Commodities, Options, and Volatility: Modelling Agricultural Futures Prices[1]

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Abstract: Recent research has concluded that commodity prices exhibit distributional characteristics inconsistent with normality or log-normality. We utilize discrete mixtures of log-normals in a GARCH framework to model soybean prices. Options premia are simulated and compared to actual premia and premia generated under standard Black-Scholes assumptions. The results indicate that flexible conditional distributions, while very significant, do not improve options price prediction. Deterministic seasonal volatility is both very significant in-sample, and reduces prediction error.

Although the genesis of derivative products as we know them today comes almost entirely from commodity markets, most research has focuses on financial derivatives. Sound reasons existed for this emphasis. Financial instruments are immune to production uncertainty and demand should be constant, or affected only by easily observable characteristics, such as volatility and the prevailing interest rate environment. Financial instruments also offer the advantage of highly liquid and transparent spot markets. The markets for financial instruments were simply easier to model.

The transition from currencies to interest rates is much smoother than that to commodities, especially agricultural products. The production cycle of agricultural commodities, combined with the important role of climate and weather, produce very distinctive behavior in commodity prices. Using observed options prices, this paper demonstrates that the inclusion of systematic seasonal variation, as in Fackler (1986), improves the ability of time-series models to predict options prices as a demonstration of the improved forecasting of the volatility process. The inclusion of a third and fourth moment-flexible condtional distribution, however, does not. Though this thesis only analyzes CBOT soybeans, it is likely that the results would be similar for other agricultural commodities, such as wheat and corn, which display similar seasonal production characteristics.

This paper has seven sections. The second section explains the time-series model used in this estimation. The third section discusses estimation of the futures price model. The fourth section reviews the theory of options pricing, while the fifth section details the procedures used to estimate the options prices. The sixth section reports the results of the study and the seventh section contains concluding remarks.

1 Time Series Model

In order to explore the effects of commodity-specific characteristics, a time-series model is first specified. The ARCH family was introduced by Engle (1982), and later, Bollerslev (1986) introduced the Generalized ARCH (GARCH) model. In the ensuing years, over 50 additional variants have been introduced. (Bollerslev, 1999) ARCH models are discrete-time models which posit that variance is a latent, deterministic function of past variances and disturbances.

A GARCH(1,1) model takes the form[1]

$$y_t = \mu + \epsilon_t \tag{1}$$

$$\epsilon_t = \sqrt{h_t}\,\eta_t \tag{2}$$

[1] GARCH(p,q) models, wherein variance is a linear function of a greater number of lags of past variances and errors, do exist. However, they are more difficult to estimate and rarely provide any additional explanatory power. (Bollerslev, Chou, and Kroner (1992)) Therefore this essay will use GARCH to denote GARCH(1,1) models.

$$h_t \;=\; \omega + \alpha h_{t-1} + \delta\epsilon_{t-1}^2 \tag{3}$$

where η_t is an i.i.d. innovation with a distribution having unit variance. In this framework, $E_{t-1}(\epsilon_t^2) = h_t$. Bollerslev showed that the unconditional variance of the GARCH process is $\omega/(1 - \alpha - \delta)$.

The role of seasonality in price volatility is well-known. Fackler (1986) modelled seasonal effects in volatility by substituting a Fourier expansion for the intercept of the GARCH volatility equation. Bollerslev and Ghysels (1996) suggested the periodic GARCH formulation to allow seasonal variation in all of the variance parameters, though they only utilized indicator variables in their study. Beller and Nofsinger (1998) independently implemented a similar scheme, again using only indicator variables. Nelson (1991) decomposed variance into deterministic and conditionally heteroskedastic components in order to study the behavior of the deterministic portion of volatility.

To estimate the seasonal and switching effects on volatility, this paper uses a polynomial expansion, ending at the point of contract expiry. The use of the Nelson decomposition and the polynomial expansion ease the interpretation of the estimated variance process. Specifically, equation 2 is modified to be

$$\epsilon_t \;=\; \eta_t \sqrt{h_t + z_t} \tag{4}$$

$$z_t \;=\; \sum_{i=1}^{K} \phi_i g_i(\tau) \tag{5}$$

where τ denotes the fractional number of years remaining until the futures contract expires. The $g_i(\tau)$ function is the i^{th} order Chebychev polynomial. Chebychev polynomials are used to reduce multicollinearity at high orders (See Miranda and Fackler, §6.2). The decomposition of variance into h_t and z_t, as in Nelson(1991), provides a simple solution for the unconditional expected variance for a given date: $E[\text{var}(\epsilon_\tau)] = z_\tau + \omega/(1-\alpha-\delta)$. K denotes the order of the expansion, and is chosen by the researcher.

Studies such as Cornew, Town and Crowsen, (1984) and Hudson, Leuthold and Sarassoro (1987) have found agricultural commodity price series to be skewed and, in the GARCH framework, Myers and Hanson (1993) found conditional leptokurtosis. Bollerslev (1987) first used the student-t distribution in GARCH modelling to capture conditional leptokurtosis in equity index prices, calling it t-GARCH. Baillie and Myers (1991) and Hsieh (1989) among many have also found non-normal conditional distributions. In order to incorporate a student-t distributed density into the GARCH model of equation (2), the t-distribution must be modified so that its variance is a function only of h_t, and not of ν. Bollerslev (1987) used the standardized t-distribution,

$$\begin{aligned} f_\nu(\epsilon_t|h_t) = {} & \Gamma\left(\frac{\nu+1}{2}\right)\Gamma\left(\frac{\nu}{2}\right)^{-1}((\nu-2)\pi h_t)^{-1/2} \\ & \times\left(1+\epsilon_t^2 h_t^{-1}(\nu-2)^{-1}\right)^{-(\nu+1)/2}, \qquad \nu > 2 \end{aligned} \tag{6}$$

This standardized t-distribution has a mean and skewness of zero, and

$$\begin{aligned} \text{Var}(\epsilon_t|h_t) &= h_t \\ E[\epsilon_t^4|h_t] &= 3(\nu-2)(\nu-4)^{-1}h_t^2, \quad \nu > 4 \end{aligned}$$

As is well known, as $\nu \to 4$, the kurtosis of the t increases toward infinity, producing the desired 'fat-tailed' behavior, and as $\nu \to \infty$, the t distribution approaches normality. Because normality of the conditional distribution is the typically desirable null hypothesis of distributional tests, $1/\nu$, the inverse of the degrees of freedom of the student-t is usually estimated instead of ν itself.

