frac{d\bar{z}}{d\zeta} - \frac{d\bar{z}}{d\tau}\bar{z}(\beta) \tag{22}$$

- i.e., the effect on labor supply when ζ and τ are changed to keep tax revenue constant

- (19) can be re-written as:

$$\bar{z}(\beta) - \bar{z} = [1-\zeta]\left[\frac{d\bar{z}}{d\tau}\bar{z}(\beta) - \frac{d\bar{z}}{d\zeta}\right] = -\left. t\frac{d\bar{z}}{d\zeta}\right|_{r=const} \tag{23}$$

** see the handout*

- which can be re-arranged as:

$$t = \frac{\bar{z} - \bar{z}(\beta)}{-\left.\frac{d\bar{z}}{d\zeta}\right|_{r=const}} \tag{24}$$

- while (24) is only an implicit expression for *t*, parametric changes can be evaluated:

(i) $$If\ \bar{z} - z(\beta) > 0,\ and\ \left.\frac{d\bar{z}}{dt}\right|_{r=const} < 0$$

an increase in disincentive effect of taxation

$\left.\frac{d\bar{z}}{dt}\right|_{r=const}$ results in reduction in optimal tax rate

(ii) As β is a decreasing function of *s*, and *z* an increasing function of *s*, the optimal tax rate would decrease if high-*s* households were given more welfare weight, i.e., equity given less weight

** see the appendix*

- from the first-order conditions (17) and (18), results on signs of tax rates can be given (Sheshinksi, 1972) and Romer(1976)

$$If\ (i)\frac{\partial z}{\partial \tau} < 0,\ (ii)\frac{\partial z}{\partial \zeta} \geq 0\ and\ R = 0,$$

$$then\ t > 0,\ and\ \tau < 0$$

** see the appendix*

Proof:

From (17) and (18), $\lambda > 0$, and $\zeta > 0$, or else $x, l = 0$ for all s. Assume then that $\zeta > 1$, (17) and (i) imply:

$$\int_0^\infty [\beta - \lambda]\gamma(s)ds \le 0 \qquad (25)$$

**** see the appendix***

As z is an increasing function of s, and β - λ is a decreasing function of *s*

$$\int_0^\infty [\beta - \lambda]z(s)\gamma(s)ds < 0 \qquad (26)$$

From (18) and (ii), inequality in (26) cannot be satisfied if $\zeta > 1$

Therefore, $\zeta < 1$, implying that $t > 0$, and $\tau < 0$ follows from zero revenue condition

(iv) Numerical Results

- Stern (1976) conducted numerical simulations of linear tax model based on CES utility function:

$$U(x,l) = (1/\rho)\{(1-\varphi)x^{-\mu} + \varphi(1-l)^{-\mu}\}^{\rho/\mu}$$

- where the elasticity of substitution $\varepsilon = 1/(1-\mu)$,

- in absence of taxation, and if $\varepsilon = 1$, labor supply function is derived from Cobb-Douglas preferences, and if $\varepsilon < 1$, labor supply is backward-bending

- Stern simulates relationship between tax rate *t*, ε, and ρ, and assumes all tax revenue *R* is used for re-distributive purposes

- Figure 3 shows tax rate is quite sensitive to elasticity of substitution and distributional preferences

- Figure 4 shows maximand as a function of tax rate, with $\varepsilon = 0.4$, and *R* = 20% of GDP