Like the normal, the student-t distribution is symmetric. Mixture distribution are a convenient third and fourth moment flexible distribution. Hsieh (1989) used discrete mixture distributions to capture skewness and leptokurtosis in GARCH conditional distributions. Though any distribution may be used as components, mixtures of normals provide relatively simple closed-form solutions for higher moments. The density function of a mixture of k discrete distributions is

$$f_k^*(x;\Theta) = \sum_{j=1}^{J} \lambda_j f_j(x;\theta_j) \qquad 0 \geq \lambda_j \geq 1 \quad \forall j, \qquad \sum_j \lambda_j = 1 \tag{7}$$

where λ_j is the weight of the jth distribution, $f_j(x)$, and x are the random variates. One disadvantage of mixture distributions is that, using the specification of equation 7 above, for mixture distributions with multiple components of the same distribution, there will be no unique maximum of the likelihood function. To avoid this situation, two normal distributions and four constraints are used. A mixture of two independent normal distributions has six parameters; two mixing parameters, two means and two standard deviations. The mixing parameters must sum to one, and the mean and variance of the conditional distribution of a GARCH process must be zero and one, respectively. These three constraints are used to eliminate the three parameters of the first distribution from estimation. Finally, λ_1 is constrained to be less than λ_2, which is equivalent to imposing the constraint that $0 \leq \lambda_2 < 0.5$. These constraints insure the uniqueness of every interior point in the parameter space.

2 Futures Data

Futures prices were drawn from the Bridge Information Services Futures Price Database. Chicago Board of Trade Soybean futures of November expiry from 1 October 1990 through 25 October 1997 were used for a total of 1,786 observations. November contracts exhibited the most liquid options market, and so were chosen for the estimation. Likewise, the CBOT Decmber Corn futures exhibited the most active options market. Corn prices from 1 November 1990 through 15 November 1997 were used, for 1,778 observations. Univariate time-series were generated by using the futures contract closest to maturity on each observation date, except during the month of expiry, when the second-shortest maturity contract was used. The omission of the final few weeks of observations is common, as the combination of decreasing liquidity and the effects of physical deliveries tends to erratically affect volatility. Table 1 provides descriptive statistics for the log changes, as well as the results of Box-Pierce (1970) Q-Tests for autocorrelation in the means and squares. Serial autocorrelation exists, especially in the squared observations.

Insert Table 1 about here.

3 Estimation

These models are all estimated using maximum likelihood (ML). If the conditional distribution is misspecified, ML estimates are consistent, but no longer efficient, therefore the quasi-maximum likelihood standard errors are reported. (See Hamilton p. 663 for more details.) The estimated model is of the form

$$\begin{aligned}
100 \times \ln(P_t/P_{t-1}) &= \mu + \epsilon_t + \phi\epsilon_{t-1}{}^2 && (8)\\
\epsilon_t &= \eta_t \sqrt{h_t + z_t} \\
h_t &= \omega + \alpha h_{t-1} + \delta\epsilon_{t-1}^2 \\
z_t &= \sum_{i=1}^{K} \phi_i g_i(\tau) && (9)
\end{aligned}$$

where η_t is a mean-zero, unit-variance iid error. The unconditional variance is used to initialize the GARCH variance, $h_0 = \omega/(1-\alpha-\delta)$. All calculations are performed in MATLAB. The GARCH estimation and simulation is performed using this author's GARCHKIT toolbox, and the numerical optimization is performed using the QNEWTON routine accompanying Miranda and Fackler.

Estimation results are presented in tables 2 through 4. For each commodity, models incorporating standard normal, student's t, and mixture distributions are estimated across a range of lengths of seasonal expansions, from K=1 to K=10 in equation 8. At K=1, variance is linearly related to the time to expiry. This linear term is always included in estimation to account for the stylized fact, known as the 'Samuelson hypothesis,' that the volatility of futures prices in inversely related to the time to maturity of the futures contract. In this model, however, it is not possible to separately identify the effects of maturity from seasonal influences because only one futures expiry month is used in estimation.

[2]In estimating the models, the residuals of the GARCH models applied to corn data display an MA(1) process. Therefore, the corn models presented here are MA(1)-GARCH(1,1), and the soybean models are GARCH(1,1).

Insert Table 2 about here.

Table 2 presents the Akaike, Bayesian (Schwarz), and Hannan-Quinn information critera (AIC, BIC, HQ) for each of the three distributional assumptions across a range of seasonal specifications. Although the three information will produce identical specifications asymptotically, in finite samples AIC penalizes parameters the least, and BIC the most, with HQ in between. In order to prevent over-fitting the models, the BIC criteria is used to choose the length of the seasonal expansion. Soybean models are estimated using K=3, and corn models are estimated using K=2.

Tables 3 and 4 contain the estimates of the soybean and corn models. Six models are estimated for each crop; seasonal and non-seasonal utilizing the three different conditional distributions. In addition to the parameter estimates, the results tables also report the kurtosis, skewness and Q-test results of the residuals of the GARCH process (ϵ_t of equation 8).

Insert Tables 3 and 4 about here.

The GARCH process is very significant for all of the estimations. The parameter estimates for α and δ correspond in both size and significance with previous studies of price series. All of the non-seasonal models also display behavior consistent with the Samuelson hypothesis, i.e. that ϕ_1 is negative and significant. The resulting unconditional expected variances are plotted for the various models in figure 1. The solid lines represent the non-seasonal variances. In each of the cases, price volatility drops as the maturity of the underlying futures contract changes. However, the plotted paths for the seasonal soybean models do not display this behavior.

As first demonstrated by Fackler, the explicit modelling of seasonal variance is very significant in all of the models. For the Gaussian, t, and mixture models of soybeans, the LR test statistic of $\phi_2 = \phi_3 = 0$ are 38.73, 12.98, and 12.28 which are distributed χ^2_2 under the null, yielding a 99% critical value of 9.21. For the corn models, the statistics are 16.69, 10.54, and 9.92 against a 99% critical value of 6.63. The plots of the seasonal variances, represented by the dashed lines in figure 1 are very similar within each crop. The variance of soybeans is highest in the summer months, May through August, and drops sharply toward harvest. This accords with the economic interpretation offered in Fackler (1986), that information arrives in the market at varying frequencies during the course of the year, but that these frequencies exhibit a pattern. In the case of soybeans, little new information arrives in the market until late winter/early spring when the planting conditions become known. As the spring progresses, and the plants begin to germinate, weather becomes critical to the size of the harvest–this continues into the summer. By late summer, the critical growth has already occurred, and expectations about harvest size have begun to converge.

The variance patterns of the t and mixture models are nearly indistinguishable from one another, and are both less variable than the normal model. These flatter patterns likely occur because outliers are more fully captured in the fatter tails of the t and mixture distributions, lessening the impact on the seasonal variance path. The similarity of the planting and harvest cycle for corn and soybeans results in similar variance paths; prices are more volatile in the summer months. Corn displays less systematic variability over the course of the year than soybeans, and the choice of conditional distribution has little effect on the seasonal variance path.

The estimation results indicate that the student's t distribution is more appropriate than either the normal or the mixture distribution. From a cursory glance at the difference in log-likelihood values or information criteria, using either a t or mixture distribution improves the fit of the model. Whether this improvement is significant is more difficult to judge. The parameter estimated in the t-GARCH model, $1/\nu$, and λ, the mixing parameter in the mixtures model, both lie on the boundary of their parameter spaces under the null hypotheses of interest ($1/\nu = 0$ and $\lambda = 0$). This violates one of the standard assumptions of hypothesis testing, that the null hypothesis is in the interior of the parameter space. Testing the mixture distribution is additionally complicated by the fact that μ_2 and σ_2^2 are undefined under the null hypothesis. Boostrap methods can generate distributions for the sample statistics. Given the large size of the LR test statistics (142 and 148 for non-seasonal t and mixture soybean models, respectively; 42 for both non-seasonal corn models) it is unlikely that bootstrap methods would fail to reject these alternative conditional distributions.

The estimates of the conditional distributions are presented in figure 2. The plots indicate that the mixture distribution's ability to incorporate skewness in not useful in this application, as the densities of the two distributions are nearly indistinguishable. The significance of skewness can be gauged by the significance of μ_2. For a non-skewed distribution, $\mu_2 = 0$. These results indicate no evidence to reject the hypothesis of symmetry, with t-statistics less than one for all of the models tested.

In sum, from the performance of the in-sample estimation, the t-GARCH model appears to embody the most parsimonious description of this commodity price data. Both the student-t and the mixture models can capture excess kurtosis, which is evident both here and in prior literature on the topic. The primary advantage of the Mixture distributions is their ability to approximate skewed distributions. However, in this application, this capability appears to be unnecessary.

4 Pricing Options

The use of time-series models to price derivatives has been oft repeated in the literature. Tilley (1993) used these methods to price options, Schwartz and Torous (1989) priced mortgage-backed securities; Duan (1995) offers a thorough review. For pricing exchange-traded options, however, better methods currently exist; for one example see Hilliard and Reis (1999). These alternative methods rely upon the existence of a price history of the underlying asset as well as the derivative product to be priced. For a new product, or one that is traded infrequently or whose transactions are not known on the open market, these assumptions are not appropriate. In these cases, exchange-traded options markets provide an ideal laboratory to explore the effects of seasonality and varying conditional distributional specifications.

The problem of pricing European-style options owes its first solution to Fisher Black and Myron Scholes(1973). The Black-Scholes (B-S) model obtains from primitive assumptions of the i.i.d. normality of log returns and frictionless trading. It is especially the former that is investigated here, though violations of the latter may influence estimation of the conditional distributions of returns, as well. The Black-Scholes model obtains regardless of the risk preferences of the investor, as it is based upon the premise that the sources of risk underlying the option price may all be hedged. According to the Black-Scholes (1973) formula, the price of a European-style call option, G_t, given the current price of the underlying asset, P_t is

$$\begin{aligned} G_t = P_t F \left(\frac{log(P_t/K) + (r + \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}} \right) \\ -K e^{-r(T-t)} F \left(\frac{log(P_t/K) + (r - \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}} \right) \end{aligned} \tag{10}$$

where $F(\cdot)$ is the normal cumulative distribution function, r is the risk free rate, K is the exercise price, and σ^2 is the variance of the prices.

However, if one allows for time-varying variance in the underlying asset, the B-S pricing formula is no longer applicable. If ARCH models, for example, are used to specify the asset price process, the variance of asset returns becomes stochastic, and unless this new risk can be hedged away, it is no longer possible to construct a risk-free portfolio containing the option.

Cox and Ross (1976) showed that as long as hedge positions can be constructed, the values of European call options can be obtained by discounting the expected payoffs of the options at maturity by the risk-free rate of return. In the case of time-varying volatility, however, perfect hedges cannot be constructed, therefore risk-neutrality must be assumed. For a risk-neutral investor, the market price of a European-style call option should be the discounted value of the right conferred by the contract:

$$G_t = e^{-r(T-t)}\, E_t(max[P_T - K, 0]) \tag{11}$$

where K is the exercise price of the option, T is the date of maturity and r is the risk-free rate of interest.

The adaptation of the options-pricing formula to the mixture of normals distribution is not unique. Ritchey (1990) showed that under risk-neutrality options prices derived from a linear combination of normal distributions are equivalent to the linear combinations of options prices derived from the Black-Scholes model. However, in the case of time-varying volatility, this finding does not obtain.

In applying the Black-Scholes method to this application, one point must be addressed. The Black-Scholes options pricing formula is for European-style options, which allow exercise only at expiry. Options on soybean futures are American-style; the buyer can exercise the option at any time prior to expiry. As such, an American-style option can be viewed as a European-style option, plus an early exercise premium, which is typically relatively small.

American-style option prices can be directly estimated, though it is a non-trivial exercise whose adaptation to the methods used here is quite complex. The primary source of difficulty in estimating American-style options via Monte Carlo estimation is that at every point in time, the option price must be greater than the equivalent European option price, as well as the value of current exercise. For an American call, $V \geq \max(P-S, V_{BS})$, where V is the American option value, P is the price of the underlying asset, S is the strike price, and V_{BS} is the value of a corresponding European call. As will be shown in the following section, the valuation of a European option is quite straightforward under almost any circumstances using Monte Carlo simulation. Valuation of the early exercise premium using Monte Carlo methods is, in Wilmott's (1998) words,

> "very, very hard. ...When we use the Monte Carlo method in its basic form for valuing a European option we only ever find the options' value at the one point, the current asset level and the current time. We have no information about the options value at any other asset level or time. So if our contract is American we have no way of knowing whether or not we violated the early-exercise constraint somewhere in the future."[3]

For this reason, previous work, especially work investigating the time-series properties of the underlying assets has on occasion simply ignored the early exercise premium. Myers and Hanson (1993), for example, makes no mention of the European/American mismatch in their Monte Carlo analysis of GARCH option pricing. It is left an open question of the exact size of the bias that this misspecification induces. Option pricing theory reveals that this bias does, however, have certain systematic characteristics. The early exercise premium increases as options become further in-the-money. The early exercise premium increases (decreases) for calls (puts) as dividends increase. Finally, the early exercise premium decreases with time to maturity.