Figure 3: ***t*** **as a function of ρ and** ϵ

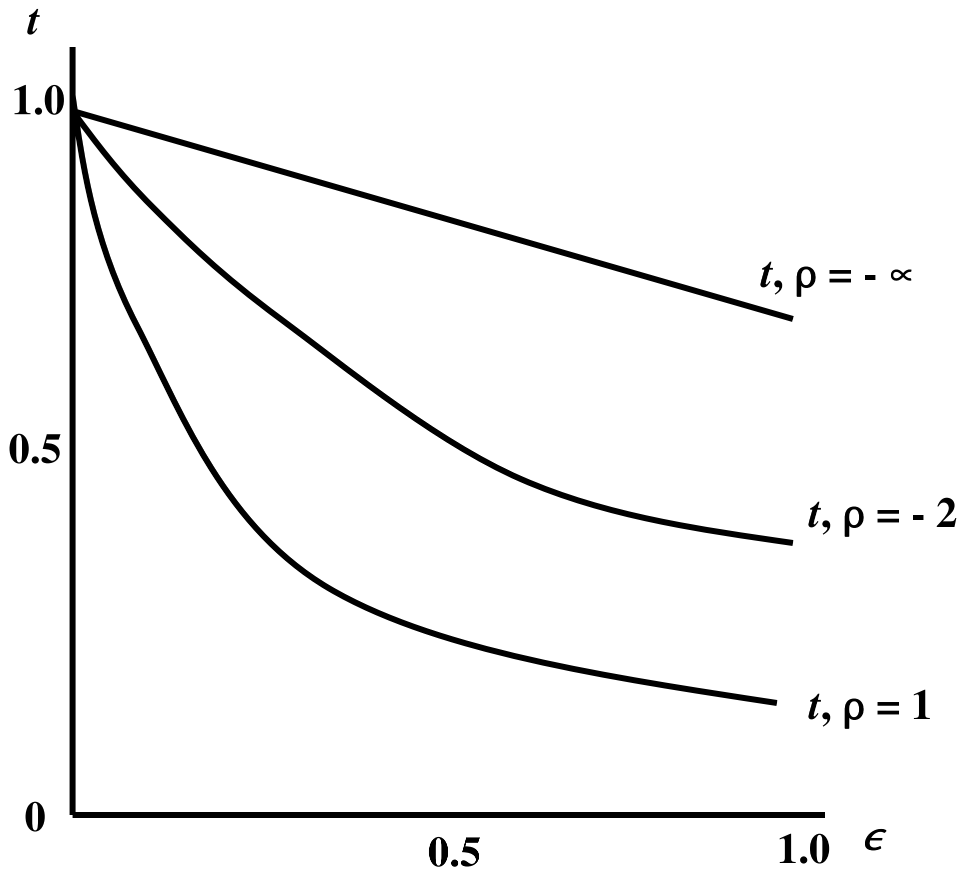
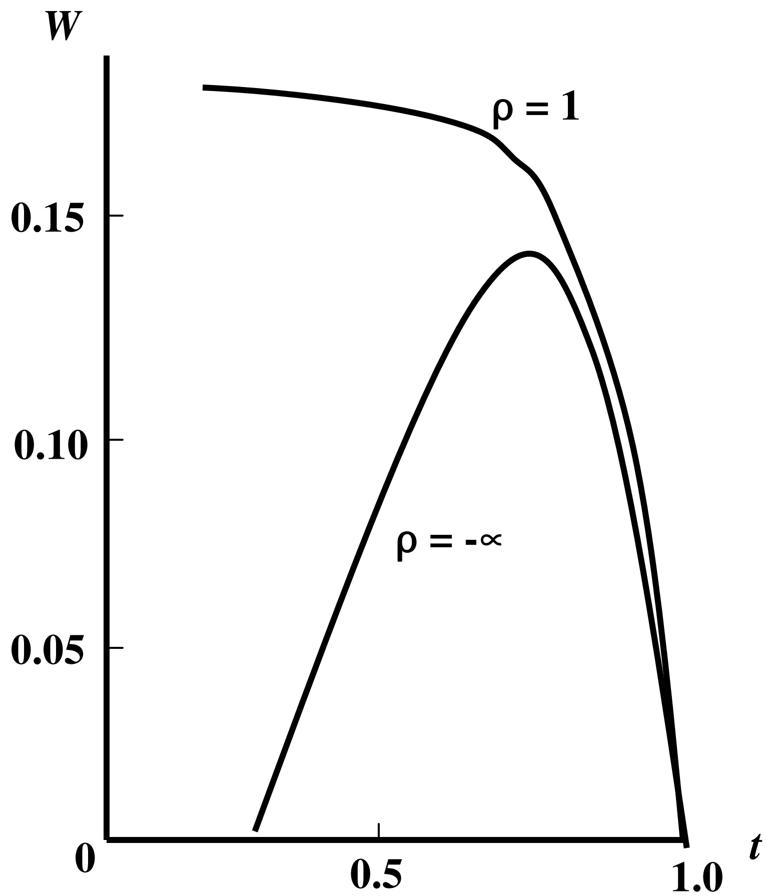