This thesis is concerned with demonstrating the value of seasonal volatility modelling, and conditional distribution choice. Therefore, for the remainder of this thesis, the

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European-style pricing model represented by equation 11 will be used.

The method used broadly parallels that of Myers and Hanson (1993). At each date that options prices are to be computed, the parameters of the model are estimated. The conditional variance of the following date is predicted using the conditional variance and the squared residual of the final date used in estimation according to equation 8. A random variate is drawn from the estimated distribution and multiplied by the square root of the conditional variance process. This method is repeated each trading day until the expiration of the option adding each day's log-change to the prior log-changes. The antilog of this sum is multiplied by the futures price on the final date used in the model's estimation, thereby creating one simulation of the futures price at the options expiration. This process is repeated 5,000 times for each date. The sample mean of these prices is subtracted, in order to simulate the martingale property of futures prices (i.e. that $E[P_T] = P_t$). Options prices are then computed according to equation 11. For the following date of observed option prices one additional closing price is added to the time-series of futures prices and the process is repeated. This yields 260 dates on which options prices are computed.

The Black-Scholes options prices are derived on the basis of the volatility of the preceding 30 trading days' prices. Interest rates are 30 day T-bill rates.

4.1 Option Data

This study utilizes CBOT November sobyean options on November futures, and CBOT December options on December corn futures. Soybean and corn options from 1 November

1996 to 25 October 1997, and 1 December 1996 to 15 November 1997 are used. The options data were provided by the USDA Economic Research Service. Option observations whose observed closing price was less than \$0.25 were omitted. Exercise prices with fewer than 10 closing prices above \$0.25 were excluded. These choices yielded a sample size of 6,530 and 7,228 total options contracts for soybeans and corn.

4.2 Option Pricing Results

The results of the simulations are reported in tables 5 and 6. The tables report the mean squared error between predicted and observed prices.

For both sets of results, and corresponding with the findings of Myers and Hanson, allowing prices to follow a GARCH process greatly reduces the prediction errors. The RMSE of the soybean call and put predictions is reduced by 71% and 77% over the B-S prediction models, the analogous statistics for corn options are 72% and 76%, which are significantly larger than those of Myers and Hanson. The difference is due to the fact that Myers and Hanson and Roberts utilized a simple binary variable to account for volatility changes resulting from changes in the maturity of futures contracts. Substituting the linear maturity structure used in this paper reduced the RMSE of the soybean non-seasonal models by approximately 40% over the simple binary results. There is no advantage in using either of the more flexible distributions for option prediction. In corn, all of the non-seasonal models have very similar RMSEs, for both calls and puts. The prediction ability of soybeans actually declines about 5% when using the more flexible conditional distributions. Seasonal models further reduce the RMSE by 15% to 18% for the models, regardless of the crop or conditional distribution used.

5 Conclusion

The use of time-series models to simulate European options prices is hardly state of the art; European options have closed-form solutions. The point of this thesis is not to offer an alternative options-pricing scheme, however, it is to investigate the effects of deterministic volatility modelling and various conditional distributions on the ability to model volatility.

The results indicate that seasonal volatility is an important component of option prices. The inclusion of seasonality in these models reduces the forecast pricing error by 15% to 18% compared to a GARCH model alone. In contrast, the inclusion of leptokurtosis and skewness in the conditional distribution of returns, though highly significant for in-sample results, adds very little to option price forecast accuracy.

All of these results must be conditioned on the circumstances in which they were obtained. The options data used in this chapter is relatively short (ca. 11 months). To generalize across even agricultural commodities is perilous. Nonetheless, time-varying volatility is a feature common to many (if not most or all) exchange-traded assets. Deterministically varying volatility, in one form or another, is likewise found in many exchange-traded assets, whether from seasonal production, as in agricultural price series, or seasonal consumption, as in the energy sector (Fackler and Roberts (1999) and Fackler and Tian (2000)), or in day-of-the-week effects (Bollerslev and Ghysels (1996)). On this basis, one could reasonably expect some amount of improvement in volatility forecasting by the incorporation of these processes. In this chapter, prediction of options prices is used to demonstrate this improvement.

The implications of this work are broad. The importance of accurate conditional distribution modelling to revenue insurance premia has been previously noted in Goodwin, Roberts and Coble, (2000) where the choice of distribution of prices affected the actuarial rate of crop insurance by a factor of two.

However the importance of deterministic, seasonal volatility modelling has been left unexplored. As mentioned previously, the most obvious application of this seasonal volatility is in asset pricing. However, insurance products themselves are very similar to options. And although the expiry on many insurance contracts is longer than those traded on the CBOT, Andersen and Bollerslev (1998) provide evidence from other price series displaying time-varying volatility that this characteristic is present even in annual data.

Table 1: Descriptive Statistics for CBOT November Soybean and December Corn Futures

CBOT November Soybean Futures				
	Mean	-0.009		
	Variance	2.227		
	Skewness	-0.158		
	Kurtosis	7.123		
Q-Test Results				
	Observations		Squared Observations	
Lags	Test Statistic	p-Value	Test Statistic	p-Value
1	3.1104	0.0778	65.2378	0.0000
2	4.2314	0.1205	86.1686	0.0000
3	4.6649	0.1980	138.4206	0.0000
5	9.3971	0.0942	252.2786	0.0000
10	21.9228	0.0155	426.5129	0.0000

CBOT December Corn Futures				
	Mean	-0.001		
	Variance	1.2986		
	Skewness	0.0381		
	Kurtosis	5.3554		
Q-Test Results				
	Observations		Squared Observations	
Lags	Test Statistic	p-Value	Test Statistic	p-Value
1	2.3192	0.1278	65.6030	0.0000
2	3.3134	0.1908	126.2128	0.0000
3	4.7452	0.1914	210.9023	0.0000
5	13.7020	0.0176	411.6510	0.0000
10	29.7254	0.0009	653.5578	0.0000

As in estimation, this series is $100 \times \ln(P_t/P_{t-1})$. Q-tests are from Box and Pierce (1970), and are performed on the observations and the demeaned squared observations.

Table 2: Information Criteria for Seasonal Model Specification

CBOT November Soybean Futures

	Gaussian			Student's t			Mixture of Normals		
	AIC	HQ	BIC	AIC	HQ	BIC	AIC	HQ	BIC
1	1.4218	1.4246	1.4294	1.3827	1.3861	1.3919	1.3824	1.3869	1.3946
2	1.4217	1.4251	1.4309	1.3825	1.3865	1.3933	1.3824	1.3875	1.3963
3	1.4126	1.4166*	1.4233*	1.3802	1.3847*	1.3925*	1.3800	1.3857*	1.3954*
4	1.4129	1.4175	1.4252	1.3803	1.3855	1.3942	1.3802	1.3864	1.3971
5	1.4129	1.4180	1.4267	1.3799*	1.3856	1.3953	1.3801*	1.3869	1.3985
6	1.4128	1.4185	1.4282	1.3801	1.3863	1.3970	1.3803	1.3877	1.4003
7	1.4114*	1.4176	1.4283	1.3805	1.3873	1.3989	1.3807	1.3887	1.4022
8	1.4119	1.4187	1.4303	1.3803	1.3877	1.4003	1.3808	1.3893	1.4038
9	1.4125	1.4198	1.4324	1.3809	1.3888	1.4024	1.3813	1.3904	1.4059
10	1.4124	1.4204	1.4339	1.3812	1.3897	1.4042	1.3818	1.3915	1.4079

CBOT December Corn Futures

	Gaussian			Student's t			Mixture of Normals		
	AIC	HQ	BIC	AIC	HQ	BIC	AIC	HQ	BIC
1	1.3898	1.3937	1.4003	1.3783	1.3829	1.3905	1.3796	1.3855	1.3953
2	1.3850	1.3896	1.3972*	1.3756	1.3807	1.3895*	1.3771	1.3835	1.3944*
3	1.3848	1.3900	1.3987	1.3751	1.3809	1.3907	1.3767	1.3839	1.3959
4	1.3842	1.3900	1.3999	1.3752	1.3817	1.3926	1.3769	1.3847	1.3978
5	1.3831	1.3896	1.4005	1.3745	1.3816	1.3936	1.3764	1.3848	1.3990
6	1.3814	1.3885	1.4005	1.3728	1.3806	1.3937	1.3748	1.3839	1.3992
7	1.3795*	1.3873*	1.4004	1.3718*	1.3802*	1.3944	1.3737*	1.3834*	1.3997
8	1.3800	1.3884	1.4026	1.3725	1.3815	1.3968	1.3743	1.3847	1.4021
9	1.3807	1.3897	1.4050	1.3731	1.3828	1.3992	1.3750	1.3860	1.4045
10	1.3798	1.3895	1.4059	1.3730	1.3834	1.4008	1.3747	1.3863	1.4059

As in estimation, this series is $100 \times \ln(P_t/P_{t-1})$.

Table 3: GARCH Parameter Estimates for CBOT November Soybean Futures.

	Gaussian				Student's t				Mixture of Normals			
	No Seasonality		Seasonality (K=3)		No Seasonality		Seasonality (K=3)		No Seasonality		Seasonality (K=3)	
	Estimate	Std. Err	Estimate	Std. Err	Estimate	Std. Err	Estimate	Std. Err	Estimate	Std. Err	Estimate	Std. Err
ω	0.0501	0.0235	0.0646	0.0305	0.0397	0.0107	0.0477	0.0143	0.0395	0.0110	0.0487	0.0158
α	0.8898	0.0357	0.8791	0.0412	0.8922	0.0164	0.8903	0.0201	0.8992	0.0174	0.8963	0.0225
δ	0.0684	0.0202	0.0599	0.0193	0.0768	0.0144	0.0655	0.0145	0.0689	0.0153	0.0579	0.0151
μ	0.0110	0.0202	0.0129	0.0200	0.0120	0.0191	0.0129	0.0192	0.0088	0.0208	0.0110	0.0207
$1/\nu$					0.1899	0.0218	0.1803	0.0216				
μ_1									0.0104	n/a	0.0143	n/a
σ_1									0.7632	n/a	0.7609	n/a
λ_1									0.8403	n/a	0.8180	n/a
μ_2									-0.0546	0.1147	-0.0641	0.1118
σ_2									1.7719	0.1482	1.6807	0.1552
λ_2									0.1597	0.0425	0.1820	0.0571
ϕ_1	-0.3361	0.1127	-0.2192	0.0826	-0.2401	0.0727	-0.1526	0.0594	-0.2208	0.0738	-0.1500	0.0648
ϕ_2			0.2058	0.0755			0.1638	0.0563			0.1443	0.0583
ϕ_3			0.2827	0.0866			0.2076	0.0665			0.2094	0.0671
Kurtosis:	5.8523		5.2557		5.9649		5.4287		5.8561		5.3694	
Skewness:	0.0037		0.0042		-0.00296		-0.0141		-0.0466		-0.0278	
LLF:	-2534.2456		-2515.8831		-2463.4805		-2456.9923		-2460.8773		-2454.7385	
	Q-Test p-values											
Lag	η	η^2	η	η^2	η	η^2	η	η^2	η	η^2	η	η^2
1	0.5938	0.7557	0.5791	0.7013	0.6391	0.3653	0.6436	0.3894	0.4955	0.8565	0.5078	0.8755
2	0.5075	0.9127	0.4274	0.7271	0.7100	0.5495	0.6382	0.4452	0.5379	0.9151	0.4652	0.8058
3	0.6660	0.9606	0.5908	0.8848	0.8726	0.7225	0.8187	0.6533	0.6869	0.9634	0.6169	0.9336
5	0.3113	0.9649	0.2245	0.9630	0.5243	0.8993	0.4339	0.8558	0.3290	0.9439	0.2437	0.9408
10	0.4535	0.9892	0.3502	0.9953	0.2204	0.9598	0.1447	0.9617	0.4424	0.9824	0.3564	0.9884

1,786 observations.

*,**,*** Significant at the 10%, 5%, and 1% level, respectively.

Table 4: GARCH Parameter Estimates for CBOT December Corn Futures.