Figure 4: Maximand and *t*



(v) Non-Linear Taxation

- major results (Mirlees, 1971; Seade, 1977; Ebert, 1992) derived for optimal non-linear taxes:

(a) marginal rate of taxation should lie between 0 and 1

(b) at highest and lowest abilities, tax rate must be zero - i.e., a tax system cannot be progressive, but interior points have re-distributive taxation

(c) may be optimal to force lowest ability households not to work

(d) pre-tax income and consumption must be increasing functions of ability

- to get further on the structure of optimal non-linear taxes, need to use numerical analysis

- how close should marginal rate be to endpoints?

- how do equity considerations affect marginal rate?

- Mirlees (1971) generated numerical results by assuming the *swf*:

$$W = \int_0^\infty \frac{1}{v} e^{-vU} \gamma(s) ds, \ v > 0, \ = \int_0^\infty U \gamma(s) ds, \ v = 0 \quad (27)$$

v can be varied parametrically in (27), higher values representing greater concern for equity

- individual utility function assumed to be Cobb-Douglas:

$$U = log\, x + log[1 - l] \quad (28)$$

- skill distribution assumed log-normal:

$$\gamma(s) = \frac{1}{s} exp\left[- \frac{[\log(s + 1)]^2}{2} \right] \quad (29)$$

with a standard deviation of $\sigma = 0.39$, derived from a distribution of income (Lydall, 1968)

- implies that a skill distribution can be inferred from an observed income distribution

- results in Table 1 suggest low marginal rates, maximal rate being 34%, and marginal rates become lower at high incomes, but do not reach 0

- negative average rates for low-income households show they are receiving an income supplement

Table 1: Optimal Tax Schedule

Income	**Consumption**	**Average tax %**	**Marginal tax**
(a) z^G=0.013, v=0			
0	0.03	-	23
0.05	0.07	-34	26
0.10	0.10	-5	24
0.20	0.18	9	21
0.30	0.26	13	19
0.40	0.34	14	18
0.50	0.43	15	16
(b) z^G=0.003, v=1			
0	0.05	-	30
0.05	0.08	-66	34
0.10	0.12	-34	32
0.20	0.19	7	28
0.30	0.26	13	25
0.40	0.34	16	22
0.50	0.41	17	20

- the results in Table 2 show the effects of increasing the dispersion of skills, which raises the marginal rate of tax across all income ranges, and also moves the maximum rate up the income range

- (note: Mirlees' top rates are still considerably lower than say the top rate in the UK in the 1970s of 70-98%)

Table 2: Optimal Tax Schedule

Increased dispersion of skills (z^G=0.013, v=1, σ=1)			
Income	**Consumption**	**Average tax %**	**Marginal tax %**
0	0.10	-	50
0.10	0.15	-50	58
0.25	0.20	20	60
0.50	0.30	40	59
1.00	0.52	48	57
1.50	0.73	51	54
2.00	0.97	51	52
3.00	1.47	51	49

- Stern (1976) has shown that Mirlees' results are sensitive to the manner in which the skill distribution is derived, and the assumption of a Cobb-Douglas utility function for households

- Stern shows that the income distribution need not reflect the skill distribution

- Stern investigated the properties of a CES utility function, and then varied the value of the elasticity of substitution, ϵ, between labor and consumption

- showed that Mirlees' choice of ϵ gives lowest possible rates of taxation, tax rates can increase to 100% as ϵ tends to zero