	Gaussian				Student's t				Mixture of Normals			
	No Seasonality		Seasonality (K=3)		No Seasonality		Seasonality (K=3)		No Seasonality		Seasonality (K=3)	
	Estimate	Std. Err	Estimate	Std. Err	Estimate	Std. Err	Estimate	Std. Err	Estimate	Std. Err	Estimate	Std. Err
ω	0.0460	0.0135	0.0343	0.0116	0.0433	0.0131	0.0324	0.0117	0.0451	0.0138	0.0336	0.0121
α	0.8495	0.0215	0.8669	0.0213	0.8526	0.0220	0.8672	0.0226	0.8497	0.0229	0.8656	0.0231
δ	0.1125	0.0166	0.0980	0.0156	0.1127	0.0169	0.1007	0.0165	0.1131	0.0175	0.1004	0.0169
μ	0.0188	0.0234	0.0222	0.0227	0.0263	0.0224	0.0273	0.0212	0.0266	0.0222	0.0282	0.0221
ψ	0.0744	0.0308	0.0724	0.0302	0.0599	0.0299	0.0591	0.0297	0.0593	0.0298	0.0590	0.0297
$1/\nu$					0.1194	0.0224	0.1098	0.0232				
μ_1									-0.0061	n/a	-0.0084	n/a
σ_1									0.7919	n/a	0.8074	n/a
λ_1									0.7014	n/a	0.7060	n/a
μ_2									0.0144	0.0832	0.0201	0.0879
σ_2									1.3748	0.1624	1.3623	0.2026
λ_2									0.2986	0.1762	0.2940	0.2260
ϕ_1	-0.2391	0.0734	-0.1532	0.0577	-0.2403	0.0687	-0.1557	0.0548	-0.2503	0.0713	-0.1673	0.0570
ϕ_2			0.1606	0.0476			0.1498	0.0463			0.1436	0.0460
Kurtosis:	4.0310		3.9431		4.0523		3.9626		4.0504		3.9625	
Skewness:	-0.0433		-0.0048		-0.0393		-0.0027		-0.0402		-0.0046	
LLF:	-2127.3854		-2119.0427		-2108.7414		-2103.4737		-2108.7584		-2103.7967	
	Q-Test p-values											
Lag	η	η^2	η	η^2	η	η^2	η	η^2	η	η^2	η	η^2
1	0.6609	0.7331	0.8145	0.7630	0.9763	0.7375	0.9882	0.8796	0.9572	0.7207	0.9806	0.7955
2	0.2774	0.9408	0.1980	0.9533	0.2830	0.9361	0.2123	0.9823	0.2853	0.9270	0.2100	0.9621
3	0.2798	0.5886	0.2551	0.8696	0.2754	0.5688	0.2649	0.8458	0.2755	0.5692	0.2597	0.8420
5	0.3736	0.8053	0.3552	0.9111	0.3613	0.8097	0.3693	0.9337	0.3607	0.8076	0.3617	0.9294
10	0.0978	0.9479	0.0498	0.9222	0.0928	0.9442	0.0565	0.9189	0.0931	0.9451	0.0547	0.9174

1,786 observations.

*,**,*** Significant at the 10%, 5%, and 1% level, respectively.

Table 5: Mean Squared Errors of Black-Scholes and GARCH Option Prices for CBOT Soybean Futures.

	Black-Scholes		Standard Normal				Student's t				Mixture of Normals				Observations	
			$K=1$		$K=2$		$K=1$		$K=2$		$K=1$		$K=2$			
	Calls	Puts	Calls	Puts	Calls	Puts	Calls	Puts	Calls	Puts	Calls	Puts	Calls	Puts	Calls	Puts
450	4.2776	6.6610	0.5480	2.4237	0.6029	2.4146	0.4609	2.3576	0.5401	2.3996	0.5124	2.4177	0.5887	2.4194	75	57
500	8.1181	-	0.9897	-	0.7790	-	1.2190	-	0.8434	-	0.9656	-	0.7180	-	78	0
525	7.0921	-	2.8989	-	2.4242	-	3.0850	-	2.5241	-	2.8888	-	2.4680	-	168	0
550	9.4851	-	2.7014	-	1.9920	-	2.9762	-	2.1379	-	2.6631	-	2.0104	-	158	0
575	13.8129	-	2.9051	-	2.3716	-	3.3034	-	2.4495	-	2.7116	-	2.1244	-	85	0
600	10.9084	15.6609	2.6466	4.3643	1.9549	3.6387	3.0127	4.5392	2.0356	3.9029	2.7338	4.8597	1.9014	4.2959	191	160
625	15.4479	16.9875	2.9376	4.3086	2.3373	3.4106	3.3880	4.6336	2.5469	3.7880	3.0748	4.9758	2.4479	4.2145	89	161
650	13.3479	18.9514	2.9417	4.3466	2.4162	3.2584	3.4017	4.8505	2.5512	3.7625	3.3947	5.1933	2.4895	4.2061	248	165
675	14.3272	19.9430	2.8314	4.1448	2.2778	2.8998	3.3567	4.7765	2.4087	3.4749	3.4527	5.1153	2.4420	3.9205	248	165
700	15.2289	21.6548	3.1594	3.5534	2.2366	2.4326	3.7177	4.2022	2.5242	2.8668	3.9060	4.4794	2.7249	3.2104	248	199
725	15.5198	20.6217	3.1971	3.1849	2.1866	2.2262	3.8016	3.7540	2.6041	2.5232	4.0201	3.9349	2.8762	2.7344	242	248
750	14.8742	18.9610	3.4922	2.9793	2.4617	2.4198	4.0694	3.5033	2.9286	2.5611	4.3036	3.5907	3.2716	2.5886	237	247
775	13.3785	17.0927	3.6570	3.0030	2.7291	2.5971	4.1435	3.4495	3.1578	2.7041	4.3773	3.4354	3.5102	2.6189	232	244
800	11.5230	15.1373	3.6363	2.7369	2.8494	2.4522	4.0016	3.1198	3.1880	2.5166	4.2290	2.9859	3.5364	2.3618	225	231
825	9.9225	13.0583	3.6319	2.4509	2.9671	2.1316	3.9040	2.8009	3.2379	2.1640	4.1057	2.5455	3.5580	1.9865	221	225
850	8.3994	-	3.5675	-	3.0288	-	3.7395	-	3.2196	-	3.9243	-	3.5092	-	219	0
875	7.0106	10.8954	3.3617	2.1938	2.9440	1.7652	3.4588	2.5049	3.0664	1.8180	3.6186	2.1725	3.3157	1.6050	216	220
900	5.9183	8.8905	3.1117	1.8508	2.7910	1.3482	3.1560	2.1181	2.8620	1.4249	3.2919	1.7938	3.0747	1.2378	213	199
950	4.5782	6.5633	2.8589	1.8226	2.6492	1.5873	2.8526	2.0548	2.6592	1.6073	2.9509	1.6492	2.8264	1.3658	150	50
1000	2.8613	4.6198	1.9054	0.5657	1.8102	0.3257	1.8632	0.8288	1.7857	0.4171	1.9303	0.6004	1.8876	0.3454	196	26
1050	2.0401	-	1.5265	-	1.4796	-	1.4826	-	1.4477	-	1.5282	-	1.5144	-	194	0
Totals	14.0124	18.6436	4.0748	4.2381	3.3926	3.6011	4.2479	4.5393	3.4977	3.7542	4.3527	4.6244	3.6639	3.8506	3933	2597

Prices are for options on CBOT November Soybean futures traded between 1 November 1996 and 25 October 1997. K denotes order of seasonal expansion. Options with closing prices less that \$0.25 are omitted. Strikes with fewer than 10 observations are omitted.

Table 6: Mean Squared Errors of Black-Scholes and GARCH Option Prices for CBOT Corn Futures.

	Black-Scholes		Standard Normal				Student's t				Mixture of Normals				Observations	
			$K=1$		$K=2$		$K=1$		$K=2$		$K=1$		$K=2$			
	Calls	Puts	Calls	Puts	Calls	Puts	Calls	Puts	Calls	Puts	Calls	Puts	Calls	Puts	Calls	Puts
180	1.4873	-	0.2923	-	0.2873	-	0.2800	-	0.2828	-	0.2964	-	0.2844	-	20	0
190	2.2222	5.2737	1.4051	1.8867	1.4123	1.8650	1.3963	1.8867	1.3929	1.8544	1.3957	1.8803	1.3998	1.8549	55	109
200	2.9365	4.0302	0.5104	1.1431	0.4301	1.0857	0.5852	1.0936	0.4910	1.0478	0.5806	1.1040	0.5139	1.0480	94	140
210	3.0193	4.4379	1.2421	0.9204	1.0912	0.8197	1.2620	1.0167	1.1140	0.8917	1.2731	1.0000	1.1399	0.9057	187	83
220	3.7383	5.1478	1.2461	1.0593	1.0601	0.9347	1.2806	1.2273	1.0990	1.0494	1.2989	1.2140	1.1454	1.1178	172	17
230	4.4404	5.6182	1.2555	1.2870	1.0181	1.1698	1.3009	1.2911	1.0754	1.1725	1.3140	1.3080	1.1278	1.1966	188	146
240	4.8223	-	1.2512	-	1.0157	-	1.3159	-	1.0771	-	1.3159	-	1.1259	-	243	0
250	6.0450	6.9801	1.4326	1.8879	1.1622	1.5518	1.4963	1.8848	1.2281	1.5729	1.4866	1.8812	1.2731	1.5812	224	175
260	7.0209	8.3120	1.6299	1.9969	1.2807	1.5782	1.6947	2.0113	1.3580	1.6214	1.6788	1.9975	1.3984	1.6348	238	189
270	7.7092	9.9682	1.7948	1.9298	1.3880	1.4680	1.8507	1.9764	1.4633	1.5448	1.8312	1.9577	1.4987	1.5676	243	243
280	8.0209	10.4958	1.8921	1.9210	1.4300	1.4453	1.9394	1.9703	1.5123	1.5322	1.9214	1.9541	1.5406	1.5599	241	243
290	7.7693	10.6829	1.8932	1.9022	1.4283	1.4399	1.9397	1.9523	1.5085	1.5280	1.9243	1.9356	1.5367	1.5580	238	243
300	7.1012	10.2832	1.8745	1.8435	1.4260	1.4156	1.9198	1.9041	1.4993	1.5028	1.9026	1.8856	1.5215	1.5387	238	242
310	6.3958	9.3246	1.8269	1.7116	1.4101	1.3548	1.8680	1.7775	1.4707	1.4363	1.8463	1.7646	1.4850	1.4792	234	238
320	5.7326	8.0240	1.8345	1.4786	1.4616	1.1809	1.8669	1.5468	1.5063	1.2533	1.8427	1.5386	1.5110	1.2999	225	227
330	5.0305	6.5386	1.7563	1.2065	1.4312	0.9751	1.7784	1.2776	1.4623	1.0384	1.7524	1.2729	1.4568	1.0837	221	217
340	4.4939	4.9895	1.6435	0.9856	1.3631	0.8182	1.6599	1.0476	1.3862	0.8637	1.6354	1.0489	1.3771	0.9071	206	213
350	3.9436	3.7967	1.5369	0.7890	1.2953	0.6159	1.5438	0.8391	1.3100	0.6492	1.5204	0.8399	1.2953	0.6846	199	194
360	3.4682	2.6515	1.4434	0.5764	1.2394	0.4291	1.4386	0.6301	1.2478	0.4593	1.4208	0.6221	1.2266	0.4871	192	150
380	2.4690	-	1.1205	-	0.9801	-	1.1049	-	0.9723	-	1.0855	-	0.9460	-	169	0
400	1.1972	1.9641	0.9188	0.4560	0.8359	0.3900	0.8704	0.5169	0.8059	0.4418	0.8624	0.4866	0.7754	0.4406	151	32
420	0.1455	-	0.1380	-	0.1375	-	0.1267	-	0.1320	-	0.1396	-	0.1404	-	20	0
440	0.6496	-	0.6198	-	0.5892	-	0.5567	-	0.5469	-	0.5730	-	0.5383	-	129	0
Totals	6.1442	8.5107	1.7908	2.0804	1.5036	1.8653	1.7954	2.0934	1.5178	1.8806	1.7939	2.1041	1.5455	1.9210	4127	3101

Prices are for options on CBOT December Corn futures traded between 1 December 1996 and 15 November 1997. K denotes order of seasonal expansion. Options with closing prices less that \$0.25 are omitted. Strikes with fewer than 10 observations are omitted.

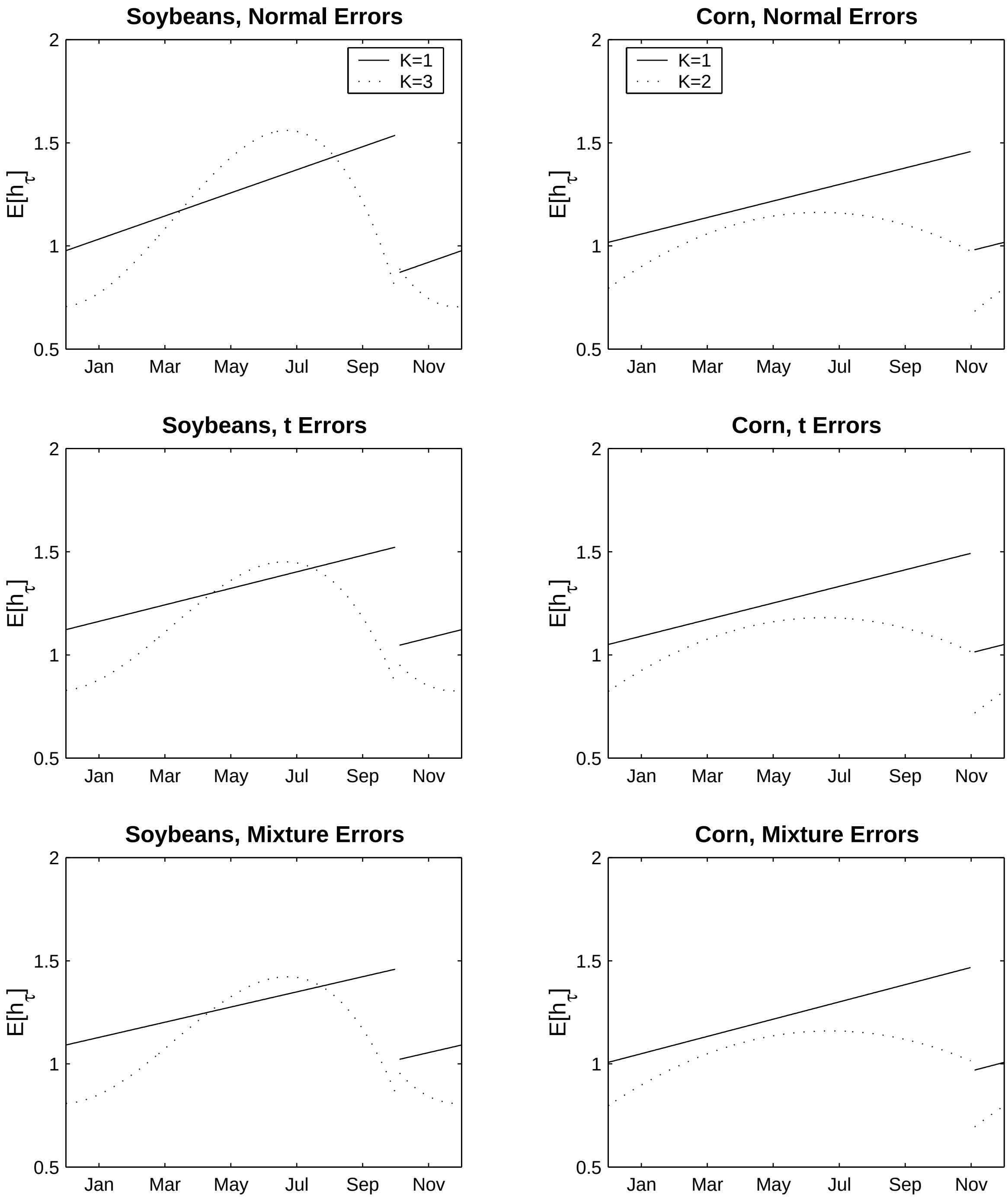

Figure 1: Unconditional Expected Variance by Date.

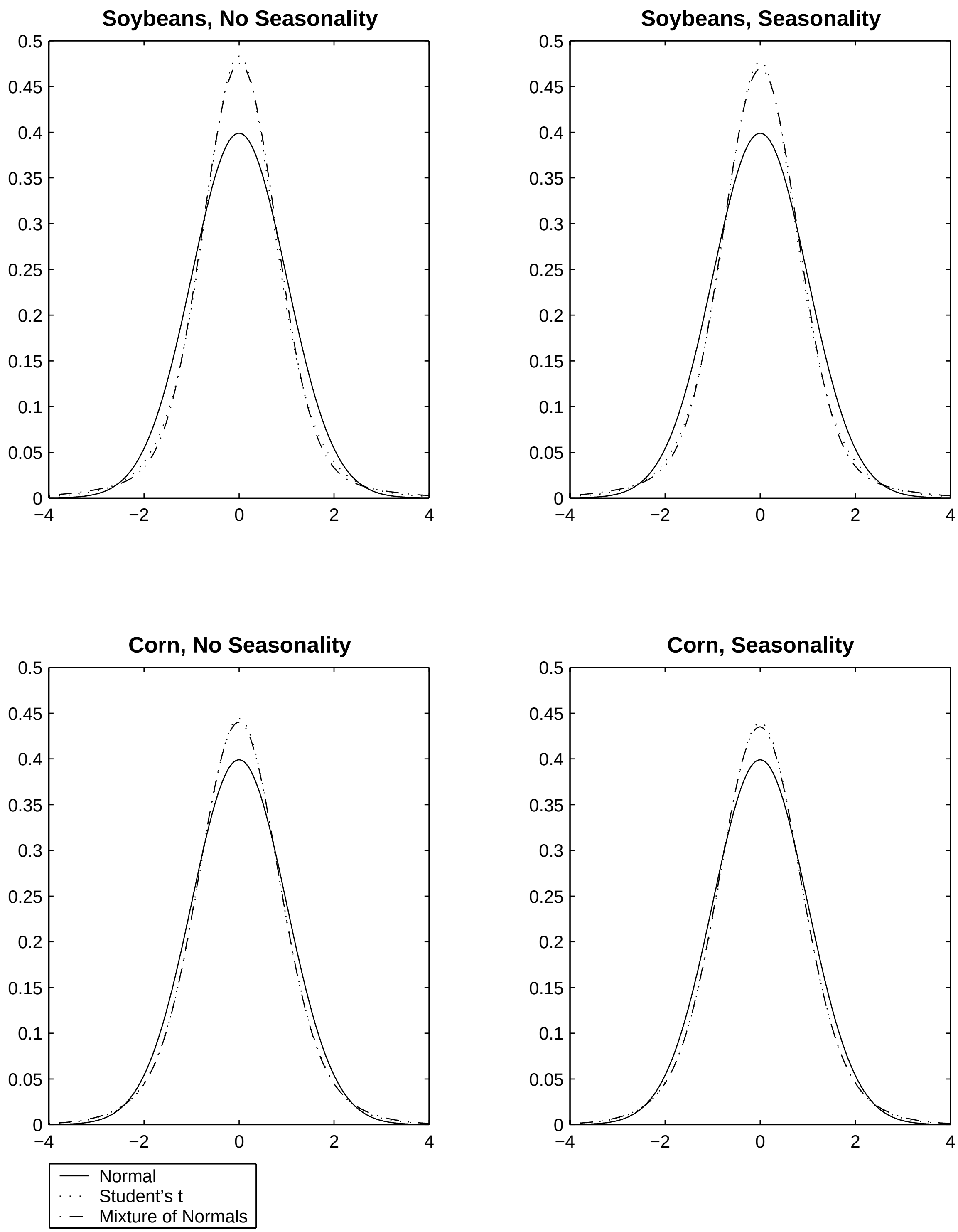

Figure 2: Estimated Conditional Distributions.

